



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

The Gift of
WILLIAM H. BUTTS, Ph.D.

A.B. 1878 A.M. 1879

Teacher of Mathematics

1898 to 1922

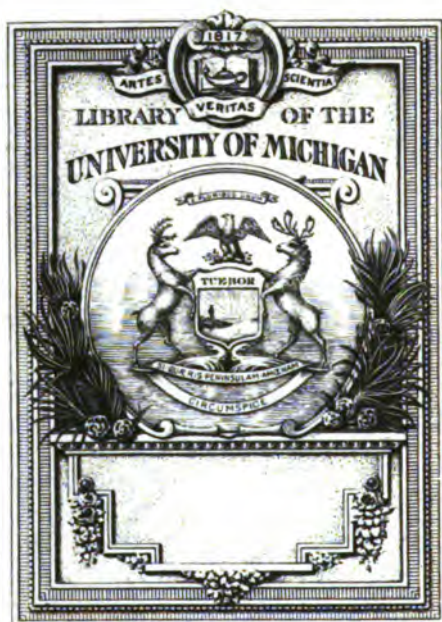
Assistant Dean, College of Engineering

1908 to 1922

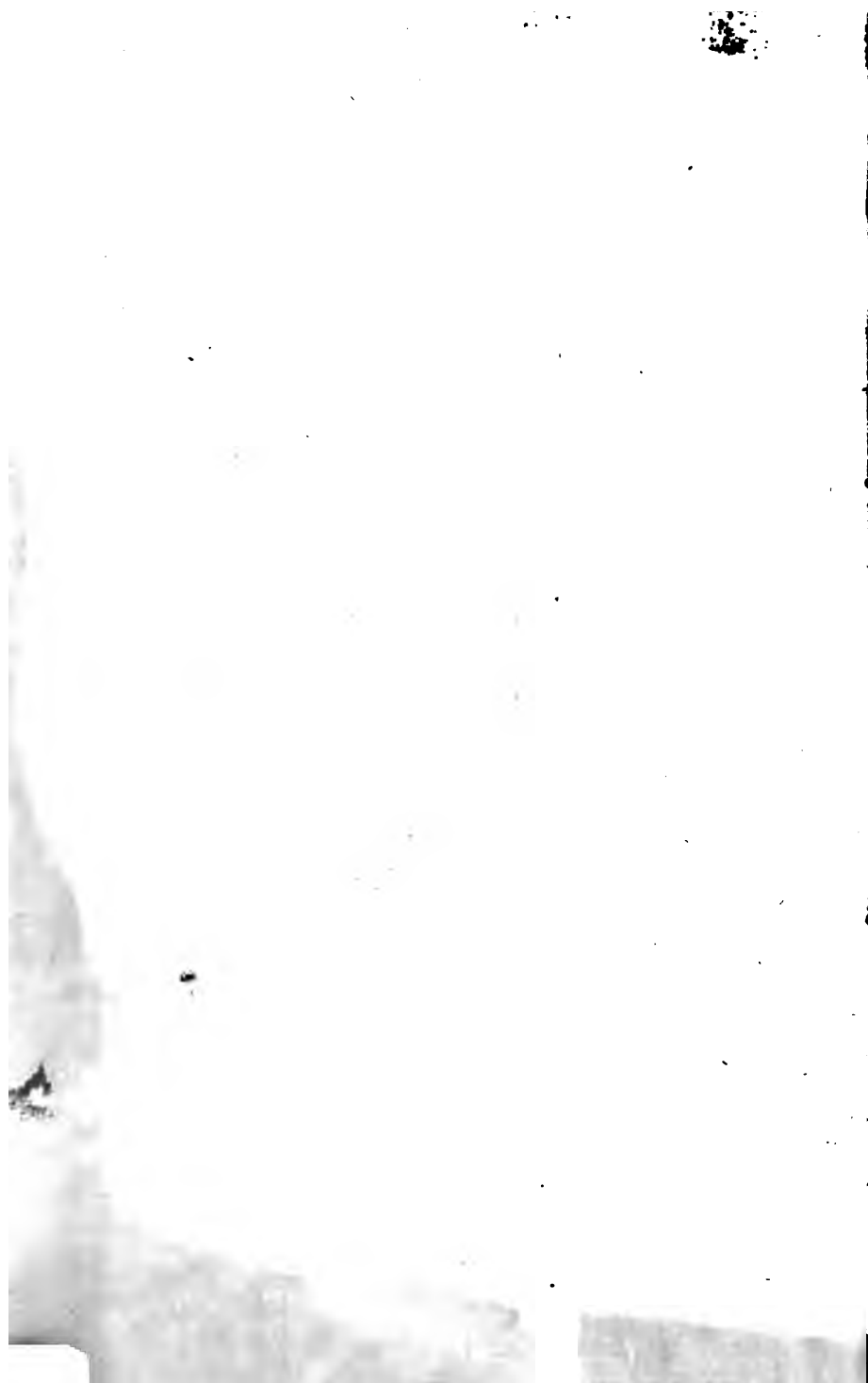
Professor Emeritus

1922

HG
875
-D14







CALCULATIONS

DEDUCED FROM

FIRST PRINCIPLES,

IN THE MOST FAMILIAR MANNER, BY

PLAIN ARITHMETIC,

FOR THE USE OF THE

S O C I E T I E S

INSTITUTED FOR THE

BENEFIT OF OLD AGE:

INTENDED AS AN

I N T R O D U C T I O N

TO THE

STUDY OF THE DOCTRINE OF ANNUITIES.

W. Dale
By a MEMBER of One of the SOCIETIES.

NUMBERS are so much the measure of every thing that is valuable, that it is not possible to demonstrate the success of any action, or the prudence of any undertaking without them.

Spectator, N° 174.

L O N D O N :

PRINTED FOR J. RIDLEY, ST. JAMES'S STREET.

M.DCC.LXXII.

[Price Six SHILLINGS in Boards.]

IN
RESPECTFUL GRATITUDE
TO
THE BEST OF MASTERS,
AND
THE MOST AMIABLE MISTRESS,

THIS
ESSAY,
INTENDED
TO INFORM THE INATTENTIVE,
TO UNDECEIVE THE CREDULOUS,
TO CAUTION THE UNWARY,
AND
TO DETECT, EXPOSE, AND SUPPRESS,
SOME NEWLY-ESTABLISHED
SCANDALOUS IMPOSITIONS ON THE PUBLIC,
IS MOST HUMBLY INSCRIBED TO THE
DUKE AND DUCHESS OF BEAUFORT,

BY THEIR GRACES, LATE DOMESTIC,
EVER MOST OBEDIENT
AND
DUTIFUL SERVANT,

THE AUTHOR.



Wm. H. Burtis
gile
11-21-39

[v]

T H E

P R E F A C E.

DR. HALLEY, Mr. DE MOIVRE, and Mr. SIMPSON, have treated of Annuities very copiously many years since. If their manner seemed obscure to the generality of Readers, there have been elucidations published ten months ago by the ingenious Dr. PRICE ;--- yet, notwithstanding this, the Directors, Managers, &c. of the Societies instituted for the Benefit of Old Age, still continue ignorant of the subject. This is the most favourable construction; for otherwise, how could **HONEST MEN** persist in publishing
pro-

promises that decoy, deceive, and plunder the unsuspecting Believer ?

When the Reader shall have perused the following sheets, he may cease to wonder at their want of knowledge ; for he will meet with more than one instance of their *unwillingness* to be INFORMED, and of their *determination* not to UNDERSTAND *.

In the 33d page of his Supplement Dr. PRICE says, *Some time ago they might have pleaded Ignorance, but this is a plea they cannot now make.* The Doctor may be assured that it is a plea they *will* make ; now, and as long as it will be hard to prove their Knowledge, or while they look on Ignorance to be a venial fault.-- For could they

* See the Author's Letter to his Fellow Members, and to the PROVIDENT SOCIETY.

they be convicted of *wilful* impositions, and of *intentionally* defrauding under FALSE PRETENCES, they are conscious they would deserve an appellation which, however just, would shock them, besides incurring some disagreeable penalties which LAW may inflict.

As to the majority of the Members who support such proceedings by their votes, they are not only parties concerned and still less conversant in the business, but also seem to enjoy the delusion like a pleasing dream; and would much more regret the being waked out of it, than be thankful to the friend who by it should prevent the most vexatious disappointment, or the most imminent misfortune.

To this may be imputed the COLD reception ALL endeavours meet with which tend to destroy the flattering hope, and their readiness to listen to
any

any thing, however false, which soothes it. Such counsel as accords with the wishes of the heart, steals imperceptibly its attention and confidence; while HONEST TRUTH is disregarded, rejected, and huffed for an intruder. -- As the eye naturally turns from any thing disgusting, to fix itself on some more pleasing object; so the mind is loth to reflect upon disagreeable circumstances; and, if it can find none REAL that are *grateful*, will amuse itself with a gay phantom of its own creation.

From such Members their censure is the praise, their reproaches are the thanks, which the Writer must expect. They may prove to be but partial Readers of this Essay, and will more eagerly collect its faults, though the catalogue be ever so long, than usefully apply one remark to the reformation

P R E F A C E. 21

mation of (what they will maintain has need of none) the plan of their Society.

Those who are not yet Members, but are desirous of becoming such, may be more impartial; and think the saving of above 50 *l. per cent.* in *some* cases, and of a cruel certain disappointment in ALL the Societies, will sufficiently repay them for the purchase of the information, and for the attention they have bestowed in the reading.

The GENTLEMEN OF CHARACTER who have countenanced these Societies, it is hoped, will take no offence at a publication that will explain to them the absurdity of their connexions.---They will, no doubt, withdraw the sanction of their respectable names, as soon as they are convinced of the iniquity of the schemes; and, through PRINCIPLE, as well as RESENT-

b

MENT

MENT for having been duped, join the HONEST part of mankind in decrying all infamous practices and their abettors.

Why the Writer engaged in the present enquiry --- why he continued it, and now publishes the result, after the appearance of so complete a Treatise as Dr. PRICE's, will be mentioned in the course of the ensuing pages. But he should here acknowledge the consciousness of his presumption in commencing AUTHOR, without having had the advantage of a liberal education, or having the happiness to boast of the least assistance ; the want of which will too frequently, he fears, betray themselves by the incorrectness of style, and too numerous grammatical errors. But, he trusts, it will appear, that he has paid particular attention to what he conceived to be the most essential part in a work of this kind, which he hopes will be found

P R E F A C E. xi

found pretty correct in Figures, (considering the multiplicity of them) however defective in the elegance of Diction.

Had his abilities been still *less*, let CASUISTS determine whether, after having prepared, he is not bound in conscience to publish, *Calculations* which he supposes are made in a manner that is plainer and easier than can be commonly met with, and when he is at the same time privy to the undue advantages daily taken of the intricacy of the subject?

On the principle and conviction that in so doing he should act the upright part, he hath attempted to execute his design in the best manner he was capable (being not unmindful that such respect was due to the Public) or that the need of a speedy information would admit of, in the hope that it will be received with candour, and the Imperfections

b 2

perfections be excused for the sake of the Intention.----Should his humble endeavours merit in some degree the approbation of the WORTHY, he certainly will more exult in receiving *that* from those he reverences, than he can possibly be affected by the censures or reproaches of such whose artful and base practices he most heartily contemns,

The Marks of Abbreviation used in this Book are,

+ Plus, or more. The sign of Addition, as $4 + 4$, that is, 4 more 4; or 4 added to 4. See also Case (21.).

— Minus, or less. The sign of Subtraction, as $4 - 4$, that is, 4 less 4; or 4 subtracted from 4.

x The sign of Multiplication, as 4×4 ; that is 4 multiplied by 4.

÷ The sign of Division, as $4 \div 4$; that is, 4 divided by 4.

) (The same; as $4)4(1$; that is, 4 divided by 4 gives 1 in the quotient.

= The sign of Equality, as $4 + 4 = 8$; that is, 4 added to 4 is equal to 8; and $4 - 4 = 0$; that is, 4 less 4, or subtracted from 4, is equal to 0, and $4 \times 4 = 16$; that is, 4 multiplied by 4 is equal to 16, and $4 \div 4 = 1$, that is, 4 divided by 4 is equal to 1.

: : : The sign of the Rule of Three, or of Proportion, for which see an explanation page xxxvi of the Introduction,

The Reader will please to observe also, that the figures he will often meet with in a Parenthesis, are meant as references to the corresponding numbers in Roman figures placed at the beginning of most Rules or Cases.

THE CONTENTS.

INTRODUCTION

Decimals	Page
Of Compound Interest	i
Of Reversion, or Discount	xli
Of the Probability of Life	lxxi
A Table of Mortality, according to the medium of 43 years burials in London from A. D. 1728 to 1770 inclusive	lxxxiii
Mr. Simpson's Table of Mortality	xcvii
Dr. Halley's	civ
	cvii

CALCULATIONS.

By the above three Tables : and also by Dr. Price's Table shewing the true probabilities of life in London	from 41 to 98
In what Time a Society, consisting of Members aged <i>Fifty</i> , would totally expend a fund (that otherwise would supply annuities during life) if <i>double</i> the annuity that was purchased should be granted	99
In what time a single Annuitant would expend his stock in the same case	104
A Table, shewing the value of an annuity for age <i>Ferty</i> , to commence immediately, or <i>after any</i> num- ber of years, not more than Ten, with the <i>interest</i> made of the purchase-money	137
	Proofs

THE CONTENTS.

	Page
Proofs of the several Societies requiring MORE than the just values for the half-yearly payments	154
A Table, shewing the value of 44 l. annuity to commence when <i>Fifty</i> , for every fifth age to <i>Eighty</i> , by the four different Tables of Mortality	170
The worth of an annuity, during life, to commence when aged <i>Twenty-one</i> — — — — —	176
Several Calculations applicable to the Societies	178
Examination of an Impartial Society — — — — —	186
the Politic Gotham Society — — — — —	194
each particular Society instituted for the Benefit of Old Age — — — — —	249
Shewing, What each Member pays, reduced to the value of a single payment in present money:	
What annuity ought to be received for the sum paid: Whether the several ages pay in due proportion to each other: Whether paying with admission be equal to paying without admission fine: What annuity each Society can afford to pay: In what proportion to those who are Members, or to the annuity the Society can pay, a person must pay to become a Member according to proposals delivered out in December 1771.	

A D D E M M A.

- ART. 1. A Plan of an IMPARTIAL GENERAL Society, in which the amount of the annuity, whether to commence immediately, or at what time hence, and the mode of paying for it, is at the choice of the Purchaser.
- ART. 2. A new method of correcting the values of annuities proposed.
- ART. 3. Mr. Simpson's value of annuities, and Mr. Demoiivre's hypothesis particularly considered.
- ART. 4. Method to clear an estate of Mortgages.
- ART. 5. A plan to provide for the Aged Poor.

The Reader is desired to correct the following ERRATA.

INTRODUCTION, Page xxi, last line of figures, for 007.1225, read 187.1225. Page xvii, line 4, for 1788663, read 1780625. Ibid. line 11, for .0157, read .0175. Page xxxii, line 17, for 17.26900, read 17.26920; last line, for $\frac{1}{2}$ read $\frac{1}{4}$. Page xxxv, note, line 6, for 363, read 365. Page xlii, line 19, for £106, read £116. Page lxxii, line 12, for 107.1859, read 107.2859. Page lxxv, at the 26th half-year, read 636949. Page xciv, line 15, read *consisting of*.

CALCULATIONS, Page 18, line 21, read *the value of the four-score payments, in present money, for 1 l. annuity for the Age Ten, &c.* For page 75, read *page 73 &c.* Page 140, line 13, for *at 56*, read *at 50*. Page 145, for 2.288394, read (in both places) 2.88394, and for 13.493743, read (in both places) 14.089289. Page 147, last line note, del. (149). Page 160, note, line 2d, for £70.4381, read £7.04381.

ADDENDA, Page 7, line 17, article 5th, read *the least value being £217 21*. Page 8, article 3d, in the introduction of the table, opposite Age 75, for 7.7347, read 4.7347.

INTRODUCTION.

IT is intended to explain the subject of this Essay, in a manner as familiar as it may admit of, to those who understand Common Arithmetic only; and lest some readers should be unacquainted with Decimal Fractions, which, on account of their *easy* operation, will be here found extremely useful, it may be proper to begin with an explanation of their nature.

D E C I M A L S.

(Said to be first used by Regiomontanus, A. D. 1464.)

Their name, derived from *decem* (ten), is expressive of their property, they always denoting parts of ten, or of ten continually multiplied by ten, such as 10, 100, 1000, 10000, &c. *ad infinitum*, which are the decimal *denominators*.

B

NOTA-

NOTATION.

I. Vulgar Fractions may have *any* number for a *denominator*, as $\frac{1}{2}$, $\frac{1}{34}$, $\frac{1}{567}$, $\frac{1}{890}$; but the denominator of a decimal is never any other than the number of *cyphers* required, with an *unit* prefixed.

II. The number of *cyphers* required in the denominator is just as many as there are figures in the numerator, or as there are *places* of figures: so .1 requires one cypher with an unit prefixed, which is ten, for a denominator, thus $\frac{1}{10}$; and .001 requires three cyphers, thus $\frac{1}{1000}$, which shews the fraction to be *one* part of a thousand: and .012034 requires six cyphers, thus $\frac{012034}{1000000}$, which is twelve thousand and thirty four parts of one million: and if these vulgar fractions, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{4}$, were written decimally, and *with denominators*, they would be thus $\frac{5}{10}$, $\frac{25}{100}$, $\frac{125}{1000}$, and $\frac{75}{100}$, which are of the same value as the vulgar fractions before; for 5 parts of ten are in the same proportion as one part of two; and 25 parts of one hundred, the same as one part of four; and 125 parts of a thousand, the same as one eighth; also 75 parts of a hundred the same as three quarters.

III. It is now *easy* to observe, that the value of the fraction would be known, if

D E C I M A L S. iii

if the denominator was, *not* inserted; for it may be *supposed* that .5 has *ten* written under it for the denominator; therefore, as the imagination can very well supply the omission, the above fractions may be written thus .5 .25 .125 .75, by which means they are left unincumbered with their denominators, and found more commodious for operation. It is for this reason that decimals never have any denominators *inserted*, they being supposed always known.

IV. As an integer, or whole number, is increased *tenfold* in value by each cypher added to the *right* hand of it, because that cypher removes its place to that of ten, hundred, thousand, &c. so cyphers placed on the *left* hand of decimals *decrease* the value in tenfold proportion also.

Whole Numbers.

Place of thousands.	
Ditto of hundreds.	
Of tens.	Unit.
	1
10	
100	
1000	

Decimals.

Tenth.	
Hundredth.	
Thousandth.	Ten Thousandth.
,1	
,01	
,001	
,0001	

V. As, in integers, cyphers *prefixed* to any number make no alteration of its value, (for 01 is but one, 0002 is but two;)

so cyphers added to, that is, following a decimal, have no significancy; for .1 is one tenth, and .10 is but one tenth of a hundred, and .100000 is but one tenth of a million; which being in the same proportion, the cyphers on the right hand of decimals may be entirely omitted, as having no expression (9); though it is necessary to retain them, till the fractions are pointed off, in Multiplication (13), and Division (16).

VI. Decimal, as well as Vulgar Fractions, may be *simple* or *compound*; or, rather, a *number* may be so: a *whole* number is simple when it has no fraction annexed to it; and a *fraction* is simple when it has no whole number preceding it: thus .1, .02, .123 are simple decimal fractions. A dot, or comma, must always be placed before a decimal fraction; for it is the *mark* that distinguishes it to be such; and any figures before the dot, or comma, are integers, and the fraction then becomes *compounded*, or is part of a compound number that consists of an integer and fraction; thus 11.75 is a compound, or mixed number, consisting of eleven wholes, and three quarters of a whole.

VII. It may not be wholly useless to remark, that *whole numbers* are, or may be considered as decimals: For what is 19? It is the same as this *compound* number 10.9,
that

DECIMALS.

v

that is, one whole ten, and nine parts of ten; and thus also 1999, wanting but one of two thousand, is, when considered *decimally*, as follows:

- 1.000 one thousand.
- .9 nine tenth parts of ditto.
- .09 nine hundredth parts of ditto.
- .009 nine thousandth parts of ditto.

1.999 wanting but one thousandth part of 2000.

Whence is seen how naturally decimals arise from whole numbers.

ADDITION.

VIII. In whole numbers, the units are placed under units; tens beneath tens, &c. to be added. Decimals may be said to be placed in the same manner; the dots under the dots; the *tenth* part under the *tenth* part; the hundredth beneath the hundredth, &c. though, in appearance, the order seems reversed: for, let 6, and 77, and 888, be added together, both as whole numbers, and as decimals.

Integers.

DECIMALS.

*Integers.**Decimals.*

6
77
888

971

Units.
Tens.
Hundreds.

.6
.77
.888

2.258

The thousandth part.
The hundredth.
The tenth.
Whole number.

IX. These may be made to have the same number of places, without altering their value, by prefixing and adding cyphers.

*Whole Numbers.**Fractions.*

006
077
888

971

.600
.770
.888

2,258

which will explain Case 5. and shew that cyphers on the right of decimals, as well as on the left of integers, have no value.

X. If the number be mixed, that is, a compound (6), still let the dots, or commas, be placed directly beneath each other, without regarding how far the fraction may extend to the right. Also the dot in the *total* must be exactly under the others, and the amount

D E C I M A L S. vii

amount carried to the left of the dot becomes an integer.

Add 6.6, and 77.77, and 888.888 together.

$$\begin{array}{r} 6.6 \\ 77.77 \\ 888.888 \\ \hline \end{array}$$

973.258 which is the same as the preceding totals added together.

$$\begin{array}{r} 971 \\ 2.258 \\ \hline 973.258 \end{array}$$

There being no other difference between addition of decimals and integers, than the *order* the numbers are to be placed in; one example more may be sufficient.

Add 12345 and 1234.5 and 1.2345 and 12.003 and 123.900402 and .005 together.

$$\begin{array}{r} 12345 \\ 1234.5 \\ 1.2345 \\ 12.003 \\ 123.900402 \\ .005 \\ \hline 13716.642902 \end{array}$$

S U B.

S U B T R A C T I O N.

XI. The reader, who comprehends the foregoing, will need but few examples to explain subtraction; which is wrought in the same manner as whole numbers, observing to place the fractions properly, as directed by the last rule.

Take the last total, and subtract from it each of its parts that were added.

$$\begin{array}{r} \text{From } 13716.642902 \\ \text{subtract } .005 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From } 13716.637902 \\ \text{subtract } 123.900402 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From } 13592.7375 \\ \text{subtract } 12.003 \end{array} \quad (5)$$

$$\begin{array}{r} \text{From } 13580.7345 \\ \text{subtract } 1.2345 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From } 13579.5 \\ \text{subtract } 1234.5 \end{array} \quad (5)$$

$$\hline 12345$$

See the last Example in Addition.

XII. When either a simple fraction, or a mixt number, is to be subtracted from a whole number, cyphers may be written, or *supposed*, as fractions to the integer.

From

From 1234 subtract .1234 1233.8766	or	1234.0000 .1234 1233.8766
--	----	--

From 1234.5 subtract 12.345 1222.155	or	1234.500 12.345 1222.155
--	----	---

M U L T I P L I C A T I O N.

XIII. Proceed as in whole numbers, observing this *general* rule: Mark off the same number of places in the product for decimals, as there are decimal *places* in both the factors, that is, in the multiplicator and multiplicand, together.

E X A M P L E S (16.)

103.5 3.5 5175 3105 362.25	103.5 .035 5175 3105 3.6225	107.1225 3.5 5356125 3213675 374.92875
---	--	---

.5 <u>20</u>	.055208 <u>20</u>	.10416 <u>12</u>
10.0	1.104160	1.24992

$$\begin{array}{r}
 107.1225 \\
 .035 \\
 \hline
 5356125 \\
 3213675 \\
 \hline
 3.7492875
 \end{array}$$

Because both the factors, in the first example, have one place of fraction each, which are *two* together, two places of figures are marked off for decimals in the product; that it may, according to the rule, have an equal number of places of fraction with what the multiplicand and multiplier have together. The last example has *seven* places of fraction in the product; for the multiplicand has *four* places, and the multiplier *three*, which are *seven* places together also.

More EXAMPLES (23.)

.006 <u>2000</u>	2000 <u>.006</u>
12.000	12.000

DECIMALS.

xi

101.75	101.75
1.75	.0175
50875	50875
71225	71225
10175	10175
178.0625	1.780625

XIV. When the product happens to have *fewer* places of figures than both the fractions have in the two factors; *prefix* as many cyphers to such product, as may make it consist of an *equal* number.

1.0175	1.071225
.07	.035
71225	5356125
	3213675
	37492875

Because the factors together have *six* places of figures for fractions in the first example, and the product has but *five*, a cypher must be prefixed to the product, thus .071225, that it may, according to the rule, have an *equal* number of places of figures in the fraction, with those in the two factors: for the like reason the last example must be made .037492875.

Also

D E C I M A L S.

Also 1.035	1.0175
<u>.035</u>	<u>.0175</u>
5175	50875
<u>3105</u>	<u>71225</u>
36225	10175
	<u>1780625</u>

which products must be made .036225,
and .01780625. Also

$$\begin{array}{r}
 .0175 \\
 \underline{4} \\
 .700 \text{ must be made } .07 \quad (5.)
 \end{array}$$

$$\begin{array}{r}
 \text{I} \\
 .07 \\
 \underline{\quad} \\
 7 \text{ must be made } .07
 \end{array}$$

And .0002

$$\begin{array}{r}
 .003 \\
 \underline{\quad}
 \end{array}$$

6 must be made .0000006.

XV. If the multiplicator be an unit with cyphers annexed, as 10. 100. 1000. 10000. there will be no need of multiplying in this manner.

D E C I M A L S.

xiii

$\begin{array}{r} .10175 \\ \hline 10 \end{array}$	$\begin{array}{r} .10175 \\ \hline 100 \end{array}$	$\begin{array}{r} .10175 \\ \hline 1000 \end{array}$	$\begin{array}{r} .10175 \\ \hline 10000 \end{array}$
$\begin{array}{r} 1.01750 \\ \hline \end{array}$	$\begin{array}{r} 10.17500 \\ \hline \end{array}$	$\begin{array}{r} 101.75000 \\ \hline \end{array}$	$\begin{array}{r} 1017.50000 \\ \hline \end{array}$

(5.)

for, as each cypher increases the sum tenfold, so removing the dot to as many places as there are cyphers does the same.

.10175 1.0175 10.175 101.75 1017.5

D I V I S I O N

XVI. Is also performed as in *whole numbers*, observing this *general rule*: Point off for decimals as many figures in the quotient, as, together with the number of fractional places in the divisor, shall be *equal* to the number of places of fractions in the dividend.

EXAMPLES (13.)

$$\begin{array}{l} 3.5)362.25(1035 \quad =103.5 \\ .035)3.6225(1035 \quad =103.5 \\ 3.5)374.92875(1071225=107.1225 \\ .035)3.7492875(1071225=007.1225 \end{array}$$

In the first example there are two places of decimals in the dividend; and because there is but *one* in the divisor, there must be *one* dotted off for a fraction in the quotient; and then the divi-
for

for and quotient together have two places of fractions, which is *equal* to the number in the dividend. In the last example there are seven places of decimals in the dividend; the divisor has three, which joined with four marked off in the quotient are seven likewise.

More EXAMPLES.

$$\begin{array}{rcl}
 178.0625 & \div 101.75 & = 1.75 \\
 178.0625 & \div 1.75 & = 101.75 \\
 1.780625 & \div .0175 & = 101.75 \\
 .071225 & \div .07 & = 1.0175 \\
 .037492875 & \div .035 & = 1.071225 \\
 .036225 & \div .035 & = 1.035 \\
 .01780625 & \div .0175 & = 1.0175
 \end{array}$$

XVII. The proof of division is by multiplication; for, multiply the quotient by the divisor, or the divisor by the quotient, and the product will be equal to the dividend, provided the remainders (if any) left by division be added to the product, and that the work is *right*.

Also the proof of multiplication is by division; for, divide the product by the multiplier or multiplicand, and the quotient will be equal to the multiplicand or multiplier, accordingly as either is made the divisor; as may be seen by comparing the examples in the last Case with those in Cases 13 and 14.

There-

D E C I M A L S. xv

Therefore the *product* in multiplication may be considered as being the *dividend* in division; and the *two factors* in multiplication, as being the *divisor and quotient* in division, thus :

Divisor.	Dividend.	Quotient.
1.75)	178.0625	(101.75
Multiplicator,		Multiplicand,
or	Product.	or
Multiplicand.		Multiplicator.

If this be duly considered, it must be evident, that when in multiplication the *product* marks off as many places of figures for decimals, as are equal to the places in *both* the factors; for the same reason, in division, the quotient and divisor *together* must have places of decimals to equal the dividend.

XVIII. When there are not so many figures in the quotient, as, with the decimals in the divisor, will be equal to the places of decimals in the dividend; prefix as many cyphers as will render them equal.

1.0175).071225(7

There are six places of decimals in the dividend, and but four in the divisor, and but one figure in the quotient; which must therefore be made .07 (14.) that the divisor and quotient, *together*, may have *six* places
of

of fractions, to equal those in the dividend. Again,

$$4).071225(17806$$

The divisor is an integer: the dividend has six fractional places; and the quotient, having but *five* figures, must have a cypher prefixed, and all the quotient pointed off as decimals.

$$.017806$$

so likewise $20)1.104160(55208$ must be made $.055208$.

XIX. When there is any remainder in division, add as many cyphers on the right hand of the dividend, as is necessary to leave no remainder; reckoning such cyphers, so added to the dividend, to be so many places of decimals added to those which might be in the dividend before; as was done in the last example.

In the second example of the last Case, there was *one* remaining: by adding a cypher to the right it becomes 10, and 4 in 10 is twice, leaving 2 remaining: to that 2 also add a cypher, and it becomes 20, in which 4 is contained 5 times, leaving no remainder: the *two* cyphers that have been thus bor-

rowed must be placed in the dividend, and accounted for among the places of fractions, thus :

$$4).07122500(1780265$$

There are now *eight* fractional places in the dividend, and but *seven* figures in the quotient; to which; by the rule, a cypher must be prefixed.

$$\underline{.01780625}$$

Divide .07 by 4 (38.)

$$4).0700(175 \text{ which must be made } .0157(18.)$$

XX. It frequently happens, that in division an *exact* quotient without *any* remainder cannot be obtained. Divide 1.25 by 12.

$$12)1.250000(.10416\phi (16.)$$

Here there would be always 8 remaining, and 12 in 80 would always give 6 to the quotient; and because of the continual repetition, any figure in such case is called a repetend, and marked thus ϕ : let the repetend be discontinued, and there will be 8 remaining for ever; and consequently, when omitted in the quotient, there will be a loss of $\frac{8}{12}$ of an unit to the quotient. This is one defect

D in

xviii DECIMALS.

in decimal fractions; but it may sometimes be corrected in part: If three different sums divided by 12, and each leaving 8 remaining, were to be added, as the above, and 2.45 and 3.65, divided by 12, thus

$$12)2.45000(.2041\bar{6}$$

$$12)3.65000(.3041\bar{6}$$

in this case, because $\frac{8}{12}$ is lost 3 times, that is, once in *each* quotient, there is a loss of $\frac{24}{12}$, which is equal to two units; therefore, when the sums are added, two units may be added to the repetends, thus:

$$.1041\bar{6}$$

$$.2041\bar{6}$$

$$.3041\bar{6}$$

$$\frac{24}{12} \text{ equal to } 2$$

.61250 the exact total, which would have been only 61248, if the remainders had been omitted.

XXI. But when the divisors and remainders are different, and when as many decimal places are obtained in the quotient, as may be thought needful, let it be observed, if the remainder be *more* or *less* than *half* the divisor; if it be *less*, omit the fraction entirely, and let it be lost to the quotient; but if it be *more*, add an unit to the quotient, as by making the
above

above repetend $\frac{1}{7}$ to be $\frac{1}{7}$; then one quotient would be too *little*, and the other would be too *much*: but when a great number of such quotients are added together, it may be supposed the defect in the *total* will be but *small*, because what fraction *one* part of it has *lost*, the *other* may have *gained*.

This rule is adhered to very strictly in the following essay; and whenever, by reason of a fraction of above one half the divisor, the quotient is increased an unit, it is marked with the sign of addition, thus $+$: and when such quotients are multiplied by their divisors, they will produce somewhat *more* than their *exact* dividends; as in those where their remainders have been *entirely omitted*, the produce will be *less*.

XXII. When unity, or any number, is to be divided by a *greater* number, annex as many cyphers as may be necessary (19.) to leave no remainder, and point them *all* off for decimals; because, when a *lesser* number is divided by a *greater*, the quotient must of necessity be a fraction. For examples (2.)

Divide *one* by *two*:

$$\begin{array}{r} 2)1.0(5 \end{array}$$

which quotient is the decimal .5.

D 2

Divide

xx D E C I M A L S.

Divide *three* by *four* :

$$4)3.00(75 \text{ which is } .75$$

Divide *one* by *four* :

$$4)1.00(25 \text{ which is } .25$$

Divide *one* by *eight* :

$$8)1.000(125 \text{ which is } .125$$

Divide *one* by *twenty* :

20)1.00(5 which is .05 (18.) and
is the decimal of a pound sterling for
one shilling.

Divide *one* by 240 :

240)1.0000000(4166 $\frac{2}{3}$ which is (18. 20.)
.004166 $\frac{2}{3}$, &c. and is the decimal of a
pound for one penny.

Divide *one* by 960 :

960)1.0000000(1041 $\frac{6}{5}$ which is (18. 20.)
.001041 $\frac{6}{5}$, and is the decimal of a pound
for one farthing.

Divide *one* by 1.0175

$$1.0175)1.00000000000(.982800\frac{1}{5} \text{ (16. 20.)}$$

there are eleven cyphers borrowed in the
dividend

D E C I M A L S. xxi

dividend for fractions ; and four in the divisor with *seven* in the quotient, are equal in number to them. The same by this example : Divide 100 by 101.75 ; but note here, that 100 is a whole number, and must be pointed off as such :

$$101.75)100.00000000(.982800\text{g}$$

Divide 7 by 20 :

$$20)7.00(35 \text{ which is } .35$$

Divide 11 by 20 :

$$20)11.00(55 \text{ which is } .55$$

XXIII. But when any whole number is to be divided by a simple fraction, or a compound number of *less* value ; add to the dividend as many cyphers for fractions, as there are fractional places in the divisor, and the quotient will be a whole number.

Divide five by 2.5 :

$$2.5)5.0(2$$

Divide 12 by .006 (13.)

$$.006(12,000(2000$$

Divide 144 by .0001 :

$$.0001)144.0000(1440000$$

Al

All which quotients are *whole* numbers; for the divisor and dividend have an equal number of fractional places: from which it may be noted, that, contrary to *whole* numbers, multiplying by simple fractions does in fact *diminish* the multiplicand, and dividing by fractions increases the dividend.

XXIV. If the divisor be unity with cyphers annexed, removing the dot only as many places backward as there are cyphers, gives the quotient (15.)

Divide 101.75 by 1000

Answer .10175

Divide 362.25 by 100

Answer 3.6225

Divide 3.5 by 100

Answer .035 (18. and 22.)

R E D U C T I O N.

XXV. Vulgar fractions are reduced to decimals by division; making the numerator with cyphers annexed to be the dividend, and the denominator to be the divisor.

Exam-

Examples of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{240}$, $\frac{1}{960}$ &c. may be seen in Case 22.

XXVI. Money is reduced to decimals by annexing cyphers to the numerator of the fraction of the least species, as *farthings*, and dividing by the denominator, as in the examples before.

Then place the numerator of the next species, as pence, before that quotient, and divide by its denominator.

Before the quotient so obtained, place the numerator of the next species, as shillings, divide that sum by the denominator, and the quotient will be the decimal fraction of a pound sterling.

A farthing in proportion to a penny is expressed by vulgar fractions thus $\frac{1}{4}$. Add cyphers to the numerator, and divide by the denominator, and the quotient will be (22.) .25.

A penny, as being a twelfth part of a shilling, is expressed by vulgar fractions thus $\frac{1}{12}$.

Place the numerator before the last quotient, and the sum will be 1.25, to which add cyphers (20.) and divide by the denominator 12, and the quotient will be .10416, which is the decimal parts of a shilling, that one penny and one farthing are.

A shilling is one twentieth part of a pound,

xxiv DECIMALS.

pound, and, in that respect, is expressed by vulgar fractions thus $\frac{1}{20}$.

Place the numerator before the last quotient, and the sum will be 1.10418, which being divided by the last denominator 20, produces .055208 (18.) which are the decimal parts of a pound for £. 0 11 $\frac{1}{4}$.

Reduce £. 3 10 0 to decimals.

The pounds are considered as being a whole number, and the shillings only are fractions, which would be expressed *vulgarly* thus, $\frac{10}{20}$, or $\frac{1}{2}$. To the numerator add a cypher, and divide by the denominator.

$$2)1.0(5 \quad \text{or} \quad 20)10.0(5$$

which quotient is .5 (22.)

Answer £. 3.5.

XXVI.* Or *any parts* may be reduced to a decimal, by reducing those parts to the *least* denomination, and placing the total for the numerator of a *vulgar* fraction, whose denominator is the unit of the integer reduced to the same denomination.

Ex-

D E C I M A L S. xxv

EXAMPLE. £. 0 1 1 $\frac{1}{4}$ reduced to the lowest denomination, is 53 farthings for a numerator.

The integer 1£. when reduced to the same denomination, is 960 farthings for a denominator, which makes the vulgar fraction $\frac{53}{960}$, and which, being reduced by the Rule (25.), gives the same answer as before.

$$960)53.000000(.055208 \quad (18.)$$

(22.)

XXVII. Reduce £. 85 $\frac{14}{20}$ $\frac{3}{12}$ $\frac{1}{7}$ to decimals.

Denominators 20 12

The numerator of the
least species with cy- }
phers is } 3.0000000 (22.)

Divide by the denominator 7) 4285714 (20.)

Prefix to the quotient }
3 pence, } 3.4285714

Divide by 12) 2857144 (20.)

Prefix the 14 shillings 14.2857144

Divide by 20, and the }
quotient will be } .71428571

The answer is £. 85.71428571

Or, £. 0 14 3 $\frac{1}{7}$, reduced to the lowest
E

lowest denomination, is 1200 for a numerator.

And 1*£*. reduced to the *same* denomination, is 1680 for a denominator.

The *vulgar* fraction $\frac{1200}{1680}$, reduced as the Rule (25.) directs, is

$$1680)1200.00000000(.71428571 \quad (20.)$$

(22)

Reduce *£*. 1 2 0 to decimals.

Answer *£*. 1.1

Reduce *£*. 3 7 0 to decimals.

Answer *£*. 3.35

for 7 shillings are expressed by vulgar fractions thus $\frac{7}{20}$; therefore $20)7.00(.35$ (22.)

Reduce eleven guineas to decimals. *They* would be expressed by vulgar fractions thus *£*. 11 $\frac{11}{20}$, which has been divided (22.) and the quotient found to be .55; therefore

The answer is *£*. 11.55

XXVIII. Reduce *£*. 20 18 2 $\frac{3}{4}$ to decimals.

The

D E C I M A L S. xxvii

The numerator of the *least* species } 3.0000
 with cyphers }
 Divide by the denominator *four*) .7500
 To the quotient prefix the *two pence* 2.7500
 Divide by 12, and the quotient } 22918
 will be }
 To the last quotient prefix the } 1822918
 18 *shillings* }
 Divide by 20, and the decimals } .911458
 will be }
 The answer is £. 20.911458

Reduce £. 75 10 3 to decimals.

The numerator of the *least* species 3.0000
 Divide by 12, the quotient is .2500
 Prefix the 10 *shillings* 1025000
 Divide by 20, the quotient is 5125
 The Answer is £. 75.5125

Reduce £. 25 4 6 to decimals.

The numerator of the *least* species 6.0000
 Divide by 12) *the quotient is* .5
 Prefix the 4 *shillings* 45
 Divide by 20, and *the quotient will be* .225
 The answer is £. 25.225

Or the answers to the three *last* questions might have been obtained in the same manner as before (26*). *viz.*

xxviii D E C I M A L S.

£. 0 18 2 $\frac{3}{4}$ reduced, is *the numerator* 875

£. 1 reduced to the *same*, is for deno- } 960
minator

The vulgar fraction reduced by the Rule
(25.)

960)875.000000(.911458 the same as be-
fore. (22.)

Also £. 0 10 3 is 123 pence.

And £. 1 is 240 pence.

The vulgar fraction reduced is (25.)

240)123.0000(.5125 the same as before.
(22.)

And also £. 0 4 6 is 54 pence.

And 1£. as before is 240 pence.

which reduced is

240)54.000(.225 the same as before.

XXIX. Decimals are reduced to shillings, pence, and farthings, by *multiplication*. The denominators that were *divisors* in the last rule, are in this taken for *multiplicators*; beginning with 20, to find if the fraction of a pound contains any *shillings*; which, being pointed off for *integers*, they are *not* to be multiplied by the next multiplicator 12; nor must the *pence*, if any, be multiplied

multiplied by 4; that is, the *fractions only* are to be multiplied: For, to understand more plainly, let the *same* sums that have been reduced to decimals, be reduced to shillings, pence, and farthings.

Reduce the decimal .055208 to shillings, pence, and farthings.

Multiply by 20, and the *product* will be 1.104160, having pointed off six fractions (13.) there will be *one shilling*. Multiply only the decimals by 12, and the product will be 1.249920, leaving one penny. Multiply the last fraction by 4, and the product will be .99968.

XXX. Here let it be observed, once for all similar cases, that .999 is so nearly 1000, that it may be reckoned as such; and in this particular case it really is 1000; for what .999 is now deficient of 1000, was lost to the quotient (20.) by omitting the *remainder* in division, which was $\frac{8}{12}$; for by taking in here the $\frac{8}{12}$ *there* lost, the sum will be exact.

The quotient 10416 multiplied by 12,

produces, without taking in the remainder, 1.24992
 To which add 8 the former remainder, 8

and the product will be 25000
 which being multiplied by 4

Produces 1.00000
 which

xxx DECIMALS.

which is *exactly one farthing.*

The answer therefore is £. 0 1 1 $\frac{1}{4}$ (26.)

To reduce £. 3.5 to shillings.

Multiply by $\frac{20}{1}$

$$\begin{array}{r} 10.0 \\ \hline \end{array} \quad (13.)$$

Answer £. 3 10 0

XXXI. Reduce £. 85.71428571 to shillings, pence, and *sevenths* of pence (27.)

Multiply the fraction

$$\begin{array}{r} .71428571 \\ \text{by} \quad 20 \\ \hline \end{array}$$

Multiply the decimals dotted off by $\frac{12}{1}$

$$\begin{array}{r} 14.28571420 \\ \hline \end{array}$$

Multiply the decimals dotted off by $\frac{7}{1}$

$$\begin{array}{r} 3.42857040 \\ \hline \end{array}$$

$$\begin{array}{r} 2.99999280 \\ \hline \end{array}$$

which product, by a similar case (30.), must be made 3, and the answer is

$$\begin{array}{r} \text{£. } 85 \quad 14 \quad 3 \quad \frac{3}{7} \\ \hline \end{array}$$

Reduce £. 1.1 to shillings.

$$\begin{array}{r} 20 \\ \hline \end{array}$$

$$2.0 \quad (13) \quad \text{Answer } \text{£. } 1 \quad 2 \quad 0$$

Reduce

D E C I M A L S.

xxxi

Reduce £. 3.35 to shillings.

$$\begin{array}{r} 20 \\ \hline 7.00 \end{array} \quad \text{Answer £. 3 7 0}$$

Reduce £. 11.55 to shillings.

$$\begin{array}{r} 20 \\ \hline 11.00 \end{array} \quad \text{Answer £. 11 11 0}$$

Reduce £. 20.911458 to shillings, pence, and farthings.

$$\begin{array}{r} 20 \\ \hline 18,229160 \\ 12 \\ \hline 2.749920 \\ 4 \\ \hline 2.999680 \quad (30.) \\ \hline \text{Answer £. 20 18 2 } \frac{3}{4} \quad (28.) \end{array}$$

Reduce £. 75.5125

$$\begin{array}{r} 20 \\ \hline 10.2500 \\ 12 \\ \hline 3.0000 \\ \hline \text{Answer £. 75 10 3} \end{array}$$

Reduce

$$\begin{array}{r}
 \text{Reduce } £.25.225 \\
 \quad \quad \quad 20 \\
 \hline
 \quad \quad 4.500 \\
 \quad \quad \quad 12 \\
 \hline
 \quad 6.000
 \end{array}$$

Answer £. 25 4 6

The several Examples before (27. and 28.) may be compared with these; and it may be observed, that such as left no remainder by division are restored to their *exact* value by multiplication.

XXXII. A few more examples (which will be referred to hereafter) are added to exercise a learner.

Reduce .86346 (39.)

$$\begin{array}{r}
 \quad \quad \quad 20 \\
 \hline
 17.26910 \\
 \quad \quad \quad 12 \\
 \hline
 \quad 3.23040 \\
 \quad \quad \quad 4 \\
 \hline
 \quad .92160 \\
 \hline
 \end{array}$$

Answer £. 0 17 3 $\frac{1}{4}$.

Reduce

DECIMALS.

xxxiii

Reduce .8634658459613 (39.)

$$\begin{array}{r}
 \hline
 17.2693169192260 \\
 \hline
 12 \\
 \hline
 3,2318030307120 \\
 \hline
 4 \\
 \hline
 .9272121228480 \\
 \hline
 \end{array}$$

Answer £. 0 17 3 $\frac{1}{4}$.

Reduce £. 3,6225

$$\begin{array}{r}
 \hline
 12,4500 \\
 \hline
 12 \\
 \hline
 5.4000 \\
 \hline
 4 \\
 \hline
 1.6000 \\
 \hline
 \end{array}$$

Answer £. 3 12 5 $\frac{1}{4}$ nearly.

Reduce .19568 (44.)

$$\begin{array}{r}
 \hline
 3.91360 \\
 \hline
 12 \\
 \hline
 10.96320 \\
 \hline
 4 \\
 \hline
 3.85280 \\
 \hline
 \end{array}$$

Answer £. 0 3 11 nearly.

F

Reduce

xxxiv D E C I M A L S.

Reduce £. 103.530625 (41.)

$$\begin{array}{r}
 20 \\
 \hline
 10,612500 \\
 12 \\
 \hline
 7.350000 \\
 4 \\
 \hline
 1.400000
 \end{array}$$

Answer £. 103 10 7 $\frac{1}{4}$ and $\frac{1}{16}$

Reduce £. 56.122148 (48.)

$$\begin{array}{r}
 20 \\
 \hline
 2.442960 \\
 12 \\
 \hline
 5,315520 \\
 4 \\
 \hline
 1.262080
 \end{array}$$

Answer £. 56 2 5 $\frac{1}{4}$

The value of *any* species, whether TIME, SPACE, WEIGHT, MEASURE, &c. may be found in the same manner; (*a*) but as *this* is not intended for a *complete Treatise*, but to explain

(*a*) A few examples may be acceptable to *some*, for whom this note is added.

What *decimal part* of a year is the time from September 30, to December 25, both days included? Having found the time

D E C I M A L S. xxxv

explain the nature of decimals so far *only* as is necessary to be known to understand the following calculations: What relates farther to them may be omitted, excepting the few examples in the Rule of Three, that follow.

XXXIII.

to be 87 days, consider it as being the *numerator* of a *vulgar* fraction, whose *denominator* is a whole year reduced to the same denomination, that is, *days*; which here suppose 365, then the fraction is this $\frac{87}{365}$, and the numerator divided by the denominator gives the answer.

$$363)87.0000(.2383$$

How many months, weeks, and days is .2383 the decimal of a year?

$$\begin{array}{r} 2383 \\ 13 \text{ Months, consisting of 23 days each.} \end{array}$$

$$\begin{array}{r} 7149 \\ 2383 \end{array}$$

$$\begin{array}{r} 3.0979 \\ 4 \text{ Weeks.} \end{array}$$

$$\begin{array}{r} .3916 \\ 7 \text{ Days.} \end{array}$$

2.7412 which, on account of the repetend 2, is equal to 3 days (30.)

$$\begin{array}{r} \text{M. W. D.} \\ \text{Answer } 3 \ 0 \ 3 \end{array}$$

$\frac{1}{8760}$ is the vulgar fraction for an hour of the *solar* year; and $\frac{1}{525948}$ for a minute; and dividing the numerator (25.) by the denominator would give the *decimal* parts.

What part of a *mile* is 880 yards?

A mile contains 1760 yards for a *denominator*, and the *numerator* being divided by it, gives .5, that is, a *half*, for answer.

XXXVI D E C I M A L S.

XXXIII. What is commonly called the *Rule of Three*, is also called the *Golden Rule* from its great *usefulness*, and is called also the *Rule of Proportion*, which it *properly* is.

The marks or signs of abbreviation for it are these dots,

: : :

which, when supplied with figures, are used thus :

As 2 : 4 : : 4 : 8

that

What decimal part of a pound (*troy weight*) is one ounce ?
Make the vulgar fraction $\frac{1}{12}$.

12)1.0000(.0833 a repetend.

Reduce .0833 to pounds, ounces, &c.
Multiply by 12 ounces in the pound.

.9996 which, because of the repetend, is equal to a unit, that is, a pound. (30.)

Oz. Pwt. Gr.

Reduce 9 18 23 to decimals : The parts are *equal* to 4775 grains for a numerator, and there are 5760 grains in a pound for a denominator. Divide the *first* by the *last*.

5760)4775.00000000000000(.82899303802083

Reduce .82899303802083 to ounces, penny-weights, and grains. 12 Ounces.

9.94791645624996

10 Penny-weights.

18.9583291249920

24 Grains.

22.9998989999808 which is 3 grains, allowing for

D E C I M A L S. xxxvii

that is, as *two* is to *four*, so is *four* to *eight*; meaning, that when *two* requires *four*, then by the same proportion, *four* will require *eight*: and this meaning is always to be understood by the reader whenever these abbreviatures present themselves.

It

for the repetend. Fewer figures would give the *same* answer nearly.

$$\begin{array}{r}
 82899 \\
 \underline{12} \\
 9.94788 \\
 \underline{20} \\
 18.95760 \\
 \underline{24} \\
 22.98240
 \end{array}$$

Lb. Oz. Pwts. Grs.
 0 9 18 23

Pecks. Gall. Pts.

Reduce 3 1 3 to the decimal of one *busℓ*. The parts are *equal* to 59 pints for a numerator; and 64 pints, being a *busℓ*, is the denominator for a *divisor*.

$$64)59.000000(.921875$$

Reduce .921875, the decimal of a *busℓ*, to pecks, gallons, and pints.

$$\begin{array}{r}
 921875 \\
 \underline{4 \text{ Pecks is a busℓ.}} \\
 3,687500 \\
 \underline{2 \text{ Gallons, a peck.}} \\
 1.375000 \\
 \underline{8 \text{ Pints is a gallon.}} \\
 3.000000
 \end{array}$$

Pecks. Gall. Pints.

Answer 3 1 3

xxxviii DECIMALS.

It is the *abbreviations* chiefly that are here meant to be explained; for the reader is supposed to understand the Rule of Three: however, an example or two in the rule by *decimals* may be useful.

If £. 85.71428571 stock will gain 3£. interest, what interest will 100£. gain?

$$85.71428571 : 3 :: 100 : 3.5$$

The Answer is £. 3 10 0

For 100£. will require more interest, in proportion, than 85.714£. therefore the largest sum is made the multiplicator, and the least is made the divisor,

$$\begin{array}{r} \text{£. } 3 \\ 100 \\ \hline 300 \end{array}$$

Because this integer is to be divided by a *compound* number that has *eight* fractional places, this whole number must have *eight* cyphers annexed to it (23) thus:

$$300.00000000$$

and the quotient will be a *whole* number.

DECIMALS.

xxxix

$$\begin{array}{r}
 85.71428571)300.00000000(3 \\
 \underline{257\ 14285713} \\
 4285714287
 \end{array}$$

And, because of the remainder, (19.) *another* cypher must be *borrowed* to add to the dividend; which, being added to the eight fractional places that were before, makes *nine* places of fractions in the dividend.

$$\begin{array}{r}
 85.71428571)300.000000000(35 \\
 \underline{257\ 14285713} \\
 .42857142870 \\
 \underline{42857142855} \\
 00000000015
 \end{array}$$

There are *eight* fractional places in the divisor, to which must be reckoned *one* in the quotient; that the number in *both* may equal the *nine* in the dividend: therefore the Answer is £. 3.5 or £. 3 10 0

The 15 remaining is counted as nothing in so many figures, especially as it is here occasioned by the divisor's being *too little* by as much as it *lost*, when £. 85.14 3 $\frac{3}{7}$ was reduced to decimals (27.)

If

xl D E C I M A L S.

If 100£. gain 3£. interest, what will 116.6666 gain?

$$\begin{array}{r}
 116.6666 \\
 \underline{\quad\quad 3} \\
 100)349.9998 \\
 \underline{3.499998} \quad (24.) \\
 \text{Answer } \underline{\underline{\pounds.3.5}} \quad (30.)
 \end{array}$$

If 100£. gain 3.5£. interest, what will 103.5£. gain?

$$\begin{array}{r}
 103.5 \\
 \underline{\quad 3.5} \\
 100)362.25 \quad (13.) \\
 \underline{3.6225} \quad (24.) \\
 \text{Answer } \underline{\underline{\pounds.3.6225}}
 \end{array}$$

If £. 12.38685 gain 1£. interest, what interest will 100£. gain?

$$12,38685)100. \quad (23.) \quad (8.073077$$

OF

O F
COMPOUND INTEREST,
 A N D
DISCOUNT or REVERSION.

The *D A T A*, from which the following Calculations
 are deduced, are,

The I N T E R E S T of Money,

A N D

The C H A N C E or P R O B A B I L I T Y of Life.

XXXIV. **I**T is not insisted on that *either* of them is *perfectly* exact, but they will *both* be found sufficiently so for the present purpose; for if they do not demonstrate the *absolute truth*, yet they will give a very great *approximation* to it. It is said, that it is not possible to draw a line *perfectly* strait, nor a circle *perfectly* round; but the *rule* makes the one, and the

G

com-

compasses makes the other, correct enough for *general* uses.

XXXV. Interest of money is herein rated at $3\frac{1}{4}\%$ per cent. per ann. because the capital stocks of the societies are to be vested in the *public funds*; where, when $\text{£. } 85\ 14\ 3\frac{1}{4}$ will purchase 100 £. stock at 3% per cent. 100 £. will purchase $\text{£. } 116\ 13\ 4$; the interest of which at 3% per cent. is $3\frac{1}{4}\%$ per ann.

$\text{£. } \text{£. } \text{£. } \text{s. } \text{d. } \text{£. } \text{s. } \text{d.}$
 As 100 : 3 : 116 13 4 : 3 10 0 (33.)

Perhaps, taking *all* times together, 85 £. or 86 £. may be as near the *general* price of 100 £. stock in the 3% per cent. annuities, as any other sum that might be fixed upon. As to the *increase* of the stock itself, no account will be taken of it; because, if in *buying*, the societies make $\text{£. } 106\ 13\ 4$ of every 100 £. so in *selling*, they may get *but* 100 £. for $\text{£. } 116\ 13\ 4$.

The rate of interest being admitted to be $3\frac{1}{4}\%$ per cent. let it be applied to find the increase of a principal at *compound* interest for any given time.

XXXVI. Compound interest is each year's interest added to the former principal; *itself* becoming thereby a part of an increased principal, gaining interest each successive year.

If 100 £. gain $3\frac{1}{4}\%$ per cent. per ann. at the

COMPOUND INTEREST. xliii

year's end, the principal and interest will be together £. 103.5 decimally (26.), which will be the new principal to remain at interest the *second* year. Let the question be, What would be the interest of *that* sum at the *same* rate, and for the *same* time?

By the Rule of Proportion.

As 100 : 3.5 : : 103.5 : 3.6225 (33.)

which is nearly £. 3 12 5 $\frac{1}{4}$ (32.) to add for a *new* principal.

If the *next* year's interest was required in the *same* manner, the operation would be still multiplying the total sum of the *last* interest *added* to the principal, by the *rate* per cent. which is here £. 3.5, and *dividing* the product by 100£.

EXAMPLE.

The last principal was	103.5
To it add the interest	<u>3.6225</u>
The new principal is	<u>107.1225</u>

What will be the interest of this *new* principal?

As 100 : 3.5 : : 107.1225 : 3.7492875

The operation at large is thus:

If 100*£*. require *£*. 3.5 interest, what will 107.1225 require?

$$\begin{array}{r}
 107.1225 \\
 \underline{3.5} \\
 5356125 \\
 3213675 \\
 \hline
 100)374.92875 \quad (13.) \\
 \underline{37492875} \quad (24.)
 \end{array}$$

The *principal* multiplied by the *rate* of interest produces *more* than *£*. 374; which, being *divided* by the 100*£*. *as the rule of three requires*, gives *£*. 3.7492875: the exact interest to be again added to the last principal, and the *total* sum to be a *new* principal, to continue at the same rate of interest the *next* year.

But if the principal be multiplied by .035 instead of 3.5, the product will be the true answer at once *without division*; for .035 is but *one hundredth* part of 3.5 (24.); consequently the product will be 100 times *less* than that produced by 3.5, and must be the *same* as multiplying by 100, and *dividing* that product by 100 would give.

For

COMPOUND INTEREST. xlv

For multiply 2
by 100

Divide the product by 100)200

The quotient is 2

Multiply 2 by *one hundredth* part of 100, which is *one*, and the product will be also 2, the *same* as the above quotient.

XXXVII. Therefore, by continually adding the *interest* to the *last* principal, and finding the interest of the *product* by multiplying it by .035, a table of compound interest may be formed for *any* number of years.

The table would be better adapted to *general* use (75.) if the calculation was for 1*l*. instead of 100*l*. in which case 1*l*. must be multiplied by .035, which is its proper *rate* of interest; and the rule of three can ~~make no~~ difference in the product, because the divisor would be always an *unit*; which (*singly*) neither *increases* nor *diminishes* as a multiplicator or divisor.

As 1 : .035 :: 1.035 : .036225

.035

5175

3105

1).036225(.036225

The

xlvi COMPOUND INTEREST.

The principal being	£. 1
The interest of one £. is	.035
What is the interest of the total	1.035?
1.035 X .035 = (14.) the answer	.036225 (32.)
What is the interest of the total	1.071225?
Ans. 1.071225 X .035 = (14.)	.037492875
Total for three years	1.108717875

There is, in reality, no difference whether the table be constructed by 1£. 10£. 100£. or any decuple of 1£. for removing the dot shews the value of any such sums; thus in three years

10£. would be worth	11.08717875
100£.	110.8717875
1000£.	1108.717875

By which the figures are all the same, and the difference in the sums is by placing the dot. (15.) (b)

Mul-

(b) Let a principal be	_____	1000£.
Add the interest	_____	35
The total for a new principal	_____	1035
		The

Multiplying by .035 takes up the space of *three* lines, that is, *two* of multiplication, and *one* of addition: a *conciser* method, that will effect the *same* purpose, is to multiply by .07, which being twice .035, divide the product by 2.

$ \begin{array}{r} 1.071225 \\ \times .035 \\ \hline 5356125 \\ 3213675 \\ \hline 037492875 \end{array} $	<i>the same as</i>	$ \begin{array}{r} 1.071225 \\ \times .07 \\ \hline 2)07498575 \\ \hline 037492875 \end{array} $
---	--------------------	---

XXXVIII.

The total of the new principal	_____	1035£.
Multiply this total by	_____	.035
	_____	5175
	_____	3105
		36.225
The interest	_____	1035
To it add the principal	_____	
		1071.225
The new principal	_____	.035
Multiply it by	_____	
	_____	5356125
	_____	3213675
		37.492875
The interest	_____	1071.225
To it add the principal	_____	
		£. 1108.717875

xlviij COMPOUND INTEREST.

XXXVIII. And when the interest is the half of 035. that is, .0175; multiply by .07, and divide by *four*, for .0175 is but one quarter of .07 (19.)

$$\begin{array}{r}
 1.03530625 \quad (42.) \\
 \underline{.0175} \\
 517653125 \\
 724714375 \\
 103530625 \\
 \hline
 .018117859375 \text{ the same as } .018117859375
 \end{array}$$

The last method greatly reduces the trouble of constructing a table for many years, with many places of decimals.

COMPOUND INTEREST.

xlix

XXXIX. A Table of Compound Interest at £. 3 $\frac{1}{2}$ per cent. per ann. calculated without the loss of a single fraction, for 20 years.

Year.	£.
	<u>.035</u>
1	1.035 <u>.036225</u>
2	1.071225 <u>.037492875 (37.)</u>
3	1.108719875 <u>.038805125625</u>
4	1.147523000625 <u>.040163305021875</u>
5	1.187686305646875 <u>.041569020697640625</u>
6	1.229255326344515625 <u>.043023936422058046875</u>
7	1.272279262766573671875 <u>.044529774196830078515625</u>
8	1.316809036963403750390625 <u>.046088316293719131263671875</u>
9	1.362897353257122881654296875 <u>.047701407363999300857900390625</u>
10	1.410598760621122182512197265625 <u>.049370956621739276387926904296875</u>
11	1.459969717242861458900124169921875 <u>.051098940103500151061504345947265625</u>
12	1.511068657346361609961628515869140625 <u>.052887403007122656348656998055419921875</u>
13	1.563256060353484266310285513924560546875 <u>.0547378462112371949320859992987359619140625</u>
14	1.618694522465856215631145506911920166015625 <u>.056654308286304967547090092741917205810546875</u>
15	1.675348830752161183178235599653837371826171875 <u>.058603209076325641411238245987884708013916015625</u>
16	1.733986039828486824589473845641721679840087890625 <u>.060689511393997038860631584597460258794403076171875</u>
17	1.794675551222483863450195430239181938634490966796875 <u>.062813644292786935220753690058371367852207183617890625</u>
18	1.857489195515270798670859120297553306486698150634765625 <u>.065012121843034477953480069210414365727034435272216796875</u>
19	1.922501317358305276624339189507967672213732585966982421875 <u>.067287546107540684681851871632778868527480640506744384765625</u>
20	1.989788863465845961306191061140746540741213226413726806640625

It may be observed in this table, that each year increases the number of figures in the interest 3 places, and that in 20 years the principal amounts to 61 places of figures; and if the table was continued, *without omitting any fraction*, for 110 years, (and for which there may be occasion) the principal would have 331 places of figures in length; which would be a tedious and *unnecessary* labour to perform.

It may be *proved unnecessary* thus: Suppose it is required to tell the amount of one million pounds in *twenty* years. Take from the table the first *twelve* figures only of the last product; the seven first figures are to be dotted off for integers, and .86346 remain for decimals; which, when reduced to money, are found to produce (32.) £. 0 17 3.9216. Again, let *twenty* figures be taken from the same last line of the table, the decimals will be .8634658459613, which, being reduced, (32.) are *also* found to be

£. 0 17 3.927212122848

The first figures of each remaining *decimal* being .92, shew there is not the *hundredth part* of a farthing deficient, when 12 figures *only* are taken, in near *two* millions of pounds; so that *twelve* figures being sufficient for common use, it is therefore *unnecessary* to extend their number farther, provided

COMPOUND INTEREST. li.

provided the table be made *correct* to that number of places: notwithstanding this, the next table will be constructed and *proved* correct to *nineteen* places of figures.

XL. Most of the societies *receive*, and are to *pay*, *half* yearly; therefore a table calculated at *half* yearly compound interest will be better adapted for *their* use than the last, which is computed by *yearly* payments.

£. 3.5 is the *annual* interest for 100£. the half is £. 1.75 for *half* yearly interest; and multiplying by .07, and dividing the product by four, produces the same interest as multiplying by .0175 for 1£. (38.)

XLI. Great care must be taken (for reasons that will frequently occur) to remember that each of the following principals and interest is for *half* a year; and that when any number of *years* are sought for, they are found at *double* that number of *half* years in the table. If it is required to know to what amount any sum will accumulate in *ten* years, it is found in the following table at *twenty* half years; so that the table is for *but* 55 *whole* years.

It may be remarked, that the following table not only serves for £. 1.75 per cent. per half yearly payments, but would also serve

for £. 1.75 per cent. per annum for 110 whole years; and for £. 7 per cent per ann. per quarterly payments for twenty seven years and a half.

This table being the *foundation* on which the future superstructures are to stand, its *formation* cannot be examined too attentively to be perfectly understood, and to be assured of its *correctness*: let it then be illustrated and ascertained by an example.

The interest of 100£. at $3\frac{1}{4}$ £. per cent. per ann. for six months is known to be certainly £. 1.75 or (not to work it *decimally*) £. 1.15 0. Quære, What will be the half year's interest of this sum added to its principal, that is, of £. 101 15 0?

To leave the answer as intelligible as possible, let it be worked at large by the common plain method, the rule of three and vulgar fractions.

If £. 100 require for half a year's interest, £. 1 15 0, what will £. 101 15 0 require?

COMPOUND INTEREST.

liii

100£. 20	£. 1 15 0 20	£. 101 15 0 20
2000	35	2035
		35
		10175
		6105
		2000)71225(35 shillings
		.1225
		12 pence
		2000)14700(7 pence
		700
		4
		2000)2800(1 farthing.
		800 or $\frac{4}{16}$.

The answer is £. 1 15 7 $\frac{1}{4}$.
To which add the first half year £. 1 15

The whole year's compound interest is 3 10 7 $\frac{1}{4}$.

The interest by *one yearly* payment } 3 10
is but

Examine next the sum total of the principal in the *second* half year in the following table, which is 103.530625 for 100£. and when reduced, is found to be (32.) exactly

as above £.103 10 7 $\frac{1}{4}$.4. As the same operation is continually repeated in the table, the same cause will ever produce the same effect.

XLII. From which it is plain, that interest paid *half* yearly will accumulate faster than by *annual* payments; but yet not so fast as those suppose, who with an exclamation say, There must be a *prodigious* difference! The whole difference is easily seen by comparing the tables; and the first year is but 7d. $\frac{1}{4}$ nearly in 100£.

In *twenty* years the *principal* and *interest* increase to £. 198 19 0, and by *half* yearly payments to £. 200, or but *little* more. So that a sum will *double* itself in *twenty* years at 3 $\frac{1}{4}$ per cent. per annum, per *half* yearly payments, and very *nearly* the same by *yearly* payments.

COMPOUND INTEREST.

lv

XLII.* A Table of Compound Interest at 1.75 per cent. per *half* year. (41.)

Half
Years.

I.	.0175 interest.
1	1.0175 .01780625 (18. and 19.)
2	1.03530625 .018117859375 (38.)
3	1.053424109375 .0184349219140625
4	1.0718590312890625 .01875753304755859375
5	1.09061656433662109375 .019085789875890869140625
6	1.109702354212511962890625 .0194197911987189593505859375
7	1.1291221454112309222412109375 .01975963754469654113922119140625
8	1.14888178295592746338043212890625 .020105431201728730609157562255859375
9	1.168987214157656193989589691162109375 .0204572762477589833948178195953369140625
10	1.1894444904054151773844075107574462890625

Having brought the table thus far without the loss of a fraction, and it being *unnecessary* to *increase* the places of figures, or even *continue* them at this number, *twenty only* are taken from hence.

lvi

COMPOUND INTEREST.

10	1.1894444904054151774 + (21.) .0208152785820947656
11	1.210259768987509943 .021179545957281424
12	1.231439314944791367 .0215501880115338489
13	1.2529895029563252159 .0219273163017356913 +
14	1.2749168192580609072 .0223110443370160659 +
15	1.2972278635950769731 .0227014876129138470 +
16	1.3199293512079908201 .0230987636461398394 +
17	1.3430281148541306595 .0235029920099472865
18	1.3665311068640779460 .0239142943701213641 +
19	1.3904454012341993101 .0243327945215984879
20	1.4147781957557977980 0247586184257264615 +
21	1.4395368141815242595 .0251918942481766745
22	1.4647287084297009340 .0256327523975197663
23	1.4903614608272207003 .0260813255644763623 +
24	1.5164427863916970626

COMPOUND INTEREST.

lvii

24	1.5164427863916970626 .0265377487618546986 +
25	1.5429805351535517612 .0270021593651871558
26	1.569982694518738917 .027474697154077931
27	1.597457391672816848 .0279555043542742948
28	1.6254128960270911428 .0284447256804740950 +
29	1.6538576217075652378 .0289425083798823917 +
30	1.6828001300874476295 .0294490022765303335
31	1.7122491323639779630 .0299643598163696144 +
32	1.7422134921803475774 .0304887361131560826
33	1.7727022282935035600 .0310222889951363141 +
34	1.8037245172886399741 .0315651790525511995
35	1.8352896963411911736 .0321175696859708455
36	1.8674072660271620191 .0326796271554753353
37	1.9000868931826373544 .0332515206306961537
38	1.9333384138133335081

38	1.9333384138133335081 <u>.0338334222417333364+</u>
39	1.9671718360550668445 <u>.0344255071309636698+</u>
40	2.0015973431860305143 <u>.035027953505755534</u>
41	2.0366252966917860483 <u>.0356409426921062558</u>
42	2.0722662393838923041 <u>.0362646591892181153</u>
43	2.1085308985731104194 <u>.0368992907250294323</u>
44	2.1454301892981398517 <u>.0375450283127174474</u>
45	2.1829752176108572991 <u>.0382020663081900027</u>
46	2.2211772839190473018 <u>.0388706024685833278+</u>
47	2.2600478863876306296 <u>.039550838011783536</u>
48	2.2995987243994141656 <u>.0402429775769897479+</u>
49	2.3398417020764039135 <u>.0409472297863370685+</u>
50	2.3807889318627409820 <u>.0416638063075979672+</u>
51	2.4224527381703389492 <u>.0423929229179809316</u>
52	2.4648456610883198808

COMPOUND INTEREST.

lit

52	2.4648456610883198808 .0431347990690455979
53	2.5079804601573654787 .0438896580527538959 +
54	2.5518701182101193746 c446577270686770891 +
55	2.5965278452787964637 .0454392372923789381
56	2.6419670825711754018 .0462344239449955695
57	2.6882015065161709713 .047043526364032992 +
58	2.7352450328802039633 .0478667880754035694 +
59	2.7831118209556075327 .0487044568667231318
60	2.8318162778223306645 .0495567848618907866
61	2.8813730626842214511 .0504240285969738754 +
62	2.9317970912811953265 .0513064490974209182
63	2.9831035403786162447 .0522043119566257843 +
64	3.035307852335242029 .0531178874158667355
65	3.0884257397511087645 .0540474504456444034 +
66	3.1424731901967531679

1x

COMPOUND INTEREST.

66	3.1424731901967531679 .0549932808284431804
67	3.1974664710251963483 .0559556632429409361 +
68	3.2534221342681372844 .0569348873496924025 +
69	3.3103570216178296869 .0579312478783120195
70	3.3682882694961417064 .0589450447161824799 +
71	3.4272333142123241863 .0599765829987156733 +
72	3.4872098972110398596 .0610261732011931975
73	3.5482360704122330571 .0620941312322140785 +
74	3.6103302016444471356 .0631807785287778249 +
75	3.6735109801732249605 .0642864421530314368
76	3.7377974223262563973 .065411454890709487 +
77	3.8032088772169658843 .066556155351296903 +
78	3.8697650325682627873 .0677208880699445988 +
79	3.9374859206382073861 .0689060036111686293 +
80	4.0063919242493760154

COMPOUND INTEREST.

lxi

80	4.0063919242493760154 .0701118586743640803 +
81	4.0765037829237400957 .0713388162011654517 +
82	4.1478425991249055474 .0725872454846858471 +
83	4.2204298446095913945 .0738575222806678494
84	4.2942873668902592439 .0751500289205795368 +
85	4.3694373958108387807 .0764651544266896787
86	4.4459025502375284594 .077803294629156748
87	4.5237058448666852074 .0791648522851669911
88	4.6028706971518521985 .0805502372001574135 +
89	4.683420934352009612 .0819598663511601682
90	4.7653808007031697802 .0833941640123054712 +
91	4.8487749647154752514 .0848535618825208169 +
92	4.9336285265979960683 .0863384992154649312 +
93	5.0199670258134609995 .0878494229517355675 +
94	5.107816448765196567

94	5.107816448765196567 .0893867878533909399
95	5.1972032366185875069 .0909510566408252814 +
96	5.2881542932594127883 .0925427001320397238
97	5.3806969933914525121 .094162197384350419 +
98	5.4748591907758029311 .0958100358385765513 +
99	5.5706692266143794824 .097486711465751641 +
100	5.6681559380801311234 .0991927289164022947 +
101	5.7673486669965334181 .1009286016724393348 +
102	5.8682772686689727529 .1026948522017070232 +
103	5.9709721208706797761 .1044920121152368961 +
104	6.0754641329859166722 .1063206223272535418
105	6.1817847553131702140 .1081812332179804787
106	6.2899659885311506927 .1100744047992951371
107	6.4000403933304458298 .112000706883282802
108	6.5120411002137286318

COMPOUND INTEREST.

lxiii

108	6.5120411002137286318
	.1139607192537402511 +
109	6.6260018194674688829
	.1159550318406807055 +
110	6.7419568513081495884

The *first* line of *each half year* is the *principal*, and the *second* is the *interest*, found by multiplying the principal by .07, and dividing the product by four. The interest that is here added to its principal might have been omitted, but is inserted, that, if the reader is curious to examine the Table, an error may be the easier detected.

XLIII. It is wished to leave neither *obscurity* nor *doubt* behind; but to clear and ascertain each particular in proceeding. Such as suspect the correctness of the table, may prove it in this manner: Take the *principal* of any two years, or half years, and *multiply* them into each other; the product will be the principal of that year which is the total sum of those *two years added* together.

Let the *second* and *fourth* half year be multiplied into each other, the product will be the principal of the *sixth* half year, for two added to four is six. The *fourth* multiplied into the *sixth* produces the principal of the *tenth*, for four added to six

lxiv COMPOUND INTEREST.

six is ten. The *tenth* half year multiplied into *itself* produces the principal of the *twentieth* half year, for ten and ten is twenty. The *twentieth* half year into *itself* produces the principal of the *fortieth*, for twenty and twenty is forty. The *fifteenth* into the *fortieth* produces the principal of the *fifty-fifth*, for forty and fifteen is fifty five; and the principal of the *fifty-fifth* multiplied into *itself* produces the *110th* half year, for 55 and 55 is 110 (*c*).

XLIV. By this table the compound interest of any *sum* for any *time* contained in it, may be found by multiplying the number of half years answerable to the time
re-

(*c*) All which have been examined, and found to be true:

The fourth *half* year multiplied by the second produces the sixth half year *exactly*; and the sixth multiplied by the fourth, produces the 41 figures of the tenth, perfectly the *same* as in the table.

The tenth *half* year multiplied into *itself* produces 4147781957557977980059, &c. the same as the 20th. The 20th multiplied into *itself* produces 200159734318603051445, &c. the same as the fortieth, to *nineteen* places of figures. The fortieth multiplied by the fifteenth, produces 2596527845278796463866, &c. the same as the fifty-fifth, to *nineteen* places of figures. The fifty fifth multiplied into *itself* produces 6741956851308149587246, &c. the same as the 110th, to *eighteen* places of figures.

The whole was intended to have been inserted at large, and is omitted only because the setting the press for such a number of figures might be thought a needless additional expence.

COMPOUND INTEREST. lxv

required, by the proposed sum, and find the value of the product by reduction (29.)

EXAMPLE. What will be the amount of 100*£*. at the rate of this table, in fifty-five years?

Fifty-five years are 110 *half* years, where is found *£*.6.7419568, &c.

By multiplication that sum produces (15.) *£*.674.19568.

By reduction (32.) the decimal is *£*.0 3 10 $\frac{1}{4}$.8528.

The Answer is *£*.674 3 10 $\frac{1}{4}$.8528.

XLV. If the second half year was *added* to the first, and the third *added* to the total of them; the fourth *added* to the total of the *three*, the fifth added to the amount of *four*, and so *adding* the next half year to the total of *all* the preceding ones; it would shew the amount of any annuity that had been forborn payment any given time within the extent of the table.

XLV. A Table to shew the amount of 2*£*. annuity that hath been forborn for any number of *half* years which it contains; or to shew the value of the payments in *present money*, when the said annuity hath been paid any given number of *half* years therein.

Half yrs.	£.		£.
1	1. 1.0175	14	15.70953251 1.27491682 +
2	2.0175 1.03530625	15	16.98444933 1.29722786
3	3.05280625 1.05342411 +	16	18.28167719 1.31992935
4	4.10623036 1.07185903	17	19.60160654 1.34302811
5	5.17808939 1.09061656	18	20.94463465 1.36653111 +
6	6.26870595 1.10970235	19	22.31116576 1.3904454
7	7.37840830 1.12912215 +	20	23.70161116 1.41478196 +
8	8.50753045 1.14888178	21	25.11639312 1.43953681
9	9.65641223 1.16898721	22	26.55592993 1.46472871 +
10	10.82539944 1.18944449	23	28.02065864 1.49036146
11	12.01484393 1.21025977 +	24	29.51102010 1.51644279 +
12	13.2251037 1.23143931	25	31.02746289 1.54298054 +
13	14.45654301 1.2529895	26	32.57044343 1.56998269
14	15.70953251	27	34.14042612

COMPOUND INTEREST.

lxvii

27	34.14042612 1.59745739	41	59.23573497 2.0366253 +
28	35.73788351 1.6254129 +	42	61.27236027 2.07226624 +
29	37.36329641 1.65385762	43	64.34462651 2.1085309 +
30	39.01715403 1.68280013	44	66.45315741 2.14543019 +
31	40.69995416 1.71224913	45	68.5985876 2.18297522 +
32	42.41220329 1.74221349	46	70.78156282 2.22117728
33	44.15441678 1.77270223 +	47	73.0027401 2.26004789 +
34	45.92711901 1.80372452	48	75.26278799 2.29959872
35	47.73084353 1.83528969	49	77.56238671 2.3398417
36	49.56613322 1.86740727 +	50	79.90222841 2.38078893
37	51.43354049 1.90008689	51	82.28301734 2.42245274 +
38	53.33362738 1.93333841	52	84.70547008 2.46484566
39	55.26696579 1.96717184 +	53	87.17031574 2.50798046
40	57.23413763 2.00159734	54	89.67829620 2.55187012 +
41	59.23573497	55	92.23016632

lxviii COMPOUND INTEREST.

55	92.23016632 2.59652784	69	133.02040495 3.31035702
56	94.82669416 2.64196708	70	136.33076197 3.36828827 +
57	97.46866124 2.68820151 +	71	139.69905024 3.42723331
58	100.15686275 2.73524503	72	143.12628355 3.4872099 +
59	102.89210778 2.78311182	73	146.61349345 3.54823607
60	105.67521960 2.83181628 +	74	150.16172952 3.6103302
61	108.50703588 2.88137306	75	153.77205972 3.67351098
62	111.38840894 2.93179709	76	157.44557070 3.73779742
63	114.32020603 2.98310354	77	161.18336812 3.80320888 +
64	117.30330957 3.03530785	78	164.9865770 3.86976503
65	120.33861742 3.08842574 +	79	168.85634203 3.93749592
66	123.42704316 3.14247319	80	172.79383795 4.00639192
67	126.56951635 3.19746647	81	176.80022987 4.07650378
68	129.76698282 3.25342213	82	180.87673365 4.1478426 +
69	133.02040495	83	185.02457625

COMPOUND INTEREST.

lxix

83	185.02457625 4.22042984	97	251.32555618 5.380697 +
84	189.24500609 4.29428737 +	98	256.70625318 5.47485919
85	193.53929346 4.3694374 +	99	262.18111237 5.57066923 +
86	197.90873086 4.44590255	100	267.75178160 5.66815593
87	202.35463341 4.52370584	101	273.41993753 5.76734867
88	206.87833925 4.6028707 +	102	279.18728620 5.86827727 +
89	211.48120995 4.68342093	103	285.05556347 5.97097212
90	216.16463088 4.7653808	104	291.02653559 6.07546413
91	220.93001168 4.84877496	105	297.10199972 6.18178476 +
92	225.77878664 4.93362853 +	106	303.28378448 6.28996599 +
93	230.71241517 5.01996703 +	107	309.57375047 6.40004039
94	235.73238220 5.10781645 +	108	315.97379086 6.5120411
95	240.84019865 5.19720324 +	109	322.48583196 6.62600182 +
96	246.03740189 5.28815429	110	329.11183378 6.74195685
97	251.32555618	111	333.85379063

lxx COMPOUND INTEREST.

Suppose B. under obligation to pay to C. 2*£*. annuity by *half* yearly payments, but had neglected paying it for the last ten years : on the day the 21st *half* yearly payment becomes due, they settle their accounts : How much should B. pay to C. for all the forborn payments?

Opposite the 21st half year in the table is *£*.25.11639312, which is the answer.

The LAST half year is paid on the day it is due; for which NO INTEREST is required, and is the same as the FIRST in the table.

And if the annuity had been *£*.6.7, that is, *£*.3.35 half yearly, then the above sum multiplied by the *half* yearly payment would be the value.

$$25.11639312 \times 3.35 = \underline{84.139916952}$$

In the same manner may be known the total present worth of sums that *have been* paid *half* yearly. B. on certain conditions, had agreed to pay to C. *£*.3.35 down in present money, and the same sum for the twenty succeeding half years : when the time is expired, and all the payments made, C. finds himself unable to fulfill his engagement, and agrees to restore to B. his money *with interest* What sum should he pay?

The

The answer is, as before, $\underline{\underline{£. 84.139916952}}$

These answers are for annuities *certain*: if payments had been to *cease* upon the *demise* of B, then B. must be allowed for the *chance* he had of *surviving*, or *not*, to receive the conditions of the agreement.

REVERSION or DISCOUNT.

XLVI. **D**iscount of money depends on the *rate* of interest, and the time *hence* that the sum and its interest are to be paid; and shews what a sum of money, to be paid at any *future* time, is worth *now*, in *present* money.

If 100£. will gain $3\frac{1}{4}$ per cent per annum, at the end of the year it will be worth 103.5£. it must be evident then that 100£. to be received a year hence, *cannot* be worth 100£. now in *present* money; for 103.5£. to be received a year hence would be worth but 100£. at *this present time*.

If 103.5£. in reversion is worth in present

lxxii REVERSION or DISCOUNT.

present money but 100*£*. what is 100*£*.
in reversion worth in present money?

As 103.5 : 100 : : 100 : 96,618357 (33.)

100*£*. at two years *compound* interest
would be worth 107.1225 (39.) what
would 100*£*. to be paid *two years hence*,
be worth in present money?

As 107.1225 : 100 : : 100 : 93,35106

In the same manner the discount for any
time by *half yearly* interest may be found.
(42.)

The first year 103.530625 : 100 : : 100 : 96.5898
The second year 107.1839 : 100 : : 100 : 93.2959

Hence as the table (42*.) shews what
interest a sum will gain in any time
within *fifty-five* years, take the sum out of
it, that is opposite the given time, for a
divisor ; and unity, with as many cyphers as
necessary, for a dividend ; and the quotient
will be the value of the reversion (*d*).

Though

(*d*) Mr. Thomas, in his *Banker's Sure Guide*, p. xx. Intro-
duction, gives an example that is an excellent illustration of
this subject.

“ A gentleman on his death bed gave his friend A. a bond
“ for 1000*£*. payable by his executor one year after his demise :
“ A. soon after, hearing of his friend's death, and being in
narrow

REVERSION OR DISCOUNT, lxxiii

Though the Table of Compound Interest is absolutely necessary to find the value of the reversion, yet the Table of Reversion being obtained, the first may be dispensed with; for, as *dividing* unity with cyphers by the compound interest gives the *reversion* in the quotient, so dividing unity with cyphers by the reversion, that is, the quotient, must give the *compound interest*, or the former divisor, in the *quotient*: by the

L two

" narrow circumstances, goes to his neighbour B. to discount
 " the said bond; B. asked him 50*£*. Very well, says A. and if
 " I allow you 50*£*. in hand, which is the full interest of 1000*£*.
 " at 5*£*. per cent. at the end of the year; who must have the
 " interest of the said 50*£*. for the present year, you or I? If
 " you will return me the interest thereof, viz. *£*. 2 10 0, I will
 " be ready to agree to your terms; if not, I must apply to ano-
 " ther. A. adds farther, If you will give me as much money for
 " my bond, supposing it were lent out on interest at 5*£*. per
 " cent. for a year, as, with the interest thereof, will, at the year's
 " end, make up my legacy 1000*£*. I will deal with you, and
 " not otherwise. B. the banker agrees, and A. calculates it
 " thus:

If 105 : 5 : : 1000

5

105)5000(47.619

" or *£*. 47 12 4 $\frac{1}{2}$ the discount due to B. which deducted
 " from 1000*£*. leaves *£*. 952 7 7 $\frac{1}{2}$, the sum A. received.
 " That B. got all he ought to have, may be proved thus; viz.
 " If B. lends out the *£*. 47 12 4 $\frac{1}{2}$, it would bring him
 " *£*. 2 7 7 $\frac{1}{2}$ interest, and increase itself to 50*£*. at the year's
 " end, which sum would be all he had a right to by law; and
 " supposing the money A. received to be put out at 5*£*. per
 " cent, it would just bring him the sum, viz. 47 12 4 $\frac{1}{2}$, he paid
 " in hand to B. and he would have his 1000*£*. entire at the
 " year's end; so that both parties would have equal justice."

lxxiv REVERSION or DISCOUNT.

two tables compound interest may be found either by *multiplication* as before (44.), or, as here, by *division*.

The more interest any sum gains, the larger that sum will be for a divisor, which makes the quotient, (*e*) that is, the reversion, proportionably less. Because 100*£*. will be worth *£*.674.1956 (44.) in *fifty-five* years, at compound interest; 100*£*. to be received *fifty-five* years hence, is worth in *present* money but *£*.14.8324.

$$\begin{array}{l} 674.1956)10000,00000000(14.8325+ \\ 14.8325+)10000.00000000(674.195(17.) \\ 6741956 \times 148325 = 1000000623700 \\ 6741956 \times 148324 = 999993881744 \quad (30.) \end{array}$$

Here the *quotient* of the first is made the *divisor* of the second, to find the *divisor* of the first. And 6741956 is multiplied by 148325, and 148324, which is an unit less, to shew that the product will be *more* or *less* than the dividend, according as the fraction remaining by division is *increased* to an unit, or *entirely omitted*. (21.)

A

(*e*) As every *whole* year in the Table (42.) by *half* yearly payments produces something *more* than by *yearly* payments in the other Table (39.) so the value of each reversion in the following table by *half* yearly payments must be something *less* than reversions found by yearly payments, though at *equal* rates of interest. (42.)

REVERSION or DISCOUNT. lxxv

XLVII. A Table of Reversion at $3\frac{1}{2}$ per cent. per annum, and by half yearly interest.

Half Yrs.		Half Yrs.	
1	.9828009 (22.)	25	.648096
2	.965898 + (21.)	26	.636940
3	.949285	27	.625995 +
4	.932959 +	28	.615228 +
5	.916912	29	.604647 +
6	.901143 +	30	.594248
7	.885644 +	31	.584027
8	.870412 +	32	.573982
9	.855441	33	.564111 +
10	.840728	34	.554408 +
11	.826269 +	35	.544873
12	.812058 +	36	.535502 +
13	.798091	37	.526292 +
14	.784364	38	.517240 +
15	.770875 +	39	.508344
16	.757616 +	40	.499601 +
17	.744586 +	41	.491007
18	.731780	42	.482563
19	.719194	43	.474263
20	.706824	44	.466107
21	.694668 +	45	.458090
22	.682721 +	46	.450212 +
23	.670978	47	.442468
24	.659438 +	48	.434858

lxxvi REVERSION OF DISCOUNT.

49	.427379	80	.249601 +
50	.420028	81	.245308
51	.412804	82	.241089
52	.405705 +	83	.236943 +
53	.398727	84	.232865 +
54	.391869	85	.228862
55	.385129	86	.224926
56	.378505	87	.221057
57	.371995	88	.217255 +
58	.365597	89	.213519
59	.359310	90	.209846
60	.353130	91	.206237
61	.347057 +	92	.202690
62	.341087	93	.199204
63	.335222 +	94	.195778 +
64	.329455	95	.192411 +
65	.323789	96	.189101
66	.318221 +	97	.185849
67	.312747	98	.182653
68	.307368	99	.179512
69	.302082	100	.176424
70	.296886 +	101	.173388
71	.291781 +	102	.170408 +
72	.286762 +	103	.167476
73	.281830	104	.164596
74	.276983 +	105	.161765
75	.272219	106	.158983
76	.267538 +	107	.156249
77	.262673 +	108	.153546
78	.258414 +	109	.150920
79	.253969 +	110	.1483249

XLVIII. It may be observed in the last table, that the *second* half year's reversion is worth less than the first, and the third is worth less than the second, &c. decreasing to the end of the table, which shews the value of any *one* half year singly: but if an annuity is to be received or paid constantly every half year, from the present time to the end of the table; then, if duly considered, it will be plain, that to find the value of the reversion of *all the* payments together, they must be all added together, and the sum would not only include each half year's payment, but also each at its just value.

Suppose a sum to be received *certain* for *two* half years, then the first and second *added* is the value of *both* in *present* money. Suppose an annuity for four half years, that is, two years, *certain*; the third added to the total of the two first, and the fourth added to the total of the three sums, would be the value. Thus, by continually adding the next half year to the sum of *all* the preceding ones, the following table is formed, that shews the value of an annuity of 2£. (which is 1£. half yearly) for any number of half years that the table contains. For example, Suppose a person is to receive, or pay an annuity of £.6 14 0 for *ten* years *certain*; the real sum that would
be

lxxviii REVERSION or DISCOUNT.

be paid would be 67*£*. but it is not of so much value *now at this time* in present money; for discount or reversion must be allowed, and therefore the just value will be found from the table, (49.) by taking the sum from the 20th half year, which is equal to *ten years*, and multiplying that sum by *£. 3 7 0*, which is the half yearly payment of *£. 6 14* annuity, thus :

At the 20th half year is found 16.75288
£. 3 7 0 is decimally 3.35

8376440
 5025864
 5025864

The worth of *all* the pay- } 56.1221480
 ments is

Which when reduced is *£. 56 2 5*¹₄(32.)

The case here is supposing an annuity *certain*: another case, when an annuity is *uncertain*, as being to be paid *during life only*, will be considered hereafter (68.)

XLIX. A Table to find the value, in *present* money, of any sum, to be paid any *certain* term of years *hence* under 55, at *£. 3*¹₂ *per cent. per annum, per half yearly payments.*

REVERSION OF DISCOUNT. lxxix

Hf. Yrs.

Hf. Yrs.

1	.982801 + .965898	15	13.092880 .757616
2	1.948699 .949285	16	13.850496 .744586
3	2.897984 .932959	17	14.595082 .731780
4	3.830943 .916912	18	15.326862 .719194
5	4.747855 .901143	19	16.046056 .706824
6	5.648998 .885644	20	16.752880 .694668
7	6.534642 .870412	21	17.447548 .682721
8	7.405054 .855441	22	18.130269 .670978
9	8.260495 .840728	23	18.801247 .659438
10	9.101223 .826269	24	19.460685 .648096
11	9.927492 .812058	25	20.108781 .636949
12	10.739550 .798091	26	20.745730 .625995
13	11.537641 .784364	27	21.371725 .615228
14	12.322005 .770875	28	21.986953 .604647
15	13.092880	29	22.591600

XXX REVERSION OF DISCOUNT.

29	22.591600 594248	43	30.042061 466107
30	23.185848 584027	44	30.508168 458090
31	23.769875 573982	45	30.966258 450212
32	24.343857 564111	46	31.416470 442468
33	24.907968 554408	47	31.858938 434858
34	25.462376 544873	48	32.293796 427379
35	26.007249 535502	49	32.721175 420028
36	26.542751 526292	50	33.141203 412804
37	27.069043 517240	51	33.554007 405705
38	27.586283 508344	52	33.959712 398727
39	28.094627 499601	53	34.358439 391869
40	28.594228 491007	54	34.750308 385129
41	29.085235 482563	55	35.135437 378505
42	29.567798 474263	56	35.513942 371995
43	30.042061	57	35.885937

REVERSION OF DISCOUNT. lxxx

57	35.885937 365597.	71	40.469669 286762
58	36.251534 359310	72	40.756431 281830
59	36.610844 353130	73	41.038261 276983
60	36.963974 347057	74	41.315244 272219
61	37.311031 341087	75	41.587463 267538
62	37.652118 335222	76	41.855001 262673
63	37.987340 329455	77	42.117674 258414
64	38.316795 323789	78	42.376088 253969
65	38.640584 318221	79	42.630057 249601
66	38.958805 312747	80	42.879658 245308
67	39.271552 307368	81	43.124966 241089
68	39.578920 302082	82	43.366055 236943
69	39.881002 296886	83	43.602998 232865
70	40.177888 291781	84	43.835863 228862
71	40.469669	85	44.064725

xxxii REVERSION OF DISCOUNT.

85	44.064725 224926	98	46.795251 179512
86	44.289651 221057	99	46.884763 176424
87	44.510708 217255	100	47.061187 173388
88	44.727963 213519	101	47.234575 170408
89	44.941482 209846	102	47.404983 167476
90	45.151328 206237	103	47.572459 164596
91	45.357565 202690	104	47.737055 161765
92	45.560255 199204	105	47.898820 158983
93	45.759459 195778	106	48.057803 156249
94	45.955237 192411	107	48.214052 153546
95	46.147648 189101	108	48.367598 150920
96	46.336749 185849	109	48.518518 148324
97	46.522598 182653	110	48.666842
98	46.705251		

Though it may seem useless to insert the lines which are added; yet, as it may afford more satisfaction of the whole being *right*, they are left as wrought.

THE
SECOND DATA,
OR
PROBABILITY of LIFE.

L. IN the quarto collection of the Yearly Bills of Mortality, printed for Millar in the Strand, is the following calculation, made by J. P. F. R. S. from ann. 1728 to 1757 inclusive; in which time, of every 1000 born, there were, computing at a medium,

	Years old.	Living.	Decrement or dead.
At	2	637	363
	5	550	87
	10	516	34
	20	485	31
	30	408	77
	40	312	96
	50	215	97
	60	135	80
	70	72	63
	80	27	45
	90	4	23
Upwards of	90	4	4
			1000
			There

lxxxiv PROBABILITY of LIFE.

There is another calculation, by John Smart, gent. in the same volume; made for the *first ten years* of the *same time*, viz. from 1728 to 1737, as follows. Born 1000.

Years old.	Living.	Decrement or dead.
At 2	614	386
5	526	88
10	490	36
20	459	31
30	385	74
40	294	91
50	204	90
60	130	74
70	69	61
80	29	40
90	5	24
Upwards of 90		.5
		1000

These years are as the burials published in the bills; and, from their consisting mostly of *ten years*, are called *decades*.

LI. It is to be regretted that there is no better guide for the *subdividing* them into *annual* burials; for, though it is *known* that 363 or 386 die before *two* years of age, it is left to *guess* how many died before *one* year old: it may be well supposed indeed, that far the *greatest* number die before *one* year

year, and therefore the aforementioned gentlemen divide them thus:

Under 1 year	250.	ditto —	290.
2 —	113.	ditto —	96.
	363		386

Though the *annual* divisions may be too inaccurate under five years of age, yet afterwards they may be made with a much greater degree of certainty, by observing that from 30 to 50 the *greatest* number die, and that the burials *decrease* gradually before and after those years; for which reason, in the *medium* of that time, the *greatest* number of *annual* burials may be expected. Also between 10 and 20 the *least* number die, and the burials *increase* from that time both ways, before and after: therefore the *least* annual number will be found between those years, which example (60.) shews. It appears then, that though the annual divisions of the decades be made at *will*, yet they are not *entirely* without reason or a *clue* to guide.

LII. It does not appear *how* the proportions to 1000 were obtained, but it is very probable they were by this method. The totals of *all* the burials in the several decades for the *first ten years* are these:

lxxvi PROBABILITY OF LIFE.

	Years old.	
Died under	2 ———	103159
	5 ———	23505
	10 ———	9775
	20 ———	8242
	30 ———	19776
	40 ———	24302
	50 ———	23989
	60 ———	19693
	70 ———	16309
	80 ———	10684
	90 ———	6450
Upwards of	90 ———	1266
		267150

The total buried in ten years being found to be 267150, the question is, How many, in proportion to *one thousand*, died before two, five, ten, or any decade? By the Rule of Three, If the total sum 267150 required 103159 to die before two, how many will 1000 require? Answer, 386: and by so working each decade the following table is acquired.

Died

PROBABILITY of LIFE. lxxxvii

	Years.	Proportion to 1000 born.	In who's numbers.
Died under	2 —	386. 39100	386
	5 —	87. 262950	88
	10 —	36. 157600	36
	20 —	30. 227500	31
	30 —	74. 6900	74
	40 —	90. 258500	91
	50 —	89. 212650	90
	60 —	73. 191050	74
	70 —	61. 12850	61
	80 —	39. 265150	40
	90 —	24. 38400	24
Upwards of	90 —	4. 197400	5
993.1870050			1000.

These agree exactly with Mr. Smart's Table.

LIII. The fractions are not *decimals*, but numerators of their *common* denominator 267150; by which number let the total of the fractions be divided, and the quotient will be just *seven*; which, being added to 993, makes the whole 1000.

Therefore those numbers that have a fraction remaining of *more value* than *half* the denominator (which half is 133575) should have

have *one whole* added to them, they being in a nearer degree to the number in the last column than to that which precedes, if the fraction be omitted (21.)

Number $36.\overset{1}{\underset{1}{\frac{1}{2}}}\overset{1}{\underset{1}{\frac{1}{4}}}\overset{1}{\underset{1}{\frac{1}{8}}}\overset{1}{\underset{1}{\frac{1}{16}}}$ is then nearer to 37 than to 36 without the fraction; but because there are only seven additional *integers* required to be added to 993, number 36 cannot be one, it being the *left* fraction of *eight*, that require *one whole* to be added to their integer.

Omitting *all* fractions increases in this manner some of the numbers, while the true value of others is decreased by so much as the *omitted* fraction is worth; but it is not possible to have the table in *whole numbers* without, and the difference is not very great.

After the same method let the table for 30 years be examined, and it will be found thus:

Under

PROBABILITY OF LIFE.

lxxxix

	Age.	Buried.	Proportion to 1000.	In whole numbers.
Under	2	272903	363.	536114
	5	64745	86.	217308
	10	25912	34.	401052
	20	22891	30.	381340
	30	58474	77.	699206
	40	71502	95.	221410
	50	73238	97.	456766
	60	59782	79.	506562
	70	47269	62.	749036
	80	33679	44.	664832
	90	16948	22.	440916
Upwards of	90	2972	3.	728034
Buried in 30 years		750322	992.600	25761000

$$6002576 \div 750322 = 8 \quad (53.)$$

This agrees also with the table of Mr. J. P. F. R. S. except in the places where that gentleman is supposed to be mistaken in adding *one* to 95, to 30, and to 86, instead of to 363, to 77, and to 97. (50.)

LIV. By comparing the two tables it may be observed, that at *different periods* there may be some *variations* in the probability of life; for in the *first ten* years the number that died under two, five, and the decades of ten, twenty, and ninety, *exceeds* the number of those that died at the same ages, when computed for 30 years together at a medium;

N

and

xc PROBABILITY OF LIFE.

and in the other decades is *deficient* in proportion.

It is *necessary then* to continue the computation to the *present* time in order to gain a *juster* medium: when *all* the burials that have been from 1728 to 1770 *inclusive* are added together, their sum *total* and proportion to a thousand is found to be

	Years.	Buried.	Proportion to the total of 1000.	In whole numbers.
Under	2	372572	357. 707591	358
	5	91755	88. 90944	88
	10	37277	35. 819705	36
	20	33459	32. 126616	32
	30	82052	78. 804314	79
	40	98319	94. 405122	94
	50	101290	97. 251211	97
	60	82062	78. 814314	79
	70	67570	64. 905232	65
	80	48264	46. 348698	46
	90	23073	22. 156986	22
Upwards of	90	3944	3. 819089	4
Dead in	43	1041637	994.6249822	1000

$$.6249822 \div 1041637 = 6 \text{ (53.)}$$

Thus are obtained the proportions, at a *medium*, for the 43 last years; but it should not be concealed, that the bills of mortality for London are very far from being perfect: it is as certainly known as that 1041637 have been

PROBABILITY of LIFE. xci

been *buried*, that no more than 673251 have been *christened*; whence it appears that there have been omissions in the *one*, or additions to the *other*; or, rather, there have been *both*.

The yearly burials at a <i>medium</i>	23608
Christenings - - - -	15657
Burials <i>more</i> than christenings	7951

The burials every year have constantly much exceeded the christenings; for the *nearest* they have been to *equality* since A. 1646, was in the year 1671, when the

Burials were	15729
Christenings	12510

And in 1758, Burials	17576
Christenings	14209

And the burials have been nearly *double* the christenings; nay, sometimes, even *more*, as in

1740	{	Burials	30811	1741	{	32169
		Christenings	15231			14957

It must be sufficiently obvious, that if 7951 natives of London were to be *buried*
N 2 every

every year more than *born*; this city, however populous, would soon be exhausted of its inhabitants: but the direct contrary seems to be the case.

The 7951 persons are then very reasonably supposed to be *aliens* (calling *all* such, who were *not born*, or *not christened* in London) and are supplied by the arrival of traders French refugees, and other foreigners;—by persons out of the country to be apprenticed—to be servants—to be workmen—to be cured of diseases—on law affairs, and on various occasions.

This is corroborated by a well known evidence, which is, that in country towns not very large, and in villages, the christenings do always exceed the burials.

Likewise there may be many omissions in the christenings, supposed to proceed from such Dissenters and Papists, who, though not *christened*, may possibly have been *buried*, according to the rites of the church of England; and consequently, though omitted in the *first*, are nevertheless numbered among the *last*, in the bills of mortality.

LV. The difficulty is, to know at what age, in what *number*, and at what *time*, these *aliens* intermingle with the natives.

The inquiry would render this *essay* more abstruse than it is intended to be; and to give the solution would require more information and judgment than the author presumes to
 insinuate

insinuate, that he thinks himself possessed of. Beside, the reader may find the question already very amply discussed by able masters of the subject. (f)

But a few remarks, that are very obvious, may perhaps be serviceable.

The aliens may be considered as constituting another and *separate* bill of mortality: then, if the *burials* of the *aliens* be united with the *natives*, so likewise should their *births*; or if the *births* of the *aliens* be separated from the *natives*, so likewise should their *burials*: but, though this is evident, still the former difficulty (55.) remains.

LVI. So far as the subject affects annuities, perhaps it might be adjusted by *one general* bill of mortality; for as annuitants may live in *all* parts of the kingdom, a calculation made from such a bill must be nearer truth than any *particular* one.

Suppose, for an instance, a city to be surrounded by 100 towns, villages, &c. which places, one with another, have 50 born yearly; and that, of the 50, *five* go annually to settle in the city, and there die. In the city, suppose 1000 births annually: it is presumed the effect would be, that the city would bury 1500 yearly, and the villages, towns, &c. only 45 each; for out of every

50

(f) See the aforementioned quarto volume, and more particularly Dr. Price, p. 167, &c.

xciv PROBABILITY OF LIFE.

50 born, the villages *lost*, and the city *gained*, *five*. In such a case a calculation made either from the city bill, or country bills, could not be just, when taken *separately*.

But if *every* place kept a register, and *all* of them were united together, the *loss* would be *restored*, and the numbers found as thus:

Buried in the city	—	1500
Buried in the country	—	4500
Total <i>buried</i>		6000

Born in the city	—	1000
Born in the country	—	5000
Total <i>born</i>		6000

By which, births and burials become *equal*.

Again, suppose 100 persons, consisting of *actual foreigners*, that is, such as arrive from *beyond the seas*, were also buried in the city, more than the 1500; then $\frac{1}{16}$ of the buried would be the said foreigners; but if no such foreigners are buried in the villages, &c. their number, in proportion to the *general bill*, would be but $\frac{1}{16}$.

LVII. These are *two* apparent advantages that might be gained by establishing bills of mortality in *every* parish throughout the nation.

LVIII.

LVIII. *One* defect in the preceding tables may be corrected with clearness and certainty; for in the 43 years 26495 *abortives* and still-born are inserted in the bills among the *burials*, but not accounted to the *births*, because *not christened*. In a calculation that compares births with burials, and the probability of life, these abortives cannot be admitted; and it is very plain that the whole must be taken from those aged under *two*, for they were included in *that* number.

In 43 years buried under two	—	372572
Abortives	—	26495
		<u>346077</u>

The difference these occasion between the following table and the preceding ones, proves, that very great defects must arise from the aliens intermingling with the natives in the number of burials, when their births are *not* brought to the account.

The reader will observe, that the following table is more correct than the preceding ones, not only by extending to, and *including* the year 1770, but also by the *rejection* of *abortives*. He is likewise so well cautioned of its *imperfection*, that it may appear strange to him that it is used at all; but he will find no *great stress* laid upon it, as the calculations will be made (not from it only, but)

xcvi PROBABILITY of LIFE.

but) from two other tables of perhaps *better* authority; and its principal use will be to *compare* with them.

LIX. A Table shewing the number that have been buried from A. D. 1728 to 1770 inclusive, *without* the *abortives*, &c.

	Years.	Buried.	Proportion to the total of 1000 bn.	In whole numbers.
Under	2	346077	340. 928720	341
	5	91755	90. 392220	90
	10	37277	36. 731888	37
	20	33459	32. 974456	33
	30	82052	80. 840640	81
	40	98319	96. 865368	97
	50	101290	99. 790942	100
	60	82062	80. 850640	81
	70	67570	66. 570628	66
	80	48264	47. 552326	47
	90	23073	22. 739876	23
Upwards of	90	3944	3. 898574	4
		1015142	991.9136278	1000

$$9136278 \div 1015142 = 9 \quad (53.)$$

LX.

PROBABILITY of LIFE. xcvii

LX. To prepare this table for calculation, let it be reduced to *annual* decrements, or deaths. (51.).

Born 1000.

Age.	Living.	Decrem. or dead.	Age.	Living.	Decrem. or dead.
Born—1000					
1 —	759	241			
2 —	659	100			
	<u>2418</u>	<u>341</u>	11 —	528	4
			12 —	525	3
3 —	611	48	13 —	522	3
4 —	585	26	14 —	519	3
5 —	569	16	15 —	517	2
	<u>1765</u>	<u>90</u>	16 —	514	3
6 —	557	12	17 —	511	3
7 —	548	9	18 —	508	3
8 —	541	7	19 —	504	4
9 —	536	5	20 —	499	5
10 —	532	4		<u>5147</u>	<u>33</u>
	<u>2714</u>	<u>37</u>			

Age.	Living.	Decrem.	Age.	Living.	Decrem.
21 —	493	6	41 —	310	11
22 —	486	7	42 —	299	11
23 —	478	8	43 —	289	10
24 —	470	8	44 —	279	10
25 —	462	8	45 —	269	10
26 —	454	8	46 —	259	10
27 —	445	9	47 —	249	10
28 —	436	9	48 —	239	10
29 —	427	9	49 —	230	9
30 —	418	9	50 —	221	9
	<u>4569</u>	<u>81</u>		<u>2644</u>	<u>100</u>
31 —	409	9	51 —	212	9
32 —	400	9	52 —	203	9
33 —	391	9	53 —	194	9
34 —	382	9	54 —	186	8
35 —	372	10	55 —	178	8
36 —	362	10	56 —	170	8
37 —	352	10	57 —	162	8
38 —	342	10	58 —	154	8
39 —	332	10	59 —	147	7
40 —	321	11	60 —	140	7
	<u>3663</u>	<u>97</u>		<u>1746</u>	<u>81</u>

Age.

PROBABILITY of LIFE.

xcix

Age.	Living.	Decrem.	Age.	Living.	Decrem.
61 —	133 —	7	81 —	24 —	3
62 —	120 —	7	82 —	21 —	3
63 —	119 —	7	83 —	18 —	3
64 —	112 —	7	84 —	15 —	3
65 —	105 —	7	85 —	13 —	2
66 —	98 —	7	86 —	11 —	2
67 —	92 —	6	87 —	9 —	2
68 —	86 —	6	88 —	7 —	2
69 —	80 —	6	89 —	5 —	2
70 —	74 —	6	90 —	4 —	1
	<u>1025</u>	<u>66</u>		<u>127</u>	<u>23</u>
71 —	68 —	6	91 —	3 —	1
72 —	63 —	5	92 —	2 —	1
73 —	58 —	5	93 —	1 —	1
74 —	53 —	5	94 —	0 —	1
75 —	48 —	5		<u>6</u>	<u>4</u>
76 —	43 —	5			
77 —	39 —	4			
78 —	35 —	4			
79 —	31 —	4			
80 —	27 —	4			
	<u>465</u>	<u>47</u>			

Total of *de-*
crements } 1000
Living at all } 26289
ages }

LXI. The formation of the last table has been shewn from its origin; that of the next will be best explained in Mr. *Simpson's* own words, who published it A. 1742.

“ The value of an annuity for life depends upon

“ upon the interest which money bears, and
“ the probability of the life continuing a
“ longer or a shorter time; the former of
“ which is generally settled by law, but the
“ latter must be determined from observa-
“ tions.

“ Of all that has been hitherto offered for
“ estimating the probability of the duration
“ of life, nothing seems deduced with
“ greater judgment and exactness, than the
“ tables published by Dr. *Halley* and Mr.
“ *Smart*, (50. 63.) for this purpose; which,
“ nevertheless, are *both* liable to considerable
“ objections.

“ The Doctor's table, being grounded on
“ observations made at Breslau, a place where
“ the generality of people live to a greater
“ age than at London (as appears by com-
“ paring the bills of mortality here with
“ those observations) can be no just measure
“ of the probability of life in this place;
“ and as to that of Mr. *Smart*, though it is
“ indeed free from this objection, and
“ founded on a very large number of obser-
“ vations, yet the great and continual
“ afflux of people from all parts up to
“ town, renders the deductions from those
“ observations considerably different, in one
“ part of life, from what they would
“ otherwise be; and this Mr. *Smart* seems
“ not in his table to have considered, or made
“ any allowance for.

“ For

“ For these reasons, though I had determined to depend on, and make use of, this
“ *last* gentleman’s observations, in the ensuing pages (as undoubtedly the best for
“ the city of London, and parts adjacent)
“ yet have I deemed it necessary to make
“ some alterations in the table of the probability of life from thence derived.

“ In doing this, I have supposed the number of persons coming to live in town,
“ after *twenty-five* years of age, to be inconsiderable with respect to the whole
“ number of inhabitants; and therefore the probabilities of life, for all ages above
“ twenty-five years, the same as this author
“ has made them; but then have increased
“ the numbers of the living corresponding to
“ all ages below twenty-five so that they may,
“ as near as possible, be in the same proportion one to another, as they would be were
“ they to be deduced from observations on
“ the mortality of those persons *only* that are
“ born within the bills: which was done
“ by comparing together the number of
“ christenings and burials, and observing, by
“ help of Dr. *Halley’s* table, the proportion
“ which there is between the degrees of mortality at *London* and *Breslau*, in the other
“ parts of life where the ages are greater
“ than twenty-five.

Mr. *Simpson* likewise observes in his *preface*, “ that it is possible the great difference

“ference which there is, in one part of life,
“between the value of an annuity accord-
“ing to the *Breslau* observations, and the
“following table, may tempt (especially those
“whose interest it is) to question the exact-
“ness of the tables, or the observations
“whereon they were grounded; for they
“may affirm, that *London* consists of too
“flux a body to admit of any certain measure
“for the probability of life, and that the
“accounts published by the company of
“parish clerks are not to be depended on.
“But to this I answer, that though the
“continual resort of people from all
“parts causes indeed a great increase in
“the bills of mortality, it will no ways
“influence the values of the annuities
“thence deduced, if the numbers of per-
“sons coming up to town at all ages be
“proportional to the whole numbers of
“the living of the same ages: and though
“this supposition is not exactly true in
“small ages, yet, as experience shews it
“to be more nearly so in greater, and as
“the number of persons that come to
“live in town after *twenty-five* or *thirty*
“years of age, is inconsiderable with re-
“spect to the whole body of inhabitants;
“it is evident that the values given in
“the tables for all ages not less than
“*twenty-five* or *thirty* years, can be but
little

PROBABILITY of LIFE. ciii

“ little affected from the cause above mention-
“ ed. It is true the values for younger lives
“ have not quite so good a foundation; but, I
“ presume, the method I have had recourse
“ to upon this occasion is such, as is not liable
“ to any reasonable objections; and as to the
“ difference that may arise from any uncer-
“ tainty or error in the accounts of the
“ parish clerks, it can be but very little,
“ because if the age happens to be given in
“ a little too high one time, there is the same
“ chance of its being put down as much
“ too low another.”

LXII. Mr. SIMPSON'S TABLE.

1280 Born.

Age.	Living.	Decrem. or dead.	Age.	Living.	Decrem. or dead.
1	870	410			
2	700	170			
	<u>2850</u>	<u>580</u>			
3	635	65	21	455	7
4	600	35	22	448	7
5	580	20	23	441	7
	<u>1815</u>	<u>120</u>	24	434	7
6	564	16	25	426	8
7	551	13	26	418	8
8	541	10	27	410	8
9	532	9	28	402	8
10	524	8	29	394	8
	<u>2712</u>	<u>56</u>	30	385	9
11	517	7		<u>4213</u>	<u>77</u>
12	510	7	31	376	9
13	504	6	32	367	9
14	498	6	33	358	9
15	492	6	34	349	9
16	486	6	35	340	9
17	480	6	36	331	9
18	474	6	37	322	9
19	468	6	38	313	9
20	462	6	39	304	9
	<u>4891</u>	<u>62</u>	40	294	10
				<u>3354</u>	<u>91</u>

PROBABILITY OF LIFE.

CV

Age.	Living.	Decremt. or dead.	Age.	Living.	Decremt. or dead.
41 —	284	10	61 —	123	7
42 —	274	10	62 —	117	6
43 —	264	10	63 —	111	6
44 —	255	9	64 —	105	6
45 —	246	9	65 —	99	6
46 —	237	9	66 —	93	6
47 —	228	9	67 —	87	6
48 —	220	8	68 —	81	6
49 —	212	8	69 —	75	6
50 —	204	8	70 —	69	6
	2424	90		960	61
51 —	196	8	71 —	64	5
52 —	188	8	72 —	59	5
53 —	180	8	73 —	54	5
54 —	172	8	74 —	49	5
55 —	165	7	75 —	45	4
56 —	158	7	76 —	41	4
57 —	151	7	77 —	38	3
58 —	144	7	78 —	35	3
59 —	137	7	79 —	32	3
60 —	130	7	80 —	29	3
	1621	74		446	40

Here Mr. Simpson's Table ends.

P—Y

The same continued.

Age.	Living.	Decremt.	Age.	Living.	Decremt.
81 —	26 —	3	91 —	4 —	1
82 —	23 —	3	92 —	3 —	1
83 —	20 —	3	93 —	2 —	1
84 —	17 —	3	94 —	1 —	1
85 —	14 —	3	95 —	0 —	1
86 —	12 —	2			
87 —	10 —	2		10	5
88 —	8 —	2			
89 —	6 —	2			
90 —	5 —	1			
	141	24			
			Total of the } living at } 25437 all ages. }		
			Total of the } 1280 decrements } (as born).		

If the undertakers, the parish clerks, or any that attend burials *constantly*, could be empowered, induced, or lawfully obliged to inquire, and return carefully the place *where* those buried were *christened*; the *natives* might be examined by their parish register, and both the number and ages of the *aliens* be obtained. Or, could there be found any city or country, in which *all* that are *born*, are *buried*; and in which none should be buried but the *natives*, and a proper account kept of their age at their demise; *there* a calculation of the probability of life for *that* place might be made with much certainty.

It

PROBABILITY of LIFE. cvii

It is said that Dr. *Halley* made choice of *Breslau*, as being an inland city that approached the nearest to such a place; and therefore he has framed a table on the bills of mortality *there*, drawn up monthly for five years, viz. from A. D. 1687 to 1691 inclusive, by Dr. *Newman* of that city, and communicated to the *Royal Society* here.

LXIII. Dr. HALLEY's TABLE.

Age.	Living.	Decremt. or dead.	Age.	Living.	Decremt. or dead.
1	— 1000	—			
2	— 855	— 145			
	<u>1855</u>	<u>145</u>			
3	— 798	— 57	11	— 653	— 8
4	— 760	— 38	12	— 646	— 7
5	— 732	— 28	13	— 640	— 6
	<u>2290</u>	<u>123</u>	14	— 634	— 6
6	— 710	— 22	15	— 628	— 6
7	— 692	— 18	16	— 622	— 6
8	— 680	— 12	17	— 616	— 6
9	— 670	— 10	18	— 610	— 6
10	— 661	— 9	19	— 604	— 6
	<u>3413</u>	<u>71</u>	20	— 598	— 6
				<u>6251</u>	<u>63</u>

Age.	Living.	Decremt.	Age.	Living	Decremt.
21 —	592 —	6	41 —	436 —	9
22 —	586 —	6	42 —	427 —	9
23 —	579 —	7	43 —	417 —	10
24 —	573 —	6	44 —	407 —	10
25 —	567 —	6	45 —	397 —	10
26 —	560 —	7	46 —	387 —	10
27 —	553 —	7	47 —	377 —	10
28 —	546 —	7	48 —	367 —	10
29 —	539 —	7	49 —	357 —	10
30 —	531 —	8	50 —	346 —	11
	<u>5626</u>	<u>67</u>		<u>3918</u>	<u>99</u>
31 —	523 —	8	51 —	335 —	11
32 —	515 —	8	52 —	324 —	11
33 —	507 —	8	53 —	313 —	11
34 —	499 —	8	54 —	302 —	11
35 —	490 —	9	55 —	292 —	10
36 —	481 —	9	56 —	282 —	10
37 —	472 —	9	57 —	272 —	10
38 —	463 —	9	58 —	262 —	10
39 —	454 —	9	59 —	252 —	10
40 —	445 —	9	60 —	242 —	10
	<u>4849</u>	<u>86</u>		<u>2876</u>	<u>104</u>

PROBABILITY of LIFE.

cix

Age.	Living.	Decremt.	Age.	Living.	Decremt.
61 —	232 —	10	81 —	34 —	7
62 —	222 —	10	82 —	28 —	6
63 —	212 —	10	83 —	23 —	5
64 —	202 —	10	84 —	20 —	3
65 —	192 —	10			
66 —	182 —	10			
67 —	172 —	10			
68 —	162 —	10			
69 —	152 —	10			
70 —	142 —	10			
	1870	100			
71 —	131 —	11	85 —	17 —	3
72 —	120 —	11	86 —	14 —	3
73 —	109 —	11	87 —	11 —	3
74 —	98 —	11	88 —	9 —	2
75 —	88 —	10	89 —	7 —	2
76 —	78 —	10	90 —	5 —	2
77 —	68 —	10			
78 —	58 —	10		168	36
79 —	49 —	9			
80 —	41 —	8			
	840	101			

Dr. Halley's Table

CONTINUED.

33962 living from one year old and upwards.
1000 dead.

* This Table was taken out of Mr. Hoyle's Doctrine of Chances (page 71.) which leaves twenty living at 84 years of age.

The

The table for forty-three years (60.) was made in the proportion of 1000 *born*, on purpose to compare with *those* that precede it, and which are in the *same* proportion. Mr. *Simpson's* table is in proportion to 1280 *born*; and Dr. *Halley's* in proportion to 1000 living at *one year old*: no just comparison can be made between the number of decrements of each age by *inspection*, unless the *three* tables were in proportion to some *one number born*, or of any particular age.

Yet there is no need to draw the reader's attention from the principal objects to adjust these proportions *now*, because they will appear in the estimate of what each should pay for an annuity; which would always be the *same*, whether it was calculated from *half*, or *double*, the number born.

But it may not be useless to remark, that the first table having 759, the second 870, and the third 1000 living at one year old; the decrements should exceed each other in the *same proportion*, and which they do in the *general total*; but not in *particular ages*, for the first has ten and eleven decrements from the 35th to the 46th year, while the second, though a greater number born, has not so many as ten, but from 40 to 43.

The reason is, that Mr. *Simpson* supposes above *half* of the born to die *before three* years old; consequently does not leave *half* to die *after* that age; whereas the first table has *half* its number to die *after nineteen*. (60.)

But

But this does not prove Mr. *Simpson* to have made a *wrong* estimate; on the contrary, it may perhaps appear, that, whenever a more accurate table shall be framed hereafter, near *one half* of those born do die under *two* years of age.

The reader has now seen, and may have examined carefully, the *second Data*; and it is presumed he scarce needs being told, that if persons die *faster* than has been calculated, the annuities might be purchased *cheaper*; but if they do not die so *fast*, the annuities cannot be purchased *so cheap* as is hereafter estimated.

LXIV. That the managers of societies, and others who have the *like* opportunity of making *practical* observations, may form a near judgment whether the tables give too little or too much, according to the *experience* had from what has happened to their *own members*; let them take notice that Mr. *Simpson* in his table allows 62 to die every year, out of 4891 living at all ages between 10 and 20; which is about *one* in .79. He allows 79 to die yearly out of 4213 of *all* ages between 20 and 30; which is about *one* in 53. He allows 92 to die yearly out of 3354, of *all* ages between 30 and 40; which is about one in $36\frac{1}{2}$. He allows 88 to die yearly out of 2424, of *all* ages between 40 and 50, which is about *one* in $27\frac{1}{2}$: and he allows 73 to die yearly out of 1621, of *all* ages between 50 and 60, which

is

ciii **PROBABILITY of LIFE.**

is about one in 22. 'Also, he allows 59 to die yearly out of 960, of all ages between 60 or 70, which is about *one* in 16. Let them examine whether more or less than these proportions *have actually died* among their own members, and let them judge accordingly (g).

Though the tables are deduced from the *experience* of past time; yet *they*, or the *best* that can be constructed, may very possibly differ from the experience of future time: by reasoning from what *has past*, a probable conjecture may be given of what *shall* come to pass; but what is expected to be, is not always *certain* of happening in this case.

LXV. Therefore the tables will always require a careful *regulation*, by which the *fund* of a society may be *secured*, and the variations in the annuities will be *lest*, or *more*, as the table agrees with the experience made from time to time. For an Example, Let an annuity be 50*l*. Of 294 members aged 40, it has been supposed by Mr. *Simpson* that 284 only will live to receive the first year, for 10 have *died*: but it shall happen in *practice* that *five* only have died

(g) As the Tables are printed they mean that opposite each age so many *are living* now, and so many *have died* within the year: but the above proportions require each decrement to be removed, or *supposed*, one place *higher*; as age 1, living 870, *will die* 170; age 2, living 700, *will die* 65; total living aged 1 and 2 is 1570; total *will die* 235 in Mr. *Simpson's* Table.

died in the year, and that therefore there are 289 who live to claim annuities.

If the fund was to pay these five *unexpected* claimants, it would be 250*£*. poorer than it should be to supply future claims, and must therefore prove deficient: but equity does not require that the fund should pay these extraordinary survivors, but rather that all of them should receive somewhat less than the expected annuity; for the annuity was granted on supposition, and on condition, that they would die in the proportion that the table shews: but if they do *not* die in such proportion, let them have the annuities that were calculated for 284 to receive; and let that sum be divided among the survivors, whether they are more or less than that number. 284 members at 50*£*. each,

$$284 \times 50 = 14200\text{£}.$$

Let the *£*. 14200, which the society, or the fund pays, be divided among 289 survivors.

$$\text{£. } 14200 \div 289 = \text{£. } 49.1349$$

Each recipient would *not* receive 18*s*. less than his expected 50*£*. and the fund continue its due stock. Or, if *more* should die than was expected, and leave but 279 claimants at the said age, let the same sum for annuities be paid, and let it be divided by the 279 survivors.

cxiv PROBABILITY OF LIFE.

✓ £. 14200 ÷ 279 = £. 50.892 each.

Here the annuitants gain more than 17s. *above* their expectation; by which it appears that in a great number of members, if *reality* differs from *supposition* even *one half*, yet it will affect the annuities but in a trifling degree; and the fund (*by this means*) not at all.

LXVI. The principal difficulty will be to keep a clear and accurate account of the several deviations, which must be *brought forward* and adjusted every payment; for when the fund has paid 284 the first year, it can pay 274 the second year: but granting that just ten members according to the table have died this year, still the five survivors of the last year (if none of *them* have died) will be claimants, and must be remembered, and accounted among the recipients of the *present* payment: in a course of years this may become a little complicated; but the difficulty may not be too great for the abilities of such as the managers would *no doubt* employ to inspect their affairs, and adjust the proportions at each payment.

It is not to be expected that the most *perfect* table would at *all* times, and at all ages, agree *exactly* with the number of decrements that *shall* happen (64.); but it is seen that *one* which shall *nearly* agree, will sufficiently
answer

answer the purpose of securing a proper fund, and giving the proportions of the annuities.

LXVII. The author has considered life as ended before the age of 95 in the *three* tables, in compliance with what Mr. Smart and Mr. J. P. had done before; for he chuses to rest upon the authority of others, in this respect, rather than his own; supposing it will have greater weight with the reader. If he might presume to make any alterations or amendments, *one* would be to extend the tables *beyond* 100 years; for, of the 1015142 buried (59.) in 43 years, 266 lived to be upward of 100 years old, and 11 of them to be 110 years old, and one of them in the year 1739 (perhaps Margaret Pattens, of St. Margaret, Westminster) to be aged 138. The tables, by supposing *all* dead before the age 95, leave no provision for these *extraordinary* ages. It is true that the chance of attaining to the age of 138 is great, so *very* great, as 1015142, and more, to one; but, that one in 1000 shall live to be 100 (*g*) years old, is (the decimal fraction) .262, that is, out of 3817 (*b*) persons born, *one* will attain the age of

100

(*g*) Viz. If 1015142 born (59.) require 266 survivors of 100 years, what will 1000 born require?

As 1015142 : 266 :: 1000 : .262

Born

(*b*) 266)1015142(3816

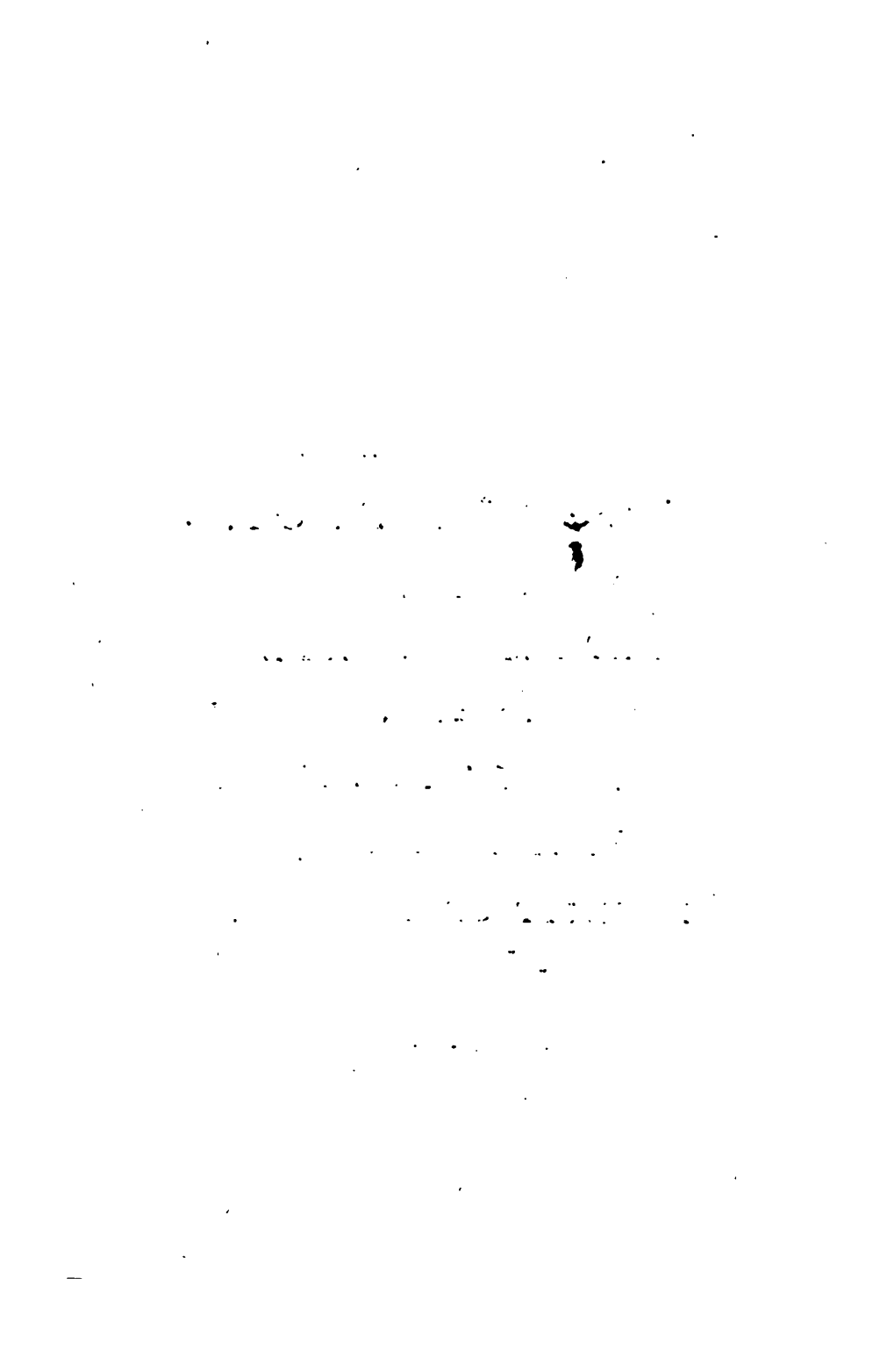
1000 (*i*). Whether the 1000 which the tables suppose born, be made 4000, and so to leave *one* living at the age of 100; or the 1000 be still supposed, and the *fourth* of a person live to the age of 100, is the *same* thing in *calculation*; though *wit* may rally, with some mirth, the supposition of dividing a person into 1000 parts, and preserving .262 of them alive (*k*).

The continuation of life to 100 years, would make an *addition* to a *capital fund* raised to pay *all* ages during life, and would be some *small* addition to the value of *every* age throughout, but especially to those of above 90, which must be as much more as the probability of attaining to 100 years is worth more than the *certainty* of dying before aged 95. This observation is made to illustrate another of a similar kind, that will be made hereafter; but the present calculations will *still* suppose the *end* of the lives of *all* to be *before ninety-five*.

(*i*) In page 147. of the quarto volume above mentioned (50.) Corbyn Morris, Esq. found that of every 3100 persons, *one* lived to be 120 years of age in the 30 years from A. D. 1728 to 1757 inclusive.

(*k*) The *annual* decrements are given in *whole* numbers; but perhaps a juster proportion would be, to allow somewhat *more* than *ten* to die at the age of 30, and somewhat *less* than *ten* to die at 35; because the deaths from 15 years old to forty gradually *increase*: also, somewhat *more* than *ten* to die aged 43, and somewhat *less* than *ten* when aged 48; because the burials *decrease* from the age of 42 to the end of life, in the Table 60.—But this may be thought a minute exactness not worth regarding.

THE GENERAL
CALCULATIONS
;
OF THE
PRESENT STATE
OF EACH
PARTICULAR SOCIETY
INSTITUTED FOR THE
BENEFIT OF OLD AGE.



CALCULATIONS

FOR THE USE OF THE

S O C I E T I E S

INSTITUTED FOR THE

BENEFIT OF OLD-AGE.

LXVIII. **W**ITH the help of the preceding Data any question, respecting annuities depending on life, may be solved, *according to the rate of these Data.* If different interest or different proportions of death be given, no doubt the answers *must* be different; but with such questions as have *other rates*, these calculations have nothing to do.

To apply the tables to practice, let it be required to know, What is the *present* value of 2*l.* immediate annuity, by half-yearly payments, for the age of *forty*; to continue 2 years, and then to *cease*?

A a

If

If the annuitant *lives*, he will receive *four* payments of 1 *l.* each ; but they being received time *hence*, are not worth 4 *l.* in *present* money. The Table (47.) shews the value of the First is

_____ .9828009

The Second is _____ .965898

The Third is _____ .949285

The Fourth is _____ .932959

The Table (49.) shews the value }
of the *whole* to be } 3.8309429

As the life is not *certain* of continuing, but may *possibly* drop before the first payment, the *proportionable chance* must be taken into consideration.

The Table (62.) shews, that of 294 persons living at forty, 289 *only* would be living six months thence; *only* 284 living a year thence; 279 *only* 18 months thence; and 274 living two years thence ; so that, if instead of *one* single annuitant, there were 294 aged 40, the above numbers *only* would remain *living* to receive the several payments.

There would be 289 recipients the *first* half year, and the present value of each 1 *l.* that they would receive is only .9828009, and the value in *present* money of the whole sum that *they* would receive is .9828009 multiplied by the 289 recipients, which produces

$$289 \times .9828009 = \text{£ } 284.0294601$$

Here

Here it is very plain that £284. 0 7½ only is the present worth of the 294*l.* which *so many* persons expect to receive among them the first half year, or for the first payment.

The second half year would have 284 living to be recipients, and the *1l.* is worth *less* in present money than the *first* was, because it is not to be received till a *year* hence, and the value is only .965898; which being multiplied by the number of recipients, *as before*, produces

$284 \times .965898 = 274.315032$,
which is the present value of the *second* 294*l.* which *as many* persons expect to receive.

As the *second* payment is worth less than the first, so the *third* is worth less than the second; its present value being only the 279 recipients, that would then be living, multiply'd into .949285, which is the value of *1l.* to be receiv'd 18 months hence, and produces

$$279 \times 949285 = 264.850515.$$

And, the fourth payment being done in the same manner, the whole will stand thus: !

Age of 40, Living 294 Persons,

—40½—	289	×	9828009	=	£. 284.02946c1
—41—	284	×	965898	=	274.315032
—41½—	279	×	949285	=	264.850515
—42—	274	×	932959	=	255.630766

To actually receive 1126*l.* Discounted for £1078 8257731.

If the 294 persons were *certain* to live, and receive the said annuities, and if money to be paid time *hence* was of *equal* value with *present* money, then the said annuities would be worth in present money the 4*l.* to be received, multiplied by the 294 recipients,

$$294 \times 4 = \text{£}1176.$$

But it is certain persons may die, and the Table (62.) supposes they will die in the given proportion there seen; and it is as certain that money paid time *hence* is not of equal value with *present* money, if interest can be made of it, but must *decrease* in its *present* value according as the distance of time in which it is to be paid *increases*: these are the reasons that the stock which the 294 annuitants might expect to receive being ———— ———— $\text{£}1176$

Is worth in present money only 1078.825773

The discount to be allowed }
for $\text{£}1176$ is ———— } $\therefore 97.174227$

Let the value of the whole be divided by the number of persons now living at 40, which is 294, and the money that each person should pay, to receive at that age 2*l.* annuity for two years only, will be found to be $\text{£}3.669475$.

$294)1078.825773(3.669475$
which is the answer required.

LXIX. The answer to this little question being *thoroughly* understood, the answer to the next question will be as plain.

What is the present value of 2 l. *immediate* annuity for the age of forty by *half-yearly* payments, to continue *ten* years, and then to cease?

The value of the *first four* payments were seen in the answer to the last question, and that same method must be continued for *ten* years, that is, for twenty *half* years.

LXX. Here it may be observed, that *half-yearly* payments require to have the tables of mortality divided into *half-yearly* decrements or deaths; and though men do not die in *fractions*, yet calculation may suppose $259\frac{1}{2}$ to live to receive the seventh payment, and $250\frac{1}{2}$ to receive the ninth; and so of any other age, as in the *second* column of the following table, in which the number of survivors at every half year is multiplied into the reversion that corresponds to the said half year; by which is seen the value of each half year's payment in present money, and the *last* found to be worth *only* £144.192096. — All these values *added* together is the total value of what will be received; which *total* being *divided* by the *number living* at 40 years old, that is 294, the *quotient* is each person's share to pay in *present* money.

LXXI. A Table to shew the Value of 2*l*. immediate Annuity by Half Yearly Payments for the Age of 40, to continue till aged 50, and then to cease; by *Mr. Simpson's* Table of Mortality.

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Value of Payments</i>	<i>Totals.</i>
40	294	(47)	to Survivors.	
40½	289	X.982801	=284.029489	
41	284	X.965398	=274.315032	
41½	279	X.949285	=264.850515	
42	274	X.932959	=255.630766	
42½	269	X.916912	=246.649328	
43	264	X.901143	=237.901752	1325.47513
43½	259½	X.885644	=229.824618	
44	255	X.870412	=221.95506	
44½	250½	X.855441	=214.289705	
45	246	X.840728	=206.819088	
45½	241½	X.826269	=199.5439635	1110.788488
46	237	X.812058	=192.457746	
46½	232½	X.798091	=185.5561575	
47	228	X.784364	=178.834992	
47½	224	X.770875	=172.676	
48	220	X.757616	=166.67552	929.068859
48½	216	X.744586	=160.830576	
49	212	X.73178	=155.13736	
49½	208	X.719194	=149.592352	
50	204	X.706824	=144.192096	

Total 4893 *l.* paid or received.

776.427904

Total of the 20 payments 4144.7603815

The Table shews, that the value of 4893*l.* which is the total of the pounds that will be received, and also is the total of the number of recipients, is only £4141.7603815 in *present* money; which being divided by

294 present members, gives for *each* person's share £14.08762 to pay in *present* money for 2 *l.* annuity for ten years.

$$294)4141.7603815(14.0876203.$$

LXXII. If the truth of this answer is doubted, it may, *now the sum is obtained*, be proved in another manner, by supposing that 294 persons, each aged 40, agree to deposite each, in present money, the sum of £14.0876203, to establish a fund of the amount of 4141.7603815; out of which every *surviving* member shall receive 1 *l.* every *half year*, as long as there is any of the stock remaining.

The product of the sum that *each* pays, multiplied by the number of members, must be the same as the dividend above,

$$14.0876203 \times 294 = 4141.76038.$$

The whole stock is 4141.76 *l.* which will gain six months interest, and then pay 289 *l.* to as many recipients who will be then living out of the 294 first establishers; there will be then remaining for stock £3925.24 to gain another six months interest, and then pay 284 *l.* to as many recipients; and so on every half year, as in the following table.

$\frac{1}{2}$ Year's

The Joint Capital Stock of 294 Persons, paying 14.08762 each,

<i>will be</i>	4141.76.		
$\frac{1}{2}$ Yrs. Int.	72.48 (38)	to pay	3340.48
			269
Total	4214.24	Age 42 $\frac{1}{2}$	3071.48
To pay	289	Interest	53.75
Age 40 $\frac{1}{2}$	3925.24	to pay	3125.23
Interest	68.69		264
	3993.93	Age 43	2861.23
to pay	284.	Interest	50.07
Age 41	3709.93	to pay	2911.3
Interest	64.92		259.5
	3774.85	Age 43 $\frac{1}{2}$	2651.8
to pay	279	Interest	46.41 +
			2698.21
Age 41 $\frac{1}{2}$	3495.85	to pay	255
Interest	61.18 +		
		Age 44	2443.21
		Interest	42.76 +
	3557.03		2485.97
to pay	274	to pay	250.5
Age 42	3283.03	Age 44 $\frac{1}{2}$	2235.47
Interest	57.45	Interest	39.12
	3340.48		2274.59
			to

(9)

to pay	2274.59 246	to pay	1231.21 224
Age 45 Interest	2028.59 35.5	Age 47½ Interest	1007.21 17.63 +
to pay	2064.09 241.5	to pay	1024.84 220
Age 45½ Interest	1822.59 31.9 +	Age 48 Interest	804.84 14.08
to pay	1854.49 237	to pay	818.92 216
Age 46 Interest	1617.49 28.31 +	Age 48½ Interest	602.92 10.55
to pay	1645.8 232.5	to pay	613.47 212
Age 46½ Interest	1413.3 24.73	Age 49 Interest	401.47 7.03
to pay	1438.03 228	to pay	408.5 208
Age 47 Interest	1210.03 21.18	Age 49½ Interest	200.5 3.5
	1231.21	to pay	204.0 204
	B b	Age 50	000 } remain- ing or deficient. Here

Here the *half-yearly* interests have been duly *added* to the stock, and the *survivors* of the annuitants have been constantly paid 1*l.* as it became due; and when the 20th payment has been made, the fund is found to be *totally* exhausted; so that there remains *nothing*; which must prove, that the stock of £4141.76 is the precise sum that will pay 294 persons aged 40 the annuity of 2*l.* for *ten* years, and no longer: therefore *that* stock divided by 294, shews the share of *each* person to pay for the said annuity.

LXXIII. By the first method (71), which is that of finding *what stock* would supply the conditions of the question, reversion took off the interest of all the sums, according to their distance from the present value: by this last method, those interests are all *restored*, as each half-year becomes present time; so that the two Tables perfectly agree in respect to interest, and the number of recipients are supposed to be the same in both,

LXXIV. £14.08762 is then *incontroversibly* the just value in present money for 2*l.* annuity on the aforesaid conditions; the value of 1*l.* annuity must then be the *half* of that sum, which is £7.04381.

LXXV. The value of 1*l.* annuity is found, because it is more *generally* useful than any other; for the value of *any* annuity whatever, on the *same* conditions, is readily found by multiplying it into this value of 1*l.* (37.).

What

What is the value of £6.7 annuity on the same conditions ?

$$£7.04381 \times 6.7 = \text{Answ. } £47.193527.$$

LXXVI. The same annuity *certain*, that is, by reversion only, and not taking the chance of life into consideration, is found by the table (49), where 20 *half-years* is £16.75288, the value of 2l. annuity; which must be multiplied by the *half* of £6.7; or otherwise £6.7 must be multiplied by the half of the value of 2l. either of which gives the product £56.122148.

The sum actually paid and received in the 20 half-years would be £67; but the value of it in present money is reduced by *reversion* to £56.122148; and that sum, when dependent on life, is farther reduced by the *bazard* of it to £47.193527.

LXXVII. The value of an annuity must be the same, whether A pays it to B, or B pays it to A; therefore, suppose B to be a Society to which A pays half-yearly, from the age of 40 until aged 50, the sum of £3.7, which is decimally £3.35 (the half of £6.7, which is the yearly sum) the value of *all the payments* which A makes to the Society in that case would be £47.193527 in *present* money; and this sum, with what other present money that A should pay, would be the *whole* of his stock in *present* money. The same method

will find the value of *any* age, and for *any* sum; of which here follow some examples.

N. B. *All questions in this Essay are supposed to be by the rates of half-yearly payments, $3\frac{1}{2}$ l. per Cent. interest, and Mr. Simpson's Table of Mortality; excepting when expressed otherwise.*

LXXVIII. What is the worth in *present money* of 1 l. *immediate annuity*, for the age of *ten*, to continue till aged 50, and then to *cease*?

It must be found by the Table (62.) how many are *living* at the age of *Ten*, and the number *surviving* of those to receive the first half year must be multiplied into the first half year's reversion: likewise the number of those surviving the second half year must be multiplied into the second half year's reversion, so of the third half year, and of every one, till the age of *Ten* attains to 50; and the total of all the *products* will pay the number of those alive at *ten* 2 l. annuity; the *half* of which sum will pay 1 l. annuity on the said conditions; so that the next Table is formed in the *same* manner as the *last*.

LXXIX.

LXXIX. A Table to shew the Value of
2l. immediate Annuity for the Age of Ten to
continue Forty years, and then to cease.

<i>Age</i>		<i>524 Living at Ten Years of Age.</i>	
10½	520½	x.9828009	=511.54786845
11	517	x.965898	=499.369266
11½	513½	x.949285	=487.4578475
12	510	x.932959	=475.80909
12½	507	x.916912	=464.874384
13	504	x.901143	=454.176072
13½	501	x.885644	=443.707644
14	498	x.870412	=433.465176
14½	495	x.855441	=423.443295
15	492	x.840728	=413.638176
15½	489	x.826269	=404.045541
16	486	x.812058	=394.660188
16½	483	x.798091	=385.477953
17	480	x.784364	=376.49472
17½	477	x.770875	=367.707375
18	474	x.757616	=359.109984
18½	471	x.744586	=350.700006
19	468	x.73178	=342.47304
19½	465	x.719194	=334.42521
20	462	x.706824	=326.552688
		<u>£1713.260928</u>	

The Total of the Twenty Payments £8249.13552395

$$8249.13552395 \div 524 = 15.74262504 \div 2 = 7.87131252$$

Age.

<i>Age.</i>		<i>462 living at Twenty Years of Age</i>	
20½	458½	x .694668	= 318.505278
21	455	x .682721	= 310.638055
21½	451½	x .670978	= 302.946567
22	448	x .659438	= 295.428224
22½	444½	x .648096	= 288.078672
23	441	x .636949	= 280.894509
23½	437½	x .625995	= 273.8728125
24	434	x .615228	= 267.008952
24½	430	x .604647	= 259.99821
25	426	x .594248	= 253.149648
25½	422	x .584027	= 246.459394
26	418	x .573982	= 239.924476
26½	414	x .564111	= 233.541954
27	410	x .554408	= 227.30728
27½	406	x .544873	= 221.218438
28	402	x .535502	= 215.271804
28½	398	x .526292	= 209.464216
29	394	x .51724	= 203.79256
29½	389½	x .508344	= 197.999988
30	385	x .499601	= 192.346385

£1018.874953

Total of the last Twenty Payments £5037.847425

Total of the Forty Payments — £15286.98294645

$$5037.847425 \div 524 = 9.6142126$$

$$15286.98294645 \div 524 = 29.3568376$$

<i>Age.</i>		<i>385 living at Thirty Years of Age.</i>	
30½	380½	X .491007	= 186.8281635
31	376	X .482563	= 181.443688
31½	371½	X .474263	= 176.1887045
32	367	X .466107	= 171.061269
32½	362½	X .45809	= 166.057625
33	358	X .450212	= 161.175896
33½	353½	X .442468	= 156.412438
34	349	X .434858	= 151.765442
34½	344½	X .427379	= 147.2320655
35	340	X .420028	= 142.80952
35½	335½	X .412804	= 138.495742
36	331	X .405705	= 134.288355
36½	326½	X .398727	= 130.1843655
37	322	X .391869	= 126.181818
37½	317½	X .385129	= 122.2784575
38	313	X .378505	= 118.472065
38½	308½	X .371995	= 114.7604575
39	304	X .365597	= 111.141488
39½	299	X .35931	= 107.43369
40	294	X .35313	= 103.82022

£555.6279205

The total of the *last* Twenty payments 2848.03147

Total of the *last* Forty payments 7885.8788925

Total of the Sixty payments £16135.01441645

$$\begin{aligned}
 2848.03147 & \div 524 = 5.43517 \\
 7885.8788925 & \div 524 = 15.0493872 \\
 16135.01441645 & \div 524 = 30.79201224
 \end{aligned}$$

Age

Age. 294 living at Forty Years of Age.

	289	X	.347057	=	100.299473	
41	284	X	.341087	=	96.868708	
	279	X	.335222	=	93.526938	
42	274	X	.329455	=	90.27067	
	269	X	.323789	=	87.099241	
43	264	X	.318221	=	84.010344	£468.06503
	259½	X	.312747	=	81.1378465	
44	255	X	.307368	=	78.37884	
	250½	X	.302082	=	75.671041	
45	246	X	.296886	=	73.033956	
	241½	X	.291781	=	70.4651115	£392.2520275
46	237	X	.286762	=	67.962594	
	232½	X	.28183	=	65.525475	
47	228	X	.276983	=	63.152124	
	224	X	.272219	=	60.977056	
48	220	X	.267538	=	58.85836	£328.0823605
	216	X	.262673	=	56.737368	
49	212	X	.258414	=	54.783768	
	208	X	.253969	=	52.825552	
50	204	X	.249601	=	50.918604	

£274.123652

The total of the *last* Twenty payments £1462.52307

Total of the *last* Forty payments ——— £4310.55454

Total of the *last* Sixty payments ——— £9348.4019625

Total of the Eighty payments ——— £17597.53748645

$$1462.52307 \div 524 = 2.791074$$

$$4310.55454 \div 524 = 8.22624$$

$$9348.4019625 \div 524 = 17.8404617$$

$$17597.53748645 \div 524 = 33.5830868 \div 2 = 167915434$$

The total of 40 years or 80 payments is £17597.53748645, which being divided by the 524 persons living when ten years old, gives the quotient 33.5836868 for the share of each to pay in *present* money for 2*l.* annuity, that is 1*l.* each half year; and that sum divided by 2 gives £16.7915434 for the value of 1*l.* annuity; which is the answer to the question.

The same Table will give answers to all questions of the same kind, for the same age, for any time within 40 years; of which the three following are examples.

LXXX. What is the value in present money of 1 *l.* *immediate* annuity for the age of *Ten*, to continue till aged 20, and then to *cease*?

By the question, the age of *Ten* will receive the annuity, if *he survives, ten years, and no longer*; therefore,

$$8249.13552395 \div 524 = 15.74262504$$

$$\text{and } 15.74262504 \div 2 = 7.87131252$$

that is, the total of the *first* 20 payments, being 8249.13552395, is divided by the 524 persons aged ten; and the quotient being for 2*l.* annuity, is again divided by 2, which gives £7.87131252 for answer to the question.

LXXXI. What is the value of 1*l.* *immediate* annuity for the age of Ten, to continue till aged 30, and then to *cease*?

By the same Table the amount of the first 20 years, that is, 40 half-yearly payments, is 13286.98294645; which being divided by the 524 aged Ten, gives 25.35683768, the half of which is 12.67841884; for the answer.

LXXXII. What is the value in *present* money of 1*l.* *immediate* annuity for the age of Ten, to continue till aged 40, and then to *cease*?

By this question the age Ten will, *if living*, receive the first 60 half-yearly payments in the Table, and no more; the total of which is, 16135.01441645 divided by 524; the quotient is equal to 30.79201224, the half of which is £15.39600612 for answer to the question.

LXXXIII. It will be useful to remark here, that the value of the fourscore payments for 1*l.* annuity is in *present* money; for the age Ten being £16.7915434 (79), if from that sum the value of the first 60 payments be deducted, the remainder would shew the value of the *last 20 payments alone*.

The value of the 80 pay- ments	}	16.7915434
Deduct the value of the first 60 payments	}	15.39600612
Value in <i>present</i> money of the last 20 payments	}	1.39553738

LXXXIV.

LXXXIV. What is the worth in *present* money of 1*l.* annuity for the age of *Ten*, to *commence* when aged 40, to continue till aged 50, and then to *cease* ?

By this question the age of *Ten* has nothing to do with those payments that are made in the last Table, till he becomes *aged 40 years and a half*, where the number of survivors are multiplied into the 61st. half-years reversion (for it would be so long before the age became recipient), and the same is continued for 10 years ; the survivors each half-year being multiplied into their *proper* reversion ; the total of all which payments is £1462.52307, which sum divided by 524 aged *Ten*, gives 2.79107 for the quotient for 2*l.* annuity, the half of which is £1.39553 for 1*l.* and is the same answer as just before given (83). In like manner the three next questions may be answered *two* ways.

LXXXV. What is the worth in *present* money of 1*l.* annuity for the age of *Ten*, to *commence* when aged *Thirty*, to continue till aged *Fifty*, and then to *cease* ?

The total of the last *Forty* payments in the last Table, is £4310.55454, which being divided by the 524 aged *Ten*, gives £8.22624, the *half* of which is £4.11312 for 1*l.* annuity. Or, from the value of

The immediate annuity } £16.7915434
till aged Fifty ———

Deduct the value of the } £12.67841884
first Forty payments ———

The answer is (*the same*) } £4.11312456
as before ———

LXXXVI. What is the worth, in *present* money, of 1 *l.* annuity for the age of *Ten*, to *commence* when aged *Twenty*, to continue till aged *Fifty*, and then to *cease*?

The total of the last Sixty payments which the age, *if it attained Fifty*, would receive, is, £9348.4019625, which being divided by the 524 aged *Ten*, gives 17.8404617, the half of which is £8.92023085 for 1 *l.* annuity. Or,

From the value of the } £16.7915434
whole 80 payments, ———
as before ———

Deduct the value of the } 7.87131252
first 20 payments ———

The answer is (*the same*) } 8.92023088
as before ———

If the age *Ten* was to receive 1 *l.* *immediate* annuity for 10 years only; or 1 *l.* annuity for 10 years only, and to *commence* when aged

aged Twenty, to *cease* when aged Thirty ; or 1 *l.* annuity for 10 years, to commence when aged Thirty, to *cease* when aged Forty ; or 1 *l.* annuity for 10 years, to commence when aged Forty, to *cease* when aged Fifty : What would be the proportion of the different values in present money ?

LXXXVII. The answer to the first (80.), and to the last (84.), has been given. The value of the said annuity from the age Twenty to Thirty may be found in the same manner. The total of those 20 payments being 5037.8474125 divided by 524, gives for 2 *l.* 9.6142126, the half of which is 4.8071063 for 1 *l.* annuity. Or, from the value of 1 *l.* immediate annuity, to *cease* when aged Thirty (81.), deduct the value of an immediate annuity, to *cease* when aged Twenty (80.).

Immediate annuity to	{	£12.67841884
<i>cease</i> when <i>Thirty</i> ,		
Ditto to <i>cease</i> when	{	£7.87131252
aged <i>Twenty</i> , —		
The answer is (<i>the same</i>)	{	£4.80710632
<i>as before</i>) —		

Likewise the total of the 20 payments from age *Thirty* to *Forty* is £2848.03147, which divided by 524 gives 5.43517, the half

half of which is £2.717585 for 1/. annuity. Or,

The immediate annuity to	}	15.39600612
cease when <i>Forty</i> (82.), is		
Ditto to cease when <i>Thirty</i>	}	12.67841884
(81.)		
The answer is, <i>the same as</i>	}	2.71758728
<i>before,</i> ———		

LXXXVIII. Thus the values are found to be for the age of *Ten* to *commence* annuitant when

Aged 40 to cease when 50	£1.39553
Ditto 30 ditto ——— 40	£2.71758728
Ditto 20 ditto ——— 30	£4.8071063
Ditto 10 ditto ——— 20	£7.87131252

Value of an immediate annuity till 50, is } 16.7915361 (78.)

LXXXIX. The Reader may now observe the difference between, and discern the reason why, an annuity received the 10 years *immediately* succeeding the purchase, is worth near six times the money as the *same* annuity for 10 years, *not* to be received till aged *Forty*, and also the proportions of any other 10 years in the time.

XC. The value of 1 *l.* annuity for the age of Ten to receive from Forty till Fifty years old might have been found by the First Table (71.), where the age Forty receives an immediate annuity till aged Fifty ; for the *same number* of survivors for the *same ages*, are in both Tables Recipients. The difference of the *total sums* arises from the last Table having multiplied the ages into Sixty-one, Sixty-two, Sixty-three, &c. half-years reversion (because the age of Ten would not become a Recipient of the several sums till after the expiration of such a number of *half-years*;) whereas the age Forty in the first Table (71.), multiplies the number of survivors into the first, second, third, &c. half-years reversion, which are of much greater value, because the annuity commences immediately. To find by the *first* Table the stock that would pay the age of Ten 1 *l.* annuity from the age of Forty to Fifty, take the *Total* stock that paid the age Forty an immediate annuity, which was (71.) £4141.76 ; multiply it into 60 half-years reversion (47.), and the product will be £1462.5, to *one* place of decimals exactly the same as the total of the *last 20* payments in the Table (79.).

$$4141.76 \times 353130 = 1462.57970880.$$

XCI.

XCI. Also the sum that would pay those aged Forty an *immediate* annuity, might have been found from the *last* Table; for taking the total of the last 20 payments, and *dividing* it by 60 half-years reversion, must yield the quotient nearly to 4141.76 (71 and 46.), which is the stock that was seen would pay that age.

$$.35313) 1462.52307 (4141.59$$

Likewise the same sum 1462.52307 *multiplied* by 2.831816, gives in the product 4141.59; which is being multiplied by *Compound Interest* (42*.).

XCH. What is the value in *present* money of 1 l. *immediate* annuity, for the age of *Twenty*, to *cease* when aged *Fifty*?

The last Table (79.) contains the number of Recipients from the age of Twenty to Fifty. They are multiplied into 20 half-years reversion *too much* for to answer this question; but if the total sum be either *divided* by 20 half-years reversion (47.), or *multiplied* into 20 half-years Compound Interest (42*), the loss would be regained, and the sum become equal to that which would pay 2 l. annuity immediately to the age of Twenty.

Thus (47.),

$$.706824) 9348.4019625 (13225.9 \text{ or,} \\ 9348.4019625 \times 1414778 = 13225.9, \&c. (42^*) \\ \text{which}$$

which sum of £13225.9 divided by 462 persons living at Twenty years of age, gives 28.6275 for 2 *l.* annuity, which is £14.3137 for 1 *l.*

XCIII. But lest this method should be found perplexing, or not *so* satisfactory as a *Table* for the Age, here follows one, shewing the value, in *present* money, of every payment the age of Twenty would receive by *immediate* annuity.

D d

A Table

A Table, shewing the Value, in present Money, of 2 l. immediate Annuity for the Age of Twenty till the Age of Fifty, and then to cease.

<i>Age.</i> 462 <i>Living at the Age of Twenty.</i>				
20½	458½	×	982801	= 450.6142585
21	455	×	965898	= 439.48359
21½	451½	×	949285	= 428.6021775
22	448	×	932959	= 417.965632
22½	444½	×	916912	= 407.567384
23	441	×	901143	= 397.494063
23½	437½	×	885644	= 387.46925
24	434	×	870412	= 377.758808
24½	430	×	855441	= 367.83963
25	426	×	840728	= 358.150128
25½	422	×	826269	= 348.685518
26	418	×	812058	= 339.440244
26½	414	×	798091	= 330.409674
27	410	×	784364	= 321.58924
27½	406	×	770875	= 312.97525
28	402	×	757616	= 304.561632
28½	398	×	744586	= 296.345228
29	394	×	73178	= 288.32132
29½	389½	×	719194	= 280.126063
30	385	×	706824	= 272.12724
				£1441 481483
The Total of the Twenty half Years				7127.43633

$$7127.43633 \div 462 = 15.42735 \div 2 = 7.713675$$

Age.

<i>Age.</i> 385 <i>living at the Age of Thirty.</i>		
30½	380½ × 694668	= 264.321174
31	376 × 682721	= 256.703096
31½	371½ × 670978	= 249.268327
32	367 × 659438	= 242.013746
32½	362½ × 648096	= 234.9348
33	358 × 636949	= 228.027742
33½	353½ × 625995	= 221.2892325
34	349 × 615228	= 214.714572
34½	344½ × 604647	= 208.3008915
35	340 × 594248	= 202.04432
35½	335½ × 584027	= 195.9410585
36	331 × 573982	= 189.988042
36½	326½ × 564111	= 184.1822415
37	322 × 554408	= 178.519376
37½	317½ × 544873	= 172.9971775
38	313 × 535502	= 167.618126
38½	308½ × 526292	= 162.361082
39	304 × 51724	= 157.24096
39½	299 × 508344	= 151.994856
40	294 × 499601	= 146.882694

£786 091718

Total of 20 Half Years, viz. from } — 4029.3375145
 Thirty to Forty ———— }

Total of 40 Half Years, viz. from } — 11156.7738445
 Twenty to Forty ———— }

2 l. Annuity for the Age Twenty, to receive when aged
 Thirty, until aged Forty.

$$4029.3375145 \div 462 = 8.7215098$$

Or, the same Age to receive *immediate*, until aged Forty,

$$11156.7738445 \div 462 = 24.1488611$$

<i>Age.</i>		<i>294 living at the Age of Forty.</i>	
40½	289	X 491007	= 141.901023
41	284	X 482563	= 137.047892
41½	279	X 474263	= 132.319377
42	274	X 466107	= 127.713318
42½	269	X 45809	= 123.22621
43	264	X 450212	= 118.855968
43½	259½	X 442468	= 114.820446
44	255	X 434858	= 110.88879
44½	250½	X 427379	= 107.0584395
45	246	X 420028	= 103.326888
45½	241½	X 412804	= 99.692166
46	237	X 405705	= 96.152085
46½	232½	X 398727	= 92.7040275
47	228	X 391869	= 89.346132
47½	224	X 385129	= 86.268896
48	220	X 378505	= 83.2711
48½	216	X 371995	= 80.35092
49	212	X 365597	= 77.506564
49½	208	X 35931	= 74.73648
50	204	X 35313	= 72.03852

£387.903584

Total of the <i>last Twenty</i> Payments	2069.225242
Total of the <i>last Forty</i> ditto	6098.5627565
Total of the <i>last Sixty</i> ditto	— 13225.9990865

$$\begin{aligned}
 2069.225242 \div 462 &= 4.478842 \\
 6098.5627565 \div 462 &= 13.2003522 \\
 13225.9990865 \div 462 &= 28.6277036
 \end{aligned}$$

The amount of the several sums paid and received in the 60 half years, in this Table, is seen to be £13225.9990865; which being divided by the 462 living aged Twenty, gives the quotient 28.6277036, the half of which is £14.3138518, for 1 l. annuity, and is very nearly the same as before (92.).

This Table will answer any questions for the age *Twenty*, as the other (79) did for the age *Ten*.

XCIV. What is the value in *present* money of 1 l. *immediate* annuity, for the age *Twenty*, till aged *Thirty*, and then to *cease*?

At the Total of the *first* Twenty Payments that sum, 7127.43633, is divided by the 462 persons living at Twenty, and the quotient is £15.42735 for 2 l. the half of which is £7.713675, for 1 l. annuity.

XCV. What is the value, in *present* money, of 1 l. *immediate* annuity, for the age *Twenty*, until aged *Forty*, and then to *cease*?

The Total of the *first* Forty payments, in this Table, being £11156, is divided by the 462 persons living at the age *Twenty*; and the quotient being 24.1488611, for 2 l. the half of it is £12.07443055, for 1 l. annuity.

XCVI. What is the value in *present* money, of 1 l. annuity, for the age *Twenty*,
to

(30)

to *commence* when aged *Forty*, to *cease* when aged *Fifty*?

$$\text{Ans. } 2069.225242 \div 462 = 4.478842 \div 2 = \pounds 2.239421.$$

That is, the total of the *last* 20 payments in the Table (93) divided by 462 living aged *Twenty*; the quotient, being for 2 *l.* is divided by 2 for 1 *l.* annuity.

Or, deduct from the value of an *immediate*

$$\begin{array}{rcl} \text{Annuity, to cease when } \} & & \\ \text{Fifty (93.) } \text{---} \} & 14.3138518 & \\ \text{The annuity to commence } \} & & \\ \text{at Forty to cease at Fifty } \} & \pounds 12.0744305 & \end{array}$$

The Answer (*as before*) is $\pounds 2.2394213$

XCVII. What is the value, in *present* money, of 1 *l.* annuity for the age *Twenty*, to *commence* when aged *Thirty*, to *cease* when aged *Fifty*?

From the value of the immediate annuity (93.), till aged Fifty, deduct the value of the immediate annuity till aged *Thirty* (94.).

Value

(31)

Value of the immediate } annuity till Fifty, is }	14.3138518
Ditto, until aged Thirty, is	7.713675

The answer is	—————	<u>£6.6001768</u>
---------------	-------	-------------------

Or, divide the *Total* of the *last Forty* half years by 462, thus :

$$6098.5627565 \div 462 = 13.2003522 \div 2 = £6.6001768$$

XCVIII. What is the value, in *present* money, of 1 *l.* *immediate* annuity, for the age of *Thirty*, to continue till aged *Fifty*, and then to *cease* ?

This question might be answered by either of the two last Tables ; but as the method is more complicated (92.) than by a Table formed on purpose for the age, a *new* one is here given.

A Table

A Table, shewing the Value in present Money of 2l. immediate Annuity for the Age of Thirty, to receive till aged Fifty, and then to cease.

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Pres. Value of Pay.</i>	<i>Total of the</i>
30	385	(47.)	<i>to Survivors.</i>	<i>Products.</i>
30½	380½	× 982801	= 373.9557805	
31	376	× 965898	= 363.177648	
31½	371½	× 949285	= 352.6593775	
32	367	× 932959	= 342.395953	
32½	362½	× 916912	= 332.3806	
33	358	× 901143	= 322.609194	£1764.569359
33½	353½	× 885644	= 313.075154	
34	349	× 870412	= 303.773788	
34½	344½	× 855441	= 294.6994245	
35	340	× 840728	= 285.84752	
35½	335½	× 826269	= 277.2132495	£1520.0050805
36	331	× 812058	= 268.791198	
36½	326½	× 798091	= 260.5767115	
37	322	× 784364	= 252.565208	
37½	317½	× 770875	= 244.7528125	
38	313	× 757616	= 237.133808	£1303.8991795
38½	308½	× 744586	= 229.704781	
39	304	× 73178	= 222.46112	
39½	299½	× 719194	= 215.039006	
40	294	× 706824	= 207.806256	

£1112.145971

Total of the 20 Half Years, is

5700.62059

$$5700.62059 \div 385 = 14.8068 \div 2 = 7.4034$$

Age.

Age.	Living.	Reversion.	Pres. Val. of Pay.	Total of the
40	294	(47.)	to Survivors.	Payments.
40½	289	X .694668	= 200.759052	
41	284	X .682721	= 193.892764	
41½	279	X .670978	= 187.202862	
42	274	X .659438	= 180.686012	
42½	269	X .648096	= 174.337824	
43	264	X .636949	= 168.154536	£936.878514
43½	259½	X .625995	= 162.4457025	
44	255	X .615228	= 156.88314	
44½	250½	X .604647	= 151.4640735	
45	246	X .594248	= 146.185008	
45½	241½	X .584027	= 141.0425205	£789.13246
46	237	X .573982	= 136.033734	
46½	232½	X .564111	= 131.1558075	
47	228	X .554408	= 126.405024	
47½	224	X .544873	= 122.051552	
48	220	X .535502	= 117.81044	£656.688638
48½	216	X .526292	= 113.679072	
49	212	X .51724	= 109.65488	
49½	208	X .508344	= 105.735552	
50	204	X .499601	= 101.918604	

11646½

£548.798548

Total of 20 Half Years, viz. from 40 to 50, 2927.49816

Total of 40 Half Years, viz. from 30 to 50, 8628.11875

$$£2927.49816 \div 385 = 7.6039 \div 2 = 3.80195$$

$$£8628.11875 \div 385 = 22.410698 \div 2 = 11.205349$$

E b

The

The total of the 40 half-yearly sums paid and received by this Table is 8628.11875; which being divided by the 385 persons living when aged Thirty, gives 22.410698, for 2 l. annuity, which is £11.205349, for 1 l. annuity, and answer to the question.

XCIX. What is the value, in *present* money, of 1 l. *immediate* annuity for the age *Thirty*, to continue till aged *Forty*, and then to *cease*?

The total of the first 20 half-yearly payments is 5700.62059; which, divided by 385, aged *Thirty*, gives 14.8068, for 2 l. which is 7.4034, for 1 l.

C. What is the value, in *present* money, of 1 l. annuity for the age *Thirty*, to commence when *Forty*, and to *cease* when aged *Fifty*?

The total of the 20 *last* payments is 2927.49816; which, divided by 385, aged *Thirty*, gives 7.6039 nearly, for 2 l. which is £3.80195, for 1 l.

Or, from the value of an *immediate* annuity for the age *Thirty*, to continue till aged *Fifty* (98.), deduct the value of 1 l. *immediate* annuity for the age *Thirty*, to *cease* when aged *Forty* (99.).

Immediate

Immediate annuity, to con-	}	11.205349
tinue till aged 50, is		
Ditto, to cease when aged 40,		7.4034

The answer is, (<i>the same as</i>	}	3.801949
<i>before,</i>) _____		

CI. What is the value of an *immediate* annuity of 1 l. for the age of *Sixty*, to continue till aged *Seventy*, and then to *cease*?

A new Table is necessary to answer this question, because the proportion of mortality between the ages *Sixty* and *Seventy* is not inserted in the preceding ones.

A Table to shew the Value, in present Money, of 1 l. immediate Annuity for the Age of Sixty, to continue Ten Years, and then to cease?

Age.	Living.	Reversion.	Pres. Val. of Pay.	Total of the
60.	130.	(47.)	to Survivors.	Products.
60½	126½	X .982801	= 124.3243265	
61	123	X .965898	= 118.805454	
61½	120	X .949285	= 113.9142	
62	117	X .932959	= 109.156203	
62½	114	X .916912	= 104.527968	
63	111	X .901143	= 100.026873	£570.7281515
63½	108	X .885644	= 95.649552	
64	105	X .870412	= 91.39326	
64½	102	X .855441	= 87.254982	
65	99	X .840728	= 83.232072	
65½	96	X .826269	= 79.321844	£457.556739
66	93	X .812058	= 75.521394	
66½	90	X .798091	= 71.82819	Total of 14 first
67	87	X .784364	= 68.239668	½ Years.
67½	84	X .770875	= 64.7535	£1323.1959665
68	81	X .757616	= 61.366896	
68½	78	X .744586	= 58.077708	
69	75	X .73178	= 54.8835	
69½	72	X .719194	= 51.781968	Total of 6 last ½
70	69	X .706824	= 48.770856	Years.
				£339.634428
Total of the Twenty payments				£1662.8303945

$$1662.8303945 \div 130 = 12.791003$$

$$12.791003 \div 2 = 6.3955015$$

The total value, in present money, of the 20 payments, is found to be £1662.8303945; and there being 130 persons living, aged Sixty, the sum must be divided by that number of persons, which gives 12.791003, for 2 l. which is £6.3955015, for 1 l. annuity.

CII. What is the value, in *present* money, of 1 l. *immediate* annuity for the age *Sixty*, to continue 7 Years, that is 14 half years, and then to *cease*?

The total of the *first* 14 payments *only*, must be taken from the Table, which is found to be £1323.1959665; which, divided by 130 persons living, aged Sixty, gives 10.1784305, the half of which is £5.08921525, for the answer.

CIII. What is the value, in *present* money, of 1 l. annuity for the age *Sixty*, to commence when aged *Sixty-seven*, to continue till aged *Seventy*, and then to *cease*?

By this question the 6 *last* payments *only*, would be received; the total of which is 339.634428; which, divided by 130, gives 2.612572 for 2 l. and 1.306286 for 1 l. annuity. If the two values, *last* found, are added together, the total will be nearly as the *first* value of 10 years *immediate* annuity (101.).

14 Half-years immediate annuity, worth	}	5.08921525
6 Ditto, to receive from the age 67 until 70	}	1.306286

Value of 10 years immediate annuity, from the age of 60 to 70	}	6.39550125
---	---	------------

CIV. What is the value, in *present* money, of 30*l.* annuity for the age *Twenty*, to commence when aged *Forty-seven*, to continue 3 years, that is, 6 payments, and then to *cease*?

By this question the age *Twenty* would receive only the *six last* half years in the Table (93.).

The total of the <i>last five</i> payments is	}	387.903584
The value of the payments opposite 47½, is	}	86.268896

The total of the <i>six</i> payments is	}	474.172480
---	---	------------

Which, being divided by the 462 living aged *Twenty*, gives 1.0263474 for 2*l.* or .5131737 for 1*l.* annuity; and .5131737, multiplied by the 30*l.* gives £15.395211, for answer to the question (75.).

CV,

CV. What is the value, in *present* money, of 30 *l.* annuity for the age of *Forty*, to commence when aged *Forty-seven*, to continue 3 years, and then to *cease*?

The age, if it survive, would receive the *last six* payments only, in the Table (71.).

The total of the *five* last is 776.427904

The sum opposite age 47 $\frac{1}{2}$ is 172.676

The total of the *six last* } 949.103904
payments is — }

Which total, divided by the 294 persons aged *Forty*, gives 3.228244 for 2 *l.* or 1.614122 for 1 *l.* annuity; and the value of 1 *l.* being multiplied by 30 *l.* gives £48.42366 for the product, which is the answer to the question (75.).

'Tis hoped that the finding answers to these queries for an annuity *limited* to a certain time, has been so far useful to the Reader, as to explain partly to him the nature of the subject, and hath convinced him that the answers must be true. In that case, he may proceed in the same *impartial* manner to find the value of an annuity *during life*.

CVI. The method to find the value of an annuity *during life*, will be like that already used to find the value of an annuity for a limited time, proceeding in the same manner to multiply the *number of survivors*

survivors into the *reversion* of the next half year, and continuing it to the end of life; that is, as long as the Tables shew that one is living. The Total of all the sums added will shew the capital stock that will pay the number of Annuitants *living* at the *commencement* of the Table, 2 l. annuity during life.

These calculations being made for the use of the Societies *in particular*, who commence annuitants at the age of Fifty, it was not thought necessary to begin the Tables at a younger age.

CVII. *A Table to find the Value, in present Money, of 1 l. immediate Annuity during Life, from Fifty to any Age upwards, at the Rate of $3\frac{1}{2}$ per Cent. per Annum, per half-yearly Payments, and the Table of Mortality calculated from the London Bills for the last Forty-three Years (60.).*

(67.) Aged.	(70.) Living.	Reversion.	Product.	Totals.
50	221			
50½	216½	X .9828009	= 212.77639485	4749.67697485
51	212	X .965898	= 204.770376	4536.96058
51½	207½	X .949285	= 196.9766375	4332.130204
52	203	X .932939	= 189.390677	4135.1535665
52½	198½	X .916912	= 182.007032	3945.7628895
53	194	X .901143	= 174.821742	3763.7558575
53½	190	X .885644	= 168.27236	3588.9341155
54	186	X .870412	= 161.896632	3420.6617555
54½	182	X .855441	= 155.690262	3258.7651235
55	178	X .840728	= 149.649524	3103.0748615
55½	174	X .826269	= 143.770806	2953.4252775
56	170	X .812058	= 138.04986	2809.6544715
56½	166	X .798091	= 132.483106	2671.6046115
57	162	X .784364	= 127.066968	2539.1215055
57½	158	X .770875	= 121.79825	2412.0545375
58	154	X .757616	= 116.672864	2290.2562875
58½	150½	X .744586	= 112.060193	2173.5834235
59	147	X .731780	= 107.57166	2061.5232305
59½	143½	X .719194	= 103.204339	1953.9515705
60	140	X .706824	= 98.95536	1850.7472315
60½	136½	X .694668	= 94.822182	1751.7918715
61	133	X .682721	= 90.801893	1656.9696895
61½	129½	X .670978	= 86.891651	1566.1677905
62	126	X .659438	= 83.089188	1479.2761455
62½	122½	X .648096	= 79.39176	1396.1869575
63	119	X .636949	= 75.796931	1316.7951975
63½	115½	X .625995	= 72.3024225	1240.9982665
64	112	X .615228	= 68.905536	1168.695844
64½	108½	X .604647	= 65.6041995	1099.790308
65	105	X .594248	= 62.39604	1034.1861085
65½	101½	X .584027	= 59.2787405	971.7900685
66	98	X .573982	= 56.250236	912.511328

Age.	Living.	Reversion.	Product.	Total.
66½	95	x .564111	= 53.590545	856.261092
67	92	x .554408	= 51.005536	802.670547
67½	89	x .544873	= 48.493697	751.665011
68	86	x .535502	= 46.053172	703.171314
68½	83	x .526292	= 43.682236	657.118142
69	80	x .517240	= 41.3792	613.435906
69½	77	x .508344	= 39.142488	572.056706
70	74	x .499601	= 36.970474	532.914218
70½	71	x .491007	= 34.861497	495.943744
71	68	x .482563	= 32.814284	461.082247
71½	65½	x .474263	= 31.0642265	428.267963
72	63	x .466107	= 29.364741	397.2037365
72½	60½	x .458090	= 27.714445	367.8389955
73	58	x .450212	= 26.112296	340.1245505
73½	55½	x .442468	= 24.556974	314.0122545
74	53	x .434858	= 23.047474	289.4552805
74½	50½	x .427379	= 21.5826395	266.4078065
75	48	x .420028	= 20.161344	244.825167
75½	45½	x .412804	= 18.782582	224.663823
76	43	x .405705	= 17.445315	205.881241
76½	41	x .398727	= 16.347807	188.435926
77	39	x .391869	= 15.282891	172.088119
77½	37	x .385129	= 14.249773	156.805228
78	35	x .378505	= 13.247675	142.555455
78½	33	x .371995	= 12.275835	129.30778
79	31	x .365597	= 11.333507	117.031945
79½	29	x .359310	= 10.41999	105.698438
80	27	x .353130	= 9.53451	95.278448
80½	25½	x .347057	= 8.8499535	85.743938
81	24	x .341087	= 8.186088	76.8939845
81½	22½	x .335222	= 7.542195	68.7078965
82	21	x .329455	= 6.918555	61.1654015
82½	19½	x .323789	= 6.3138855	54.2468465
83	18	x .318221	= 5.727978	47.9329610
83½	16½	x .312747	= 5.1603255	42.2049830
84	15	x .307368	= 4.61052	37.0546575
84½	14	x .302082	= 4.229148	32.4341375
85	13	x .296886	= 3.859518	28.2049895
85½	12	x .291781	= 3.501372	24.3454715
86	11	x .286762	= 3.154382	20.8440995
86½	10	x .281830	= 2.8183	17.6897175
87	9	x .276983	= 2.492847	14.8714175
87½	8	x .272219	= 2.177752	12.3785705
88	7	x .267538	= 1.872766	10.2008185
88½	6	x .262673	= 1.576038	8.3280525

Age.

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Product.</i>	<i>Totals.</i>
89	5	× .258414	= 1.29207	6:7520145
89½	4	× .253969	= 1.1428605	5:4599445
90	4	× .249601	= .998404	4:317084
90	3½	× .245308	= .858578	3:31868
91	3	× .241089	= .723267	2 460102
91½	2½	× .236943	= .5923575	1:736833
92	2	× .232865	= .46573	1:1444775
92½	1½	× .228862	= .343293	.6787475
93	1	× .224926	= .224926	.3354545
93½	½	× .221057	= .1105285	.1105285

CVIII. The totals in the Fifth or last column are added together in this manner: The *product* at the age 93 is added to that opposite the age 93 ½; the product opposite 92 ½ is added to the total of the *two* last; the product opposite the age 92 is added to the total of the *three* last; the product of the age 91 ½ is added to the total of the *four* last; and so on, in like manner, from the *end* to the *beginning*; where the total of *all* the payments is found to be 4749.67697485, which is the capital stock that will pay 221 persons, aged Fifty, an immediate annuity of 2 *l.* during life, according to *this* Table of Mortality.

CIX. The use of this method is to shew the sum that would pay the same number of persons, of the *same* age, the same annuity, after any number of years reversion. Suppose the age Fifty *not* to commence Annuitant till the expiration of 10 years, it would receive none of the *first* 20 payments (107.); and opposite the age 60 ½, when he

F f 2

would

would become Recipient of the *first* payment, is the sum of £1751.7918715, which is the capital stock that would pay 221 persons aged Fifty, an annuity of 2 l. during life, to commence when aged Sixty. That stock divided by the 221 persons gives 7.92666 for 2 l. or £3.96333 for the value of 1 l. annuity; and in like manner for any other time of reversion.

By this Table also the value of an *immediate* annuity for any age above *Fifty* may be found thus.

CX. What is the worth of an *immediate* annuity of 1 l. for the age *Sixty*, during life?

The total of all the payments *this* Age would have a chance of receiving is opposite the age $60\frac{1}{2}$ in the fifth column, the same as before, 1751.7918715; but the sums are all multiplied into 20 half-years reversion more than should be (92.), for an *immediate* annuity for *this* age; therefore the total sum must be *divided* by 20 half-years *reversion* (46.), or *multiplied* by 20 half-years *Compound Interest* (42*), to restore it to *present* value; for the age is supposed by the *Question* to receive the *first* payment at the expiration of 6 months, *not* at the expiration of 21 half-years.

$$\begin{aligned} & .706824) 1751.7918715 (2478.398 \text{ or} \\ & 17517918715 \times 141477819 = 2478.398 \end{aligned}$$

Let

Let this be compared with the sum opposite the age 60 in the *next* Table, and let it be divided by the 140 persons supposed by this Table to be living at Sixty, and the quotient will be 17.7028 for 2 *l.* annuity, or £8.8514 for answer to the Question.

CXI. The next Table will exhibit a proof of the last, and will furnish the Reader with a conciser and *simpler* method of finding the present value of an *immediate* annuity, the which will be explained hereafter.

It is said (108.), that £4749.67697485 present money will pay 221 persons aged Fifty, an immediate annuity of 2 *l.* during life. Let this be *evidently* explained.

CXII. A Table to prove the foregoing :

If 221 persons pay down equal sums that together will amount to £4749.677, the half-year's interest will be £83.119 (38.), which added to the stock will be £4832.796, and £216. 10 s. 0 d. will be paid to so many surviving members; which being deducted, there will remain the principal £4616.296, at interest for the next half-year, which interest will be added to the principal, as before; and 212 members will be living to receive 212l. which being deducted, and the same method continued to the end of life, the deficiency or overplus of the stock will be seen, as in the following Table (72.).

Stock	£4749.677+				4563.570
Interest	83.119		to pay		207.5
Total	4832.796		4th		4356.070
to pay	216.5		Interest		76.231
2d	} 4616.296.				4432.301
principal					203
Interest	80.785		to pay		
	4697.081		5th		4229.301
to pay	212		Interest		74.013
3d	4485.081				4303.314
Interest	78.489+				
	4563.570				to

to pay	4303.314 198.5	to pay	3690.935 178
6th principal }	4104.814	Age 55	3512.935
Interest }	71.834	Interest	61.477 +
	4176.648		3574.412
to pay	194.	to pay	174
Age 53	3982.648	Age 55½	3400.412
Interest	69.696	Interest	59.507
	4052.344		3459.919
to pay	190	to pay	170
Age 53½	3862.344	Age 56	3289.919
Interest	67.591	Interest	57.574 +
	3929.935		3347.493
to pay	186	to pay	166
Age 54	3743.935	Age 56½	3181.493
Interest	65.519 +	Interest	55.676
	3809.454		3237.169
to pay	182	to pay	162
Age 54½	3627.454	Age 57	3075.169
Interest	63.481 +	Interest	53.815
	3690.935		3128.984
			to

to pay	3128.984 158	to pay	2618.396 140
Age 57 $\frac{1}{2}$ Interest	2970.984 51.992	Age 60 $\frac{1}{2}$ Interest	2478.396 ⁽¹⁰⁰⁾ 43.372 +
to pay	3022.976 154	to pay	2521.768 136.5
Age 58 Interest	2868.976 50.207	Age 60 $\frac{1}{2}$ Interest	2385.268 41.742
to pay	2919.183 150.5	to pay	2427.010 133
Age 58 $\frac{1}{2}$ Interest	2768.683 48.452 +	Age 61 Interest	2294.010 40.145
to pay	2817.135 147	to pay	2334.155 129.5
Age 59 Interest	2670.135 46.727	Age 61 $\frac{1}{2}$ Interest	2204.655 38.582
to pay	2716.862 143.5	to pay	2243.237 126
Age 59 $\frac{1}{2}$ Interest	2573.362 45.034 +	Age 62 Interest	2117.237 37.052 +
	2618.396		2154.289

to

(49)

to pay	<u>2154.289</u> 122.5	to pay	<u>1740.328</u> 105
Age 62½	2031.789	Age 65	1635.328
Interest	<u>35.556</u>	Interest	<u>28.618</u>
to pay	<u>2067.345</u> 119	to pay	<u>1663.946</u> 101.5
Age 63	1948.345	Age 65½	1562.446
Interest	<u>34.096</u>	Interest	<u>27.043+</u>
to pay	<u>1982.441</u> 115.5	to pay	<u>1589.789</u> 98
Age 63½	1866.941	Age 66	1491.789
Interest	<u>32.672</u>	Interest	<u>26.106</u>
to pay	<u>1899.613</u> 112.	to pay	<u>1517.895</u> 95
Age 64	1787.613	Age 66½	1422.895
Interest	<u>31.283</u>	Interest	<u>24.901</u>
to pay	<u>1818.896</u> 108.5	to pay	<u>1447.796</u> 92
Age 64½	1710.396	Age 67	1355.796
Interest	<u>29.932+</u>	Interest	<u>23.726</u>
	<u>1740.328</u>		<u>1379.522</u>

G g

to

(50)

to pay	1379.522 89	to pay	1066.679 74
Age 67½ Interest	1290.522 22.584	Age 70 Interest	992.679 17.372
to pay	1313.106 86	to pay	1010.051 71
Age 68 Interest	1227.106 21.474	Age 70½ Interest	939.051 16.433
to pay	1248.580 83	to pay	955.484 68
Age 68½ Interest	1165.580 20.398 +	Age 71 Interest	887.484 15.531 +
to pay	1185.978 80	to pay	903.015 65.5
Age 69 Interest	1105.978 19.355 +	Age 71½ Interest	837.515 14.657 +
to pay	1125.333 77	to pay	852.172 63
Age 69½ Interest	1048.333 18.346 +	Age 72 Interest	789.172 13.811 +
	1066.679		802.983

to

(51)

to pay	802.983 60.5	to pay	582.876 48
Age 72½	742.483	Age 75	534.876
Interest	12.993	Interest	9.360
to pay	755.476 58	to pay	544.236 45.5
Age 73	697.476	Age 75½	498.736
Interest	12.206 +	Interest	8.728 +
to pay	709.682 55.5	to pay	507.464 43
Age 73½	654.182	Age 76	464.464
Interest	11.448	Interest	8.128
to pay	665.630 53	to pay	472.592 41.
Age 74	612.630	Age 76½	431.592
Interest	10.721	Interest	7.553 +
to pay	623.351 50.5	to pay	439.145 39
Age 74½	572.851	Age 77	400.145
Interest	10.025 +	Interest	7.003 +
	582.876		407.148
	G g 2		to

to pay	407.148 37	to pay	269.809 27
Age 77½	370.148	Age 80	242.809
Interest	6.478+	Interest	4.249
	376.626		247.058
to pay	35	to pay	25.5
Age 78	341.626	Age 80½	221.358
Interest	5.978	Interest	3.877
	347.604		225.435
to pay	33	to pay	24
Age 78½	314.604	Age 81	201.435
Interest	5.506+	Interest	3.523+
	320.110		204.960
to pay	31	to pay	22.5
Age 79	289.110	Age 81½	182.460
Interest	5.059	Interest	3.193
	294.169		185.653
to pay	29	to pay	21
Age 79½	265.169	Age 82	164.653
Interest	4.640	Interest	2.882
	269.809		167.535
			to

to pay	167.535 19.5	to pay	95.001 13
Age 82½	148.035	Age 85	82.001
Interest	2.598 +	Interest	1.435
to pay	150.626 18	to pay	83.436 12
Age 83	132.626	Age 85½	71.436
Interest	2.321 +	Interest	1.25
to pay	134.947 16.5	to pay	72.686 11
Age 83½	118.447	Age 86	61.686
Interest	2.073 +	Interest	1.08 +
to pay	120.520 15	to pay	62.766 10
Age 84	105.520	Age 86½	52.766
Interest	1.847 +	Interest	.923
to pay	107.367 14	to pay	53.689 9
Age 84½	93.367	Age 87	44.689
Interest	1.634 +	Interest	.782
	95.001		45.471
			to

to pay	<u>45.471</u> 8	to pay	<u>17.288</u> 4
Age 87½	37.471	Age 90	13.288
Interest	.656+	Interest	.233+
to pay	<u>38.127</u> 7	to pay	<u>13.521</u> 3.5
Age 88	31.127	Age 90½	10.021
Interest	.545+	Interest	.175
to pay	<u>31.672</u> 6	to pay	<u>10.196</u> 3
Age 88½	25.672	Age 91	7.196
Interest	.449	Interest	.126+
to pay	<u>26.121</u> 5	to pay	<u>7.322</u> 2.5
Age 89	22.121	Age 91½	4.822
Interest	.37. +	Interest	.084
to pay	<u>21.491</u> 4.5	to pay	<u>4.906</u> 2.
Age 89½	16.991	Age 92	2.906
Interest	.297	Interest	.051+
	<u>17.288</u>		<u>2.957</u> to

to pay	2.957	to pay	1.482
	1.5		1
Age 92½	1.457	Age 93	.482
Interest	.025	Interest	.008
	1.482		.490 deficient
		to pay	.5
		(Supposed the End of Life.)	

The sum that would pay 221 persons aged Fifty, an immediate annuity of 2 *l.* during life, was first found (107.) by adding the value in reversion, when multiplied by the number of Recipients living, at each payment together : *That* sum has *here* been supposed to be the Capital Stock of the 221 persons, and every survivor has been seen to receive 1 *l.* half-yearly ; and every half-year's interest, as it became due, has been added to the Stock. It is evident, then, that this Stock of £4749.677 is the true sum that should be deposited for a fund to supply the conditions ; for the two Tables agree so far to prove each other, that the difference is but $\frac{1}{10}$ deficient to pay the last 10 Shillings. This being the whole of the errors in the several Tables of Interest, Reversion, and of working the several parts into each other, the Tables may be deemed sufficiently *correct*.

Such Proofs, *jointly*, must be so powerful as to overcome all Doubts ; therefore proceed in the *same* manner with Dr. HALLEY's Table, and Mr. SIMPSON's,

*Dr. HALLEY's Table of Mortality, from
the Age of Fifty; with the half-yearly
Survivors multiplied into the present Value
of the Payments (47.).*

<i>Age.</i>	<i>Surv.</i>	<i>Reversion.</i> (47.)	<i>Pres. Value of Pay- to Survivors.</i>	<i>Total of the Products.</i>
50	346			
50½	340½	X 982801	= 334.6437405	8051.8452185
51	335	X 965898	= 323.57583	7717.201478
51½	329½	X 949285	= 312.7894075	7393.625648
52	324	X 932959	= 302.278716	7080.8362405
52½	318½	X 916912	= 292.036472	6778.5578245
53	313	X 901143	= 282.057759	6486.5210525
53½	307½	X 885644	= 272.33553	6204.4632935
54	302	X 870412	= 262.864424	5932.1277635
54½	297	X 855441	= 254.065977	5669.2633395
55	292	X 840728	= 245.492576	5415.1973625
55½	287	X 826269	= 237.139203	5169.7047865
56	282	X 812058	= 229.000356	4932.5655835
56½	277	X 798091	= 221.071207	4703.5652275
57	272	X 784364	= 213.347008	4482.4940205
57½	267	X 770875	= 205.823625	4269.1470125
58	262	X 757616	= 198.495392	4063.3233875
58½	257	X 744586	= 191.358602	3864.8279955
59	252	X 731780	= 184.40856	3673.4693935
59½	247	X 719194	= 177.640918	3489.0608335
60	242	X 706824	= 171.051408	3311.4199155
60½	237	X 694668	= 164.636316	3140.3685075
61	232	X 682721	= 158.391272	2975.7321915
61½	227	X 670978	= 152.312006	2817.3409199
62	222	X 659438	= 146.395236	2665.0289135
62½	217	X 648096	= 140.636832	2518.6336775
63	212	X 636949	= 135.033188	2377.9968455
63½	207	X 625995	= 129.580965	2242.9636575
64	202	X 615228	= 124.276056	2113.3826925
64½	197	X 604647	= 119.115459	1989.1066369
65	192	X 594248	= 114.095616	1869.9911775
65½	187	X 584027	= 109.213049	1755.8955615
66	182	X 573982	= 104.464724	1646.6825125
66½	177	X 564111	= 99.847647	1542.2177885
67	172	X 554408	= 95.348176	1442.3702415
67½	167	X 544873	= 90.993791	1347.0119655
68	162	X 535502	= 86.751324	1256.0181745

Age.	Living	Reversion.	Pres. Value of Pay. to Survivors.	Total of the y r r
68½	157	X 526292	= 82.627844	1109.2668505
69	152	X 517240	= 78.62048	1086.6390065
69½	147	X 508344	= 74.726568	1008.0185265
70	142	X 499601	= 70.943342	933.2919584
70½	136½	X 491007	= 67.0224555	862.3486163
71	131	X 482563	= 63.215753	795.326161
71½	125½	X 474263	= 59.5200065	732.110408
72	120	X 466107	= 55.93284	672.5904015
72½	114½	X 458090	= 52.451305	616.6575615
73	109	X 450212	= 49.073108	564.2062565
73½	103½	X 442468	= 45.795438	515.1331485
74	98	X 434858	= 42.616084	469.3377105
74½	93	X 427379	= 39.746247	426.7216265
75	88	X 420028	= 36.962464	386.9753795
75½	83	X 412804	= 34.262732	350.0129153
76	78	X 405705	= 31.64499	315.7501835
76½	73	X 398727	= 29.107071	284.1051935
77	68	X 391869	= 26.647092	254.9981225
77½	63	X 385129	= 24.263127	228.3510305
78	58	X 378505	= 21.95329	204.0879035
78½	53½	X 371995	= 19.9017325	182.1346135
79	49	X 365597	= 17.914253	162.232881
79½	45	X 359310	= 16.16895	144.318628
80	41	X 353130	= 14.47833	128.149678
80½	37½	X 347057	= 13.0146375	113.671348
81	34	X 341087	= 11.596958	100.6567105
81½	31	X 335222	= 10.391882	89.0597525
82	28	X 329455	= 9.22474	78.6678705
82½	25½	X 323789	= 8.2566195	69.4431305
83	23	X 318221	= 7.319083	61.186511
83½	21½	X 312747	= 6.7240605	53.867428
84	20	X 307368	= 6.14736	47.1431675
84½	18½	X 302082	= 5.588517	40.9960075
85	17	X 296886	= 5.047062	35.4074903
85½	15½	X 291781	= 4.5226055	30.5604285
86	14	X 286762	= 4.014668	25.837823
86½	12½	X 281830	= 3.522875	21.823155
87	11	X 276983	= 3.046813	18.30028
87½	10	X 272219	= 2.72219	15.253467
88	9	X 267538	= 2.407842	12.531277
88½	8	X 262673	= 2.101384	10.123435
89	7	X 258414	= 1.808898	8.022051
89½	6	X 253969	= 1.523814	6.213153
			H h	100

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Pres. Value of Pay. to Survivors.</i>	<i>Total of the Payments.</i>
90	5	x 249601	= 1.248005	4.689339
90½	4	x 245308	= .981232	3.441334
91	3	x 241089	= .723267	2.460102
91½	2½	x 236943	= .5923575	1.736835
92	2	x 232865	= .465730	1.1444775
92½	1½	x 228862	= .343293	.6787475
93	1	x 224926	= .224926	.3354545
93½	½	x 221057	= .1105285	.1105285

CXIV. The proof of this Table is (as before) by supposing 346 persons Fifty years old, possessed of the Stock of £8051.84522 + the *half-year's* interest will be £140.90729, which, added to the first principal, will make it £8192.75251; and 340 ½ members will be *living* to receive as many pounds; after which there will remain £7852.253 + to gain interest, and to pay each half-year, as follows :

The

The Stock at 50	7392.808
Years old is	to pay 318.5
$\text{£}8051.845$	
$\frac{1}{4}$ Yrs Int. 140.907 ⁽³⁸⁾	Age 52 $\frac{1}{2}$ 7074.308
	Interest 123.8
Total 8192.753+	
to pay 340.5	7198.108
	to pay 313
Age 50 $\frac{1}{2}$ 7852.253	Age 53 6885.108
Interest 137.414	Interest 120.489
7989.667	7005.597
to pay 335	to pay 307.5
Age 51 7654.667	Age 53 $\frac{1}{2}$ 6698.097
Interest 133.957+	Interest 117.217+
7788.624	6815.314
to pay 329.5	to pay 302
Age 51 $\frac{1}{2}$ 7459.124	Age 54 6513.314
Interest 130.535+	Interest 113.983+
7589.659	6627.297
to pay 324	to pay 297
Age 52 7265.659	Age 54 $\frac{1}{2}$ 6330.297
Interest 127.149	Interest 110.78
7392.808	6441.077

(60)

to pay	6441.077 292	to pay	5538.057 267
Age 55	6149.077	Age 57½	5271.057
Interest	107.609 +	Interest	92.243
	6256.686		5363.300
to pay	287	to pay	262
Age 55½	5969.686	Age 58	5101.3
Interest	104.47 +	Interest	89.273 +
	6074.156		5190.573
to pay	282	to pay	257
Age 56	5792.156	Age 58½	4933.573
Interest	101.363 +	Interest	86.338 +
	5893.519		5019.911
to pay	277	to pay	252
Age 56½	5616.519	Age 59	4767.911
Interest	98.289	Interest	83.438
	5714.808		4851.349
to pay	272	to pay	247
Age 57	5442.808	Age 59½	4604.349
Interest	95.249	Interest	80.576
	5538.057		4684.925

to

to pay	4684.925 242	to pay	3886.202 217
Age 60 Interest	4442.925 77.751	Age 62½ Interest	3669.202 64.211
to pay	4520.676 237	to pay	3733.413 212
Age 60½ Interest	4283.676 74.964	Age 63 Interest	3521.413 61.625 +
to pay	4358.640 232	to pay	3583.038 207
Age 61 Interest	4126.64 72.216	Age 63½ Interest	3376.038 59.081 +
to pay	4198.856 227	to pay	3435.119 202
Age 61½ Interest	3971.856 69.507	Age 64 Interest	3233.119 56.58 +
to pay	4041.363 222	to pay	3289.699 197
Age 62 Interest	3819.363 66.839 +	Age 64½ Interest	3092.699 54.122
	3886.202		3146.821

to

to pay	3146.821 192	to pay	2472.157 167
Age 65 Interest	2954.821 51.709	Age 67½ Interest	2305.157 40.34
to pay	3006.53 187	to pay	2345.497 162
Age 65½ Interest	2819.53 49.342 +	Age 68 Interest	2183.497 38.211
to pay	2868.872 182	to pay	2221.708 157
Age 66 Interest	2686.872 47.02	Age 68½ Interest	2064.708 36.132
to pay	2733.892 177	to pay	2100.84 152
Age 66½ Interest	2556.892 44.746 +	Age 69 Interest	1948.84 34.105 +
to pay	2601.638 172	to pay	1982.945 147
Age 67 Interest	2429.638 42.519 +	Age 69½ Interest	1835.945 32.129
	2472.157		1868.074

to pay	1868.074 142	to pay	1346.146 114.5
Age 70 Interest	1726.074 30.206	Age 72½ Interest	1231.646 21.554 +
to pay	1756.280 136.5	to pay	1253.2 109
Age 70½ Interest	1619.78 28.346	Age 73 Interest	1144.2 20.023
to pay	1648.126 131	to pay	1164.223 103.5
Age 71 Interest	1517.126 26.55 +	Age 73½ Interest	1060.723 18.563 +
to pay	1543.676 125.5	to pay	1079.286 98
Age 71½ Interest	1418.176 24.818	Age 74 Interest	981.286 17.173 +
to pay	1442.994 120	to pay	998.459 93
Age 72 Interest	1322.994 23.152	Age 74½ Interest	905.459 15.846 +
	1346.146		921.305

to

to pay	<u>921.305</u> 88	to pay	<u>592.919</u> 63
Age 75	833.305	Age 77½	529.919
Interest	<u>14.583 +</u>	Interest	<u>9.274 +</u>
	847.888		539.193
to pay	<u>83</u>	to pay	<u>58</u>
Age 75½	764.888	Age 78	481.193
Interest	<u>13.386 +</u>	Interest	<u>8.421 +</u>
	778.274		489.614
to pay	<u>78</u>	to pay	<u>53.5</u>
Age 76	700.274	Age 78½	436.114
Interest	<u>12.255 +</u>	Interest	<u>7.632 +</u>
	712.529		443.746
to pay	<u>73.</u>	to pay	<u>49</u>
Age 76½	639.529	Age 79	394.746
Interest	<u>11.192 +</u>	Interest	<u>6.908</u>
	650.721		461.654
to pay	<u>68</u>	to pay	<u>45</u>
Age 77	582.721	Age 79½	356.654
Interest	<u>10.198 +</u>	Interest	<u>6.241.</u>
	592.919		362.895

to

(65)

to pay	362.895 41	to pay	214.468 25.5
Age 80 Interest	321.895 5.633	Age 82½ Interest	188.968 3.307+
to pay	327.528 37.5	to pay	192.275 23
Age 80½ Interest	290.028 5.075	Age 83 Interest	169.275 2.962
to pay	295.103 34	to pay	172.237 21.5
Age 81 Interest	261.103 4.569	Age 83½ Interest	150.737 2.638+
to pay	265.672 31	to pay	153.375 20
Age 81½ Interest	234.672 4.107+	Age 84 Interest	133.375 2.334
to pay	238.779 28	to pay	135.709 18.5
Age 82 Interest	210.779 3.689+	Age 84½ Interest	117.209 2.051
	214.468		119.26

I i

to

to pay	119.26 17	to pay	56.032 10
Age 85 Interest	102.26 1.79 +	Age 87½ Interest	46.032 806 +
to pay	104.05 15.5	to pay	46.838 9
Age 85½ Interest	88.55 1.55 +	Age 88 Interest	37.838 662
to pay	90.1 14	to pay	38.5 8
Age 86 Interest	76.1 1.332 +	Age 88½ Interest	30.5 534
to pay	77.432 12.5	to pay	31.034 7
Age 86½ Interest	64.932 1.136	Age 89 Interest	24.034 421 +
to pay	66.068 11	to pay	24.455 6
Age 87 Interest	55.068 964	Age 89½ Interest	18.455 323 +
	56.032		18.778

to

(67)

to pay	18.778 5	to pay	4.904 2
Age 90 Interest	13.778 241	Age 92 Interest	2.904 .051 +
to pay	14.019 4	to pay	2.955 1.5
Age 90½ Interest	10.019 175	Age 92½ Interest	1.455 .025
to pay	10.194 3	to pay	1.480 1
Age 91 Interest	7.194 126	Age 93 Interest	.480 .008
to pay	7.320 2.5	to pay	.488 .5
Age 91½ Interest	4.82 .084	Deficient	.012
	4.904		

CXV. It is intended to make the *principal* Calculations from Mr. *Simpson's* Table, not so much from conviction that it is preferable to Dr. *Halley's*, but because it is from about Seventeen as far as the age of near Sixty, the *medium* between the Two other Tables ; and as most people seem of opinion, that an Annuity on the conditions of the several Societies cannot be worth *so much* present money as will appear from Calculations made hereafter ; *such* will please to observe, that if the *same* Calculations were made from Dr. *Halley's* Table, the *worth* would be *considerably more* from Childhood to near Sixty ; though afterward it would be *considerably less* ; and, if the *majority* of the Members are aged *less* than Sixty (as no question but they are abundantly so), then it follows that the Calculations are made at the *lowest* rate ; which is far from *exaggerating* the difference, when comparison is made between the Calculations from the said Table, and the Proposals of the Societies ; and for this reason this Table is chosen, and commences at *Forty*, it being necessary that it should, to answer some particular questions.

CKVI, Mr. Simpson's Table from Forty Years old; with the half-yearly Survivors multiplied into the present Value of the Payments.

Age.	(70.) Living.	Reversion.	Present Value.	Added Payments.
40	294	(47.)	(71.)	(108.)
40 $\frac{1}{2}$	289	X .982801	= 284.029489	7283.474286
41	284	X .965898	= 274.315032	6999.4447975
41 $\frac{1}{2}$	279	X .949285	= 264.850515	6725.1297655
42	274	X .932959	= 255.630766	6460.2792505
42 $\frac{1}{2}$	269	X .916912	= 246.649328	6204.6484845
43	264	X .901143	= 237.901752	5957.9991565
43 $\frac{1}{2}$	259 $\frac{1}{2}$	X .885644	= 229.824618	5720.0974045
44	255	X .870412	= 221.95506	5490.2727865
44 $\frac{1}{2}$	250 $\frac{1}{2}$	X .855441	= 214.2879705	5268.3177265
45	246	X .840728	= 206.819088	5054.029756
45 $\frac{1}{2}$	241 $\frac{1}{2}$	X .826269	= 199.5439635	4847.210668
46	237	X .812058	= 192.457746	4647.6667045
46 $\frac{1}{2}$	232 $\frac{1}{2}$	X .798091	= 185.5561575	4455.2089585
47	228	X .784364	= 178.834992	4269.652801
47 $\frac{1}{2}$	224	X .770875	= 172.676	4090.817809
48	220	X .757616	= 166.67552	3918.141809
48 $\frac{1}{2}$	216	X .744586	= 160.830576	3751.466289
49	212	X .731780	= 155.13736	3590.635713
49 $\frac{1}{2}$	208	X .719194	= 149.592352	3435.498353
50	204	X .706824	= 144.192096	3285.906001
50 $\frac{1}{2}$	200	X .694668	= 138.9336	3141.719905
51	196	X .682721	= 133.813516	3002.780305
51 $\frac{1}{2}$	192	X .670978	= 128.827776	2868.966989
52	188	X .659438	= 123.974344	2740.139213
52 $\frac{1}{2}$	184	X .648096	= 119.249664	2616.164869
53	180	X .636949	= 114.65082	2496.915205
53 $\frac{1}{2}$	176	X .625995	= 110.17512	2382.264385
54	172	X .615228	= 105.819216	2272.089265
54 $\frac{1}{2}$	168 $\frac{1}{2}$	X .604647	= 101.8830195	2166.270049
55	165	X .594248	= 98.05092	2064.3870295
55 $\frac{1}{2}$	161 $\frac{1}{2}$	X .584027	= 94.3203605	1966.3361095
56	158	X .573982	= 90.689156	1872.015749

Age

Age.	Living.	Reversion.	Present Value.	Added Payments.
56½	154½	X .564111	= 87.1551495	1781.326593
57	151	X .554408	= 83.715608	1694.1714435
57½	147½	X .544873	= 80.3687675	1610.4558355
58	144	X .535502	= 77.112288	1530.087068
58½	140½	X .526292	= 73.944026	1452.97478
59	137	X .51724	= 70.86188	1379.030754
59½	133½	X .508344	= 67.863924	1308.168874
60	130	X .499601	= 64.94813	1240.30495
60½	126½	X .491007	= 62.1123855	1175.35682
61	123	X .482563	= 59.355249	1113.2444345
61½	120	X .474263	= 56.91156	1053.8891855
62	117	X .466107	= 54.534519	996.9776255
62½	114	X .45809	= 52.22226	942.4431065
63	111	X .450212	= 49.973532	890.2208465
63½	108	X .442468	= 47.786544	840.2473145
64	105	X .434858	= 45.66009	792.4607705
64½	102	X .427379	= 43.592658	746.8001805
65	99	X .420028	= 41.582772	703.208025
65½	96	X .412804	= 39.629184	661.6252505
66	93	X .405705	= 37.730565	621.9960665
66½	90	X .398727	= 35.88543	584.2655015
67	87	X .391869	= 34.092603	548.3800715
67½	84	X .385129	= 32.350836	514.2874685
68	81	X .378505	= 30.658905	481.9366325
68½	78	X .371995	= 29.01561	451.2777275
69	75	X .365597	= 27.419775	422.2621175
69½	72	X .35931	= 25.87032	394.8423425
70	69	X .35313	= 24.36597	368.9720225
70½	66½	X .347057	= 23.0792905	344.6060525
71	64	X .341087	= 21.829568	321.526762
71½	61½	X .335222	= 20.616153	299.697194
72	59	X .329455	= 19.437845	279.081041
72½	56½	X .323789	= 18.2940785	259.643196
73	54	X .318221	= 17.183934	241.3491175
73½	51½	X .312747	= 16.1064705	224.1651835
74	49	X .307368	= 15.061032	208.058713
74½	47	X .302082	= 14.197854	192.997681
75	45	X .296886	= 13.35987	178.799827
75½	43	X .291781	= 12.546583	165.439957
76	41	X .286762	= 11.757242	152.893374
76½	39½	X .281830	= 11.132285	141.136132
77	38	X .276983	= 10.525354	130.003847
77½	36½	X .272219	= 9.9359935	119.478493

Age.

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Present Value.</i>	<i>Added Payments.</i>
78	35	X .267538	= 9.36383	109.5424995
78½	33½	X .262673	= 8.7995455	100.1786695
79	32	X .258414	= 8.269248	91.379124
79½	30½	X .253969	= 7.7460545	83.109876
80	29	X .249601	= 7.238429	75.3638215
80½	27½	X .245308	= 6.74597	68.1253925
81	26	X .241089	= 6.268314	61.3794225
81½	24½	X .236943	= 5.8051035	55.1111085
82	23	X .232865	= 5.355895	49.306005
82½	21½	X .228862	= 4.920533	43.95011
83	20	X .224926	= 4.49852	39.029577
83½	18½	X .221057	= 4.0895545	34.531057
84	17	X .217255	= 3.693335	30.4415025
84½	15½	X .213519	= 3.3095445	26.7481675
85	14	X .209846	= 2.937844	23.438623
85½	13	X .206237	= 2.681081	20.500779
86	12	X .20269	= 2.43228	17.819698
86½	11	X .199204	= 2.191244	15.387418
87	10	X .195778	= 1.95778	13.196174
87½	9	X .192411	= 1.731699	11.238394
88	8	X .189101	= 1.512808	9.506695
88½	7	X .185849	= 1.300943	7.993887
89	6	X .182653	= 1.095918	6.692944
89½	5½	X .179512	= .987316	5.697026
90	5	X .176424	= .88212	4.50971
90½	4½	X .173388	= .780246	3.72759
91	4	X .170408	= .681632	2.947344
91½	3½	X .167476	= .586166	2.265712
92	3	X .164596	= .493788	1.679546
92½	2½	X .161765	= .4044125	1.185758
93	2	X .158983	= .317966	.7813455
93½	1½	X .156249	= .2343735	.4633795
94	1	X .153546	= .153546	.229006
94½	½	X .15092	= .07546	.07546

Let

Let this Table also be proved, in like manner as the *other Two*, that is, by *adding* Interest to the Stock, and *paying* 2l. *Annuity* out of it as long as any Part of it is *remaining*.

CXVII. 294 Persons, 40 Years old, subscribe *equally* towards the (iii.)

Stock of	£7283.474				6924.508
1st ½ Ys. Int.	127.46 (38)	to pay			274
to pay	7410.934	Age 42		6650.508	
	289	Interest		116.384+	
Age 40½	7121.934				
Interest	124.634+				
		to pay			6766.892
	7246.568				269
to pay	284	Age 42½		6497.892	
		Interest		113.713	
Age 41	6962.568				
Interest	121.845+				
		to pay			6611.605
	7084.413				264
to pay	279	Age 43		6347.605	
		Interest		111.083	
Age 41½	6805.413				
Interest	119.095+				
	6924.508				6458.688

to

to pay	6458.688 259.5	to pay	5723.319 237
Age 43½	6199.188	Age 46	5486.319
Interest	108.486 +	Interest	96.011 +
to pay	6307.674 255	to pay	5582.330 232.5
Age 44	6052.674	Age 46½	5349.83
Interest	105.922 +	Interest	93.622
to pay	6158.596 250.5	to pay	5443.452 228
Age 44½	5908.096	Age 47	5215.452
Interest	103.392 +	Interest	91.27
to pay	6011.488 246	to pay	5306.722 224
Age 45	5765.488	Age 47½	5082.722
Interest	100.896	Interest	88.948 +
to pay	5866.384 241.5	to pay	5171.67 220
Age 45½	5624.884	Age 48	4951.67
Interest	98.435	Interest	86.654
	5723.319		5038.324

K k

to

to pay	5038.324 216	to pay	4398.258 196
Age 48½	4822.324	Age 51	4202.258
Interest	84.391 +	Interest	73.54
to pay	4906.715 212	to pay	4275.798 192
Age 49	4694.715	Age 51½	4083.798
Interest	82.158 +	Interest	71.466
to pay	4776.873 208	to pay	4155.264 188
Age 49½	4568.873	Age 52	3967.264
Interest	79.955	Interest	69.427
to pay	4648.828 204	to pay	4036.691 184
Age 50	4444.828	Age 52½	3852.691
Interest	77.784	Interest	67.422
to pay	4522.612 200	to pay	3920.113 180
Age 50½	4322.612	Age 53	3740.113
Interest	75.646 +	Interest	65.452 +
	4398.258		3805.565

to

to pay	3805.565 176	to pay	3261.451 158
Age 53½	3629.565	Age 56	3103.451
Interest	63.517	Interest	54.31
to pay	3693.082 172	to pay	3157.761 154.5
Age 54	3521.082	Age 56½	3003.261
Interest	61.619 +	Interest	52.557
to pay	3582.701 168.5	to pay	3055.818 151
Age 54½	3414.201	Age 57	2904.818
Interest	59.749 +	Interest	50.834
to pay	3473.959 165	to pay	2955.652 147.5
Age 55	3308.95	Age 57½	2808.152
Interest	57.907 +	Interest	49.143 +
to pay	3366.857 161.5	to pay	2857.295 144
Age 55½	3205.357	Age 58	2713.295
Interest	56.094 +	Interest	47.483 +
	3261.451		2760.778

to pay	<u>2760.778</u> 146.5	to pay	<u>2306.937</u> 123
Age 58½	2620.278	Age 61	2183.937
Interest	45.854	Interest	38.219 +
	<u>2666.132</u>		<u>2222.156</u>
to pay	<u>137</u>	to pay	<u>120</u>
Age 59	2529.132	Age 61½	2102.156
Interest	44.26 +	Interest	36.788 +
	<u>2573.392</u>		<u>2138.944</u>
to pay	<u>133.5</u>	to pay	<u>117</u>
Age 59½	2439.892	Age 62	2021.944
Interest	42.698	Interest	35.384
	<u>2482.59</u>		<u>2057.328</u>
to pay	<u>130</u>	to pay	<u>114</u>
Age 60	2352.59	Age 62½	1943.328
Interest	41.17	Interest	34.008
	<u>2393.76</u>		<u>1977.336</u>
to pay	<u>126.5</u>	to pay	<u>111</u>
Age 60½	2267.26	Age 63	1866.336
Interest	39.677	Interest	32.661 +
	<u>2306.937</u>		<u>1898.997</u>

to pay	1898.997 108	to pay	1533.12 93
Age 63½ Interest	1790.997 31.342	Age 66 Interest	1440.12 25.202
to pay	1822.339 105	to pay	1465.322 90
Age 64 Interest	1717.339 30.053	Age 66½ Interest	1375.322 24.068
to pay	1747.392 102	to pay	1399.39 87
Age 64½ Interest	1645.392 28.794	Age 67 Interest	1312.39 22.967+
to pay	1674.186 99	to pay	1335.357 84
Age 65 Interest	1575.186 27.566+	Age 67½ Interest	1251.357 21.899+
to pay	1602.752 96	to pay	1273.256 81
Age 65½ Interest	1506.752 26.368	Age 68 Interest	1192.256 20.864
	1533.12		1213.12

to

(80)

to pay	1213.12 78	to pay	942.645 64
Age 68½	1135.12	Age 71	878.645
Interest	19.865 +	Interest	15.376
to pay	1154.985 75	to pay	894.021 61.5
Age 69	1079.985	Age 71½	832.521
Interest	18.9 +	Interest	14.569
to pay	1098.885 72	to pay	847.09 59
Age 69½	1026.885	Age 72	788.09
Interest	17.97	Interest	13.792 +
to pay	1044.855 69	to pay	801.882 56.5
Age 70	975.855	Age 72½	745.382
Interest	17.077	Interest	13.044
to pay	992.932 66.5	to pay	758.426 54
Age 70½	926.432	Age 73	704.426
Interest	16.213 +	Interest	12.327
	942.645		716.753

to

to pay	<u>716.753</u> 51.5	to pay	<u>533.163</u> 41
Age 73½	665.253	Age 76	492.163
Interest	11.642 +	Interest	8.613 +
	<u>676.895</u>		<u>500.776</u>
to pay	<u>49</u>	to pay	<u>39.5</u>
Age 74	627.895	Age 76½	461.276
Interest	10.988	Interest	8.092
	<u>638.883</u>		<u>469.348</u>
to pay	<u>47</u>	to pay	<u>38</u>
Age 74½	591.883	Age 77	431.348
Interest	10.358 +	Interest	7.549 +
	<u>602.241</u>		<u>438.897</u>
to pay	<u>45</u>	to pay	<u>36.5</u>
Age 75	557.241	Age 77½	402.397
Interest	9.752	Interest	7.042 +
	<u>566.993</u>		<u>409.439</u>
to pay	<u>43</u>	to pay	<u>35</u>
Age 75½	523.993	Age 78	374.439
Interest	9.17 +	Interest	6.553 +
	<u>533.163</u>		<u>380.992</u>

to pay	380.992 33.5	to pay	254.547 26
Age 78½	347.492	Age 81	228.547
Interest	6.081	Interest	4 +
to pay	353.573 32	to pay	232.547 24.5
Age 79	321.573	Age 81½	208.047
Interest	5.628 +	Interest	3.641 +
to pay	327.201 30.5	to pay	211.688 23
Age 79½	296.701	Age 82	188.688
Interest	5.192	Interest	3.302
to pay	301.893 29	to pay	191.99 21.5
Age 80	272.893	Age 82½	170.49
Interest	4.776 +	Interest	2.984 +
to pay	277.669 + 27.5	to pay	173.474 26
Age 80½	250.169	Age 83	153.474
Interest	4.378 +	Interest	2.686
	254.547		156.16

(83)

to pay	156.16 18.5	to pay	87.863 12
Age 83½	137.66	Age 86	75.863
Interest	2.409	Interest	1.328+
	140.069		77.191
to pay	17	to pay	11
Age 84	123.069	Age 86½	66.191
Interest	2.154+	Interest	1.158
	125.223		67.349
to pay	15.5	to pay	10
Age 84½	109.723	Age 87	57.349
Interest	1.92	Interest	1.004+
	111.643		58.353
to pay	14	to pay	9
Age 85	97.643	Age 87½	49.353
Interest	1.709+	Interest	.864+
	99.352		50.217
to pay	13	to pay	8
Age 85½	86.352	Age 88	42.217
Interest	1.511	Interest	.739+
	87.863		42.956

L 1

to

to pay	42.956 7	to pay	17.234 4
Age 88½ Interest	35.956 .63 +	Age 91 Interest	13.234 .232 +
to pay	36.586 6	to pay	13.466 3.5
Age 89 Interest	30.586 .535	Age 91½ Interest	9.966 .174
to pay	31.121 5.5	to pay	10.14 3
Age 89½ Interest	25.621 .448	Age 92 Interest	7.14 .125 +
to pay	26.069 5	to pay	7.265 2.5
Age 90 Interest	21.069 .369 +	Age 92½ Interest	4.765 .083
to pay	21.438 4.5	to pay	4.848 2
Age 90½ Interest	16.938 .296	Age 93 Interest	2.848 .05 +
	17.234		2.898

to

	2.898	Age 94	.422
to pay	1.5	Interest	.007
Age 93½	1.398		.429
Interest	.024	to pay	.5
	1.422	Deficient	.071
to pay	1		
	.422		

Most of the Calculations were made not only *thus* far, but also beyond, *before* the publication of Dr. Price's excellent *Observations on Reversionary Payments, &c.* in which (Page 333.) there is a Table of the *true* probabilities of Life, in *London*, which the Reader, no doubt, would wish to see compared with the preceding ones. It is therefore here inserted, and it will be found to give the purchase-money *more* than Mr. *Simpson's* Table from the Age of Five to Forty, though afterward *less* (162.), for Annuities by *Reversion*; but, for *immediate* Annuities it gives the purchase-money *more* than Mr. *Simpson's* Table till above Fifty years of age (131.).

CXVIII. *Shewing the true Probabilities of Life in London for all Ages. Formed from the Bills for Ten Years, from 1759 to 1768.*

<i>Ages.</i>	<i>Persons living.</i>	<i>Decr. of Life.</i>	<i>Ages.</i>	<i>Persons living.</i>	<i>Decr. of Life.</i>	<i>Ages.</i>	<i>Persons living.</i>	<i>Decr. of Life.</i>
0	1518	486	31	404	9	62	132	7
1	1032	200	32	395	9	63	125	7
2	832	85	33	386	9	64	118	7
3	747	59	34	377	9	65	111	7
4	688	42	35	368	9	66	104	7
5	646	23	36	359	9	67	97	7
6	623	20	37	350	9	68	90	7
7	603	14	38	341	9	69	83	7
8	589	12	39	332	10	70	76	6
9	577	10	40	322	10	71	70	6
10	567	9	41	312	10	72	64	6
11	558	9	42	302	10	73	58	5
12	549	8	43	292	10	74	53	5
13	541	7	44	282	10	75	48	5
14	534	6	45	272	10	76	43	5
15	528	6	46	262	10	77	38	5
16	522	7	47	252	10	78	33	4
17	515	7	48	242	9	79	29	4
18	508	7	49	233	9	80	25	3
19	501	7	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

CXIX. Let this Table be divided into *half-yearly* Decrements, as the others have been, and let the *Numbers living* be multiplied into each *half Year's* Reversion, as follows :

(70.)

Age.	Living.	Reversion.	Pres. Value of Pay.	Total of the
		(4-.)	to Survivors.	Products.
50	224			
50½	219½	X .982801	= 215.7248195	
51	215	X .965898	= 207.66807	
51½	210½	X .949285	= 199.8244925	
52	206	X .932959	= 192.189554	
52½	202	X .916912	= 185.216224	
53	198	X .901143	= 178.426314	£1000.62326
53½	194	X .885644	= 171.814936	
54	190	X .870412	= 165.37828	
54½	186½	X .855441	= 159.5397465	
55	183	X .840728	= 153.853224	
55½	179½	X .826269	= 148.3152855	£829.0125005
56	176	X .812058	= 142.922208	
56½	172½	X .798091	= 137.6706975	
57	169	X .784364	= 132.557516	
57½	165½	X .770875	= 127.5798125	
58	162	X .757616	= 122.733792	£689.0455195
58½	158½	X .744586	= 118.016881	
59	155	X .73178	= 113.4259	
59½	151	X .719194	= 108.598294	
60	147	X .706824	= 103.903128	
60½	143	X .694668	= 99.337524	£566.677995
61	139	X .682721	= 94.898219	
61½	135½	X .670978	= 90.917519	
62	132	X .659438	= 87.045816	
62½	128½	X .648096	= 83.280336	
63	125	X .636949	= 79.618625	£455.479414
63½	121½	X .625995	= 76.0583925	
64	118	X .615228	= 72.596904	
64½	114½	X .604647	= 69.2320815	
65	111	X .594248	= 65.961528	
65½	107½	X .584027	= 62.7829025	£363.467531
66	104	X .573982	= 59.694128	
66½	100½	X .564111	= 56.6931555	
67	97	X .554408	= 53.777576	
67½	93½	X .544873	= 50.9456255	

£282.87.3875

Age.	Living	Reversion.	Pres. Value of Pay. to Survivors.	Total of the Products.
68	90	X .535502	= 48.19518	
68½	86½	X .526292	= 45.524258	
69	83	X .51724	= 42.93092	
69½	79½	X .508344	= 40.413348	
70	76	X .499601	= 37.969676	
70	73	X .491007	= 35.843511	£215.033382
71	70	X .482563	= 33.77941	
71½	67	X .474263	= 31.775621	
72	64	X .466107	= 29.830848	
72½	61	X .45809	= 27.94349	
73	58	X .450212	= 26.112296	£159.47288
73½	55½	X .442468	= 24.556974	
74	53	X .434858	= 23.047474	
74½	50½	X .427379	= 21.5826395	
75	48	X .420028	= 20.161344	
75½	45½	X .412804	= 18.782582	£115.4607275
76	43	X .405705	= 17.445315	
76½	40½	X .398727	= 16.1484435	
77	38	X .391869	= 14.891022	
77½	35½	X .385129	= 13.6720795	
78	33	X .378505	= 12.490665	£80.939442
78½	31	X .371995	= 11.531845	
79	29	X .365597	= 10.602313	
79½	27	X .35931	= 9.70137	
80	25	X .35313	= 8.82825	
80½	23½	X .347057	= 8.1558395	£53.154443
81	22	X .341087	= 7.503914	
81½	20½	X .335222	= 6.872051	
82	19	X .329455	= 6.259645	
82½	17½	X .323789	= 5.6663075	
83	16	X .318221	= 5.091536	£34.457757
83½	14½	X .312747	= 4.5348315	
84	13	X .307368	= 3.995784	
84½	12	X .302082	= 3.624984	
85	11	X .296886	= 3.265746	
85½	10	X .291781	= 2.9178100	£20.5128815
86	9	X .286762	= 2.580858	
86½	8	X .28183	= 2.25464	
87	7	X .276983	= 1.938881	
87½	6	X .272219	= 1.633314	

£11.325503

Age.

<i>Age.</i>	<i>Living.</i>	<i>Reversion.</i>	<i>Pres. Value of Pay. to Survivors.</i>	<i>Total of the Payments.</i>
88	5	X .267538	= 1.33769	
88½	4½	X .262673	= 1.1820285	
89	4	X .258414	= 1.033656	
89½	3½	X .253969	= .8888915	
90	3	X .249601	= .748803	
90½	2½	X .245308	= .61327	£5.191069
91	2	X .241089	= .482178	
91½	1½	X .236943	= .3554145	
92	1	X .232865	= .232865	
92½	0½	X .228862	= .114431	
				£1.7981585
				Total £4885.245751

CXX. The *Stock* being thus obtained, let it be examined by adding each *half* year's Interest to it, and *paying* each *surviving* Member .1l. every *half* year.

The Capital Stock		4438.634	
for		to pay	202
Age 50 is £4885.246 +			
Interest 85.492		Age 52½	4236.634
		Interest	74.141
Total 4970.738 +			4310.775
to pay 219.5		to pay	198
Age 50½ 4761.238		Age 53	4112.775
Interest 83.147 +		Interest	71.974 +
4834.385			4184.749
to pay 215		to pay	194
Age 51 4619.385		Age 53½	3990.749
Interest 80.839		Interest	69.838
4700.224			4060.587
to pay 210.5		to pay	190
Age 51½ 4489.724		Age 54	3870.587
Interest 78.57		Interest	67.735
4568.294			3938.322
to pay 206		to pay	186.5
Age 52 4362.294		Age 54½	3751.822
Interest 76.34		Interest	65.657 +
4438.634			3817.479

to

to pay	3817.479 183	to pay	3235.474 165.5
Age 55 Interest	3634.479 63.603	Age 57½ Interest	3069.974 53.725+
to pay	3698.082 179.5	to pay	3123.699 162
Age 55½ Interest	3518.582 61.575	Age 58 Interest	2961.699 51.83 +
to pay	3580.157 176	to pay	3013.529 158.5
Age 56 Interest	3404.157 59.573+	Age 58½ Interest	2855.029 49.963
to pay	3463.73 172.5	to pay	2904.992 155
Age 56½ Interest	3291.23 57.597+	Age 59 Interest	2749.992 48.125+
to pay	3348.827 169	to pay	2798.117 151
Age 57 Interest	3179.827 55.647+	Age 59½ Interest	2647.117 46.325+
	3235.474		2693.442

M m

to

to pay	2693.442 147	to pay	2202.896 128.5
Age 60 Interest	2546.442 44.563 +	Age 62½ Interest	2074.396 36.302 +
to pay	2591.005 143	to pay	2110.698 125
Age 60½ Interest	2448.005 42.84	Age 63 Interest	1985.698 34.75 +
to pay	2490.845 139	to pay	2020.448 121.5
Age 61 Interest	2351.845 41.157	Age 63½ Interest	1898.948 33.232 +
to pay	2393.002 135.5	to pay	1932.18 118
Age 61½ Interest	2257.502 39.506	Age 64 Interest	1814.18 31.748
to pay	2297.008 132	to pay	1845.928 114.5
Age 62 Interest	2165.008 37.888 +	Age 64½ Interest	1731.428 30.3 +
	2202.896		1761.728

to

to pay	1761.728 111	to pay	1372.785 93.5
Age 65 Interest	1650.728 28.888 +	Age 67½ Interest	1279.285 22.387
to pay	1679.616 107.5	to pay	1301.672 90
Age 65½ Interest	1572.116 27.512	Age 68 Interest	1211.672 21.204
to pay	1599.628 104	to pay	1232.876 86.5
Age 66 Interest	1495.628 26.173	Age 68½ Interest	1146.376 20.062 +
to pay	1521.801 100.5	to pay	1166.438 83
Age 66½ Interest	1421.301 24.873 +	Age 69 Interest	1083.438 18.96
to pay	1446.174 97	to pay	1102.398 79.5
Age 67 Interest	1349.174 23.611 +	Age 69½ Interest	1022.898 17.901 +
	1372.785		1040.799

to pay	1040.799 76	to pay	765.754 61
Age 70 Interest	964.799 16.884 +	Age 72½ Interest	704.754 12.333
to pay	981.683 73	to pay	717.087 58
Age 70½ Interest	908.683 15.902	Age 73 Interest	659.087 11.534
to pay	924.585 70	to pay	670.621 55.5
Age 71 Interest	854.585 14.955	Age 73½ Interest	615.121 10.765 +
to pay	869.54 67	to pay	625.886 53
Age 71½ Interest	802.54 14.044	Age 74 Interest	572.886 10.026 +
to pay	816.584 64	to pay	582.912 50.5
Age 72 Interest	752.584 13.17	Age 74½ Interest	532.412 9.317
	765.754		541.729

to pay	541.729 48	to pay	363.807 35.5
Age 75 Interest	493.729 8.64	Age 77½ Interest	328.307 5.745
to pay	502.369 45.5	to pay	334.052 33
Age 75½ Interest	456.869 7.995	Age 78 Interest	301.052 5.268
to pay	464.864 43	to pay	306.32 31
Age 76 Interest	421.864 7.383+	Age 78½ Interest	275.32 4.818
to pay	429.247 40.5	to pay	280.138 29
Age 76½ Interest	388.747 6.803	Age 79 Interest	251.138 4.395+
to pay	395.55 38	to pay	255.533 27
Age 77 Interest	357.55 6.257	Age 79½ Interest	228.533 3.999
	363.807		232.532

(96)

to pay	232.532 25	to pay	137.418 17.5
Age 80 Interest	207.532 3.632+	Age 82½ Interest	119.918 2.094
	211.164		122.012
to pay	23.5	to pay	16
Age 80½ Interest	187.664 3.284	Age 83 Interest	106.012 1.855
	190.948		107.867
to pay	22	to pay	14.5
Age 81 Interest	168.948 2.957	Age 83½ Interest	93.367 1.634+
	171.905		95.001
to pay	20.5	to pay	13
Age 81½ Interest	151.405 2.65	Age 84 Interest	82.001 1.435
	154.055		83.436
to pay	19	to pay	12
Age 82 Interest	135.055 2.363	Age 84½ Interest	71.436 1.25
	137.418		72.686

to pay 72.686
11

Age 85 61.686
Interest 1.08 +

62.766
to pay 10

Age 85½ 52.766
Interest .923

53.689
to pay 9

Age 86 44.689
Interest .782

45.471
to pay 8

Age 86½ 37.471
Interest .656 +

38.127
to pay 7

Age 87 31.127
Interest .545 +

31.672

to pay 31.672
6

Age 87½ 25.672
Interest .449

26.121
to pay 5

Age 88 21.121
Interest .37 +

21.491
to pay 4.5

Age 88½ 16.991
Interest .297

17.288
to pay 4

Age 89 13.288
Interest .233 +

13.521
to pay 3.5

Age 89½ 10.021
Interest .175

10.196

to

(98)

to pay	10.196 3	to pay	2.957 1.5
Age 90 Interest	7.196 .126 +	Age 91½ Interest	1.457 .025
to pay	7.322 2.5	to pay	1.482 1
Age 90½ Interest	4.822 .084	Age 92 Interest	.482 .008
to pay	4.906 2	Last payment	.490 .5
Age 91 Interest	2.906 .051	Deficient	.01
	2.957		

CXXX

CXXI. If a Society should pay *double* the Annuity that has been *purchased*, it is worth enquiring *how long* it might subsist. The Table (117.) shews that £4444.828 would pay 204 Persons aged *Fifty*, an annuity of 2*l.* during *Life*: but if the Society was to pay *Four Pounds Annuity*; then, the *Ages*, *Number of Recipients*, and *Money* to be paid, would be *half-yearly* as follows:

<i>Ages.</i>	<i>Recipients.</i>	<i>To pay double Number of Pounds.</i>
50		
50½	— 200	— 400 <i>£</i>
51	— 196	— 392
51½	— 192	— 384
52	— 188	— 376
52½	— 184	— 368
53	— 180	— 360
53½	— 176	— 352
54	— 172	— 344
54½	— 168½	— 337
55	— 165	— 330
55½	— 161½	— 323
56	— 158	— 316
56½	— 154½	— 309
57	— 151	— 302
57½	— 147½	— 295

(100)

Stock	£4444.828	Age 52½	2857.779
Interest	77.784	Interest	50.011
	4522.612		2907.79
to pay	400	to pay	360
Age 50½	4122.612	Age 53	2547.79
Interest	72.146+	Interest	44.586
	4194.758		2592.376
to pay	392	to pay	352
Age 51	3802.758	Age 53½	2240.376
Interest	66.548	Interest	39.207+
	3869.306		2279.583
to pay	384	to pay	344
Age 51½	3485.306	Age 54	1935.583
Interest	60.993+	Interest	33.873+
	3546.299		1969.456
to pay	376	to pay	337
Age 52	3170.299	Age 54½	1632.456
Interest	55.48	Interest	28.568+
	3225.779		1661.024
to pay	368	to pay	330
	2857.779		1331.024

to

Age 55	1331.024				746.199
Interest	23.293 +	to pay			309
	1354.317	Age 56½			437.199
to pay	323	Interest			7.651 +
Age 55½	1031.317				444.85
Interest	18.048	to pay			302
	1049.365	Age 57			142.85
to pay	316	Interest			2.5 +
Age 56	733.365				145.35
Interest	12.834 +	to pay			295
	746.199	Deficient			149.65

It is evident, in *this* case, that there would be *wanting* £149.65 to make good the *Fifteenth half-yearly Payment*; so that the Members would subsist no longer than *Seven years and a Half* upon their *own* money. — The Society might consist of *younger* Members, who, not being yet Recipients, *their* money makes a part of the Capital Stock, that, if suffered to increase by Interest, would be of *equal value* with what the Age Fifty had. Nothing can be plainer, than that the Stock belonging to these *younger* Members should be *inviolably* preserved entire, that it might grow to a *sufficiency* to pay *each of* THEM the Annuity he purchased

purchased; and which it may be seen (*a*) it would do. Those Members who were the *first* Recipients, and, by receiving *double* the Annuity they purchased, should have exhausted their *own* Stock, could not, with the least JUSTICE, apply the money belonging to the *younger* Members to their *own* use; or if they did, they would spend the Stock of the *same* number of *young* Members as the *old* ones were, in *much less* time; and then certainly there could be *nothing* remaining for the *youngest* Members;—they would spend it in *much less* time; because the Stock of a young Member, suppose of *Twenty* years of age, is worth *not near* so much as it would be, if it continued at Compound Interest till he became aged *Fifty*. For Example, the Stock of *three* Members, at *Fifty* years old, would be for 1 l. Annuity,

$$(b) 10.894 \times 3 = 32.682 \quad (131.)$$

Age 15 worth only (<i>a</i>)	1.341	(162.)
25	2.1913	+ (162.)
35	3.8843	(162.)
	7.4166	

(*b*) By Mr. SYMPSON'S Table.

(*a*) Though these sums appear small, when compared with the £10.894, which Age 50 pays for an *immediate* annuity; yet they

The total of the Stock of all the *three* above young Members is not a *quarter* of what it would be when they should be aged *Fifty*; by which it may be easily conceived what havock the *first* Recipients would make with sums that are *as yet* so *small*.

they are in the *very same proportion*; for the sum that either of the young ages pay, will accumulate *interest*; and, by the chance of *Mortality*, become of the *same value*, when the age attains to 50, as what 50 now pays. Though this must be evident from many following instances, yet an Example *here* may be useful.

492 Persons living, aged 15 years, pay each £ 1.341; the Product is,

$$1.341 \times 492 = 659.772$$

This product will be accumulating Interest 35 years; which number of years the ages *want* of 50; therefore, the sum must be *divided* by 35 years Reversion (47.) to shew what it gains by Compound Interest (46.).

$$.296886) 659.772 (2222.311$$

There is found only 204 who attain to the age of *Fifty* (62.), and the accumulated sum is to be *divided* among the *Survivors*:

$$204 \rightarrow 2222.311 (10.894 (131.)$$

And thus the *small* sum of 1.341 (if suffered to *increase*) would be worth (when aged *Fifty*) the *same* money that the Age of *Fifty* pays for a *present* Annuity. If the *old* Age converts that small sum to his *own* use, he will be but *little* benefited by it himself, though he totally deprives the *young* Age of his expectation.

CXXII. Annuitants have been hitherto considered as *forming a Society*; and how necessary it is that men should *associate*, may be illustrated by supposing *single* annuitants in a particular case or two.

Suppose a *single* Annuitant to receive *double* the value he had actually purchased; *how long* would the Stock pay such Annuity?

The age of Fifty, on paying £21.788^(131.) Mr. *Sympson's* Table, *present* money, ought to receive 2 *l.* annually. If instead of 2 *l.* he should receive 4 *l.* that is 2 *l.* *half-yearly*, the consequence would be found as follows:

The Stock of Age 50, present money	—	—	} £21.788
The <i>first</i> Half-year's Interest is,			
			.381
Total, when added,	—		22.169
To pay Half-year's Annuity			£ 2
The Stock remaining the <i>second</i> half-year	—	—	} 20.169
Interest	—	—	
			.353 +
			20.522
To pay	—		2
Age 51 Stock			18.522
Interest	—		.324
			18.846
			to

(105)

to pay	18.846 2	Interest	8.017 .1
Age 51½	16.846	to pay	8.157
Interest	.295	Age 54½	6.157
	17.141	Interest	.108
to pay	2		6.265
Age 52	15.141	to pay	2
Interest	.265	Age 55	4.265
	15.406	Interest	.075
to pay	2		4.34
Age 52½	13.406	to pay	2
Interest	.235	Age 55½	2.34
	13.641	Interest	.041
to pay	2		2.381
Age 53	11.641	to pay	2
Interest	.204	Age 56	0.381
	11.845	Interest	.007+
to pay	2		0.388
Age 53½	9.845	to pay	2
Interest	.172	Deficient	£1.612
	10.017		
to pay	2		
Age 54	8.017		

There

There is but a trifle remaining after paying *six* years ; whereas, by the last case, the Stock paid an annuity of *double* value *seven years and a quarter*. The reason of the difference is, that the *single* Annuitant, *if he survives*, always receives here the *same full* sum ; but in a Society the Recipients and sums paid are *less and less each half year*.

CXXIII. Suppose one *single* person, only, to have an annuity of 2 *l.* at the age of *Fifty*, for which he pays (131.)

$$£10.894 \times 2 = £21.788,$$

and is to receive 1 *l.* every *half* year. By the Table (116.) there are 204 Members living when *Fifty* years old ; and there are 102, which is one *half*, living when aged *Sixty-four and half* ; then it is, an *even chance* if a person aged *Fifty*, shall be among the *half* that is *dead*, or the *half* that is *living*, at the end of 14 years and *half* ; and therefore, it is an *even chance* whether a person aged *Fifty* shall live 14 years and a *half*, or not ; consequently the money that is paid for his Annuity should be sufficient to pay that Annuity *fourteen years and a half*, and *no longer*. Let this be put in practice, by *adding* the *Interest* to the Stock, and paying 1 *l.* out of it every *half* year. For 2 *l.* annuity the Stock is for Age *Fifty*, £21.788.

Age

Age 50	£21.788	Age 52½	18.5842
Interest	.3813+	Interest	.3252
	22.1693		18.9094
to pay	I	to pay	I
Age 50½	21.1693	Age 53	17.9094
Interest	.3705+	Interest	.3134
	21.5398		18.2228
to pay	I	to pay	I
Age 51	20.5398	Age 53½	17.2228
Interest	.3594	Interest	.3014
	20.8992		17.5242
to pay	I	to pay	I
Age 51½	19.8992	Age 54	16.5242
Interest	.3482	Interest	.2892
	20.2474		16.8134
to pay	I	to pay	I
Age 52	19.2474	Age 54½	15.8134
Interest	.3368	Interest	.2767
	19.5842		16.0901
to pay	I	to pay	I
	18.5842		15.0901

Age 55	15.0901	Age 57½	11.2795
Interest	.2641 +	Interest	.1974 +
to pay	15.3542	to pay	11.4769
Age 55½	14.3542	Age 58	10.4769
Interest	.2512 +	Interest	.1833
to pay	14.6054	to pay	10.6602
Age 56	13.6054	Age 58½	9.6602
Interest	.2381 +	Interest	.1691 +
to pay	13.8435	to pay	9.8293
Age 56½	12.8435	Age 59	8.8293
Interest	.2248 +	Interest	.1545 +
to pay	13.0683	to pay	8.9838
Age 57	12.0683	Age 59½	7.9838
Interest	.2112 +	Interest	.1397
to pay	12.2795	to pay	8.1235
	11.2795		7.1235
			Age

(109)

Age 60	7.1235	Age 62	3.5292
Interest	.1247 +	Interest	.0618 +
to pay	7.2482	to pay	3.5910
	I		I
Age 60½	6.2482	Age 62½	2.591
Interest	.1093	Interest	.045
to pay	6.3575	to pay	2.636
	I		I
Age 61	5.3575	Age 63	1.636
Interest	.0938 +	Interest	.029 +
to pay	5.4513	to pay	1.665
	I		I
Age 61½	4.4513	Age 63½	0.665
Interest	.0779 +	Interest	.012 +
to pay	4.5292	to pay	0.677
	I		I
	3.5292	Deficient	.323

By the Table (117.) the *same* sum would pay the *same* annuity to Survivors, to *very old* Age, when a number are *associated* together; whereas it appears *here* that a person, *unconnected* with any other, would expend the whole Stock, that should last 14 years and a *half*, before 14 years * complete; it is also plain that if the Assured lives more than 14 years, the Assurer must *lose*, and that if he dies in less time, the Assurer will *gain* in proportion †.

CXXIV. In Society it is supposed that chance will be more *equal*, and for every one that dies before the time of *equal chance*, another will survive; also when a person has survived one chance, he will have an equal chance for surviving the next: Thus if Age 50 attains to 64½ he will have an equal chance of living 9 years longer; and if he attains to 73½, he will have an equal chance of living 7½ years; and so on to extreme old age,

CXXV. Suppose A and B were two Annuity-tants at the Age of 50, and that A should die immediately, without receiving the least part of the Annuity; he would die 14 years and a half *before* the time that he had an *equal chance* of living; this being the *extreme* chance on the side of *death*, the *extreme* chance

* See (73.)

† See the Addenda.

chance of *life* would not only be for *twice* 14 years and a half, that is, till Age 79, but also to the *extremest* age; so that the person who first dies of a Society being A, the last Survivor may be called B; and, in this case, B would have A's capital Stock, as well as his own, to furnish him with 2 *l.* annuity *during life*. Let it be examined how long the Stock would pay that annuity.

The Stock of A <i>deceased</i> , for }	21.788
2 <i>l.</i> annuity, — }	
Ditto of B the <i>Survivor</i> , for }	21.788
2 <i>l.</i> annuity, — }	
Total — —	£43.576
Half-year's Interest	.763 +
	44.339
To pay $\frac{1}{2}$ Year's Annuity, I	
Age 50 $\frac{1}{2}$	43.339
Interest	.758
	44.097
to pay I	
Age 51	43.097
Interest	.754
	43.851
	to

to pay	<u>43.851</u> I	to pay	<u>42.557</u> I
Age 51½	42.851	Age 54	41.557
Interest	.75 +	Interest	.727
	<u>43.601</u>		<u>42.284</u>
to pay	<u>I</u>	to pay	<u>I</u>
Age 52	42.601	Age 54½	41.284
Interest	.746 +	Interest	.722
	<u>43.347</u>		<u>42.006</u>
to pay	<u>I</u>	to pay	<u>I</u>
Age 52½	42.347	Age 55	41.006
Interest	.741	Interest	.718 +
	<u>43.088</u>		<u>41.724</u>
to pay	<u>I</u>	to pay	<u>I</u>
Age 53	42.088	Age 55½	40.724
Interest	.737 +	Interest	.713 +
	<u>42.825</u>		<u>41.437</u>
to pay	<u>I</u>	to pay	<u>I</u>
Age 53½	41.825	Age 56	40.437
Interest	.732 +	Interest	.708 +
	<u>42.557</u>		<u>41.145</u>

to pay	<u>41.145</u> 1	to pay	<u>39.605</u> 1
Age 56½	40.145	Age 59	38.605
Interest	.703 +	Interest	.676 +
to pay	<u>40.848</u> 1	to pay	<u>39.281</u> 1
Age 57	39.848	Age 59½	38.281
Interest	.697	Interest	.67 +
to pay	<u>40.545</u> 1	to pay	<u>38.951</u> 1
Age 57½	39.545	Age 60	37.951
Interest	.692	Interest	.664
to pay	<u>40.237</u> 1	to pay	<u>38.615</u> 1
Age 58	39.237	Age 60½	37.615
Interest	.687 +	Interest	.658
to pay	<u>39.924</u> 1	to pay	<u>38.273</u> 1
Age 58½	38.924	Age 61	37.273
Interest	.681	Interest	.652
	<u>39.605</u>		<u>37.925</u>

to

to pay	37.925 I	Age 62½	36.211
		Interest	.634+
Age 61½	36.925	to pay	36.845
Interest	.646		I
	37.571	Age 63	35.845
to pay	I	Interest	.627
			36.472
		to pay	I
Age 62	36.571	Age 63½	35.472
Interest	.64 +	Interest	.621
	37.211		36.093
to pay	I	to pay	I
	36.211	Age 64	35.093

It was seen before (123.) that the Stock of a *single* Annuitant would be totally expended in 28 *half* years; and yet *here* B has received 28 payments, and has £35.093 still remaining. The reason of this is, that, although B has actually expended the whole of his *own* Stock; yet that of A's not only remains *untouched*, but has been *gaining* 14 years Compound Interest.

A's Stock was £21.788; and 14 years, or 28 half years (42*.) Compound Interest is 1.6254; which two sums, multiplied into each other, produce £35.4142, which is rather

rather more than the now *remaining* Stock of B, and £13.6262 more than the Stock *was* when left him *at first* by A—to proceed,

Age 64	£35.093	Age 66	33.507
Interest	.614	Interest	.586
to pay	35.707	to pay	34.093
Age 64½	34.707	Age 66½	33.093
Interest	.607	Interest	.579
to pay	35.314	to pay	33.672
Age 65	34.314	Age 67	32.672
Interest	.6 +	Interest	.572 +
to pay	34.914	to pay	33.244
Age 65½	33.914	Age 67½	32.244
Interest	.593	Interest	.564
to pay	34.507	to pay	32.808
	33.507		31.808

(116)

Age 68	31.808	Age 70½	29.513
Interest	.557	Interest	.516
to pay	32.365	to pay	30.029
	I		I
Age 68½	31.365	Age 71	29.029
Interest	.549+	Interest	.508
to pay	31.914	to pay	29.537
	I		I
Age 69	30.914	Age 71½	28.537
Interest	.541	Interest	.499
to pay	31.455	to pay	29.036
	I		I
Age 69½	30.455	Age 72	28.036
Interest	.533	Interest	.491
to pay	30.988	to pay	28.527
	I		I
Age 70	29.988	Age 72½	27.527
Interest	.525+	Interest	.482+
to pay	30.513	to pay	28.009
	I		I
	29.513		27.009

Age

Age 73	27.009	Age 75½	24.278
Interest	.473+	Interest	.425+
to pay	27.482	to pay	24.703
Age 73½	26.482	Age 76	23.703
Interest	.463	Interest	.415+
to pay	26.945	to pay	24.118
Age 74	25.945	Age 76½	23.118
Interest	.454	Interest	.405+
to pay	26.399	to pay	23.523
Age 74½	25.399	Age 77	22.523
Interest	.444	Interest	.394
to pay	25.843	to pay	22.917
Age 75	24.843	Age 77½	21.917
Interest	.435	Interest	.384+
to pay	25.278	to pay	22.301
	24.278		21.301

to

Age 78	21.301	Age 80½	18.054
Interest	.373	Interest	.316+
	21.674		18.37
to pay	I	to pay	I
Age 78½	20.674	Age 81	17.37
Interest	.362	Interest	.304+
	21.036		17.674
to pay	I	to pay	I
Age 79	20.036	Age 81½	16.674
Interest	.351	Interest	.292+
	20.387		16.966
to pay	I	to pay	I
Age 79½	19.387	Age 82	15.966
Interest	.339	Interest	.279
	19.726		16.245
to pay	I	to pay	I
Age 80	18.726	Age 82½	15.245
Interest	.328	Interest	.267+
	19.054		15.512
to pay	I	to pay	I
	18.054		14.512

Age 83	14.512	Age 85½	10.649
Interest	.254 +	Interest	.186
	14.766		10.835
to pay	I	to pay	I
Age 83½	13.766	Age 86	9.835
Interest	.241 +	Interest	.172
	14.007		10.007
to pay	I	to pay	I
Age 84	13.007	Age 86½	9.007
Interest	.228	Interest	.158 +
	13.235		9.165
to pay	I	to pay	I
Age 84½	12.235	Age 87	8.165
Interest	.214	Interest	.143 +
	12.449		8.308
to pay	I	to pay	I
Age 85	11.449	Age 87½	7.308
Interest	.2	Interest	.128
	11.649		7.436
to pay	I	to pay	I
	10.649		6.436

to

Age 88	6.436				
Interest	.113	to pay	3.792		
			I		
	6.549				
to pay	I	Age 90	2.792		
		Interest	49 +		
Age 88½	5.549		2.841		
Interest	97	to pay	I		
	5.646	Age 90½	1.841		
to pay	I	Interest	32		
Age 89	4.646		1.873		
Interest	81	to pay	I		
	4.727	Age 91	.873		
to pay	I	Interest	15		
Age 89½	3.727		.888		
Interest	65	to pay	I		
	3.792	91½ Deficient	.112		

It is now seen, that although the Stock of a single Annuitant aged 50, would not last quite 14 years, yet, by having the whole Stock of *another* Annuitant added to his *own*, it will supply the Annuity 41 years and near a half; that is, *nearly* the extent of life.

CXXVI. Again, let it be supposed that of the two Annuitants, A and B, aged 50; the Annuitant

nuitant A does *not* die *immediately*, as before, but lives till aged 57, and *then* dies, leaving the remainder of his Stock to the Survivor B: Let it be examined how long the *joint* Stock would supply an Annuity of 2 l. *per annum*?

Stock of A (<i>as before</i>) (123.)	21.788
Ditto of B —	21.788
Total of the joint Stock	43.576
Half a year's Interest —	.763+
	44.339
To pay $\frac{1}{2}$ years Annuity to two Members — }	2
	42.339
Stock, when aged 50 $\frac{1}{2}$	42.339
Interest .	.741
	43.08
to pay —	2
	41.08
Age 51	41.08
Interest	.719
	41.799
to pay —	2
	39.799
Age 51 $\frac{1}{2}$	39.799
Interest ..	.696
	40.495
	to

to pay	40.495 2	to pay	33.625 2
Age 52 Interest	38.495 .674	Age 54½ Interest	31.625 .553
to pay	39.169 2	to pay	32.178 2
Age 52½ Interest	37.169 .65	Age 55 Interest	30.178 .528
to pay	37.819 2	to pay	30.706 2
Age 53 Interest	35.819 .627	Age 55½ Interest	28.706 .502
to pay	36.446 2	to pay	29.208 2
Age 53½ Interest	34.446 .601	Age 56 Interest	27.208 .476
to pay	35.047 2	to pay	27.684 2
Age 54 Interest	33.047 .578	Age 56½ Interest	25.684 .449
33.625		to pay	26.133 2
		Age 57	24.133

Here

Here A, having received *fourteen half* years, is supposed to *die*, and leave his Stock for the Benefit of B; and what he leaves is the *same* as a *single* Annuitant would leave at the end of 7 years (123, Age 57.); viz. £12.0683 :—for the half of the joint Stock remaining above is £12.0665, very nearly the same; the small difference being occasioned only by taking *four decimal* places in the one, and but *three* in the other.

It is known that B's *own* Stock will support the Annuity 7 years (123.) longer; and therefore the Stock left by A will be, untouched, *seven* years accumulating Compound Interest; and, consequently, must become of *greater* value than B's own Stock: — B has now the Stock of £24.133; and there is no need of seeking how long this would last, because it is known by the last Table (125.); where, at opposite the age of 76, is £23.703, which paid nearly 31 half years; therefore £24.133 will pay *something* more; for B's own Stock will supply him fourteen *half* years from *this* time; and A's *improved* Stock will pay *eighteen* nearly; for the £12.0665, left by A, at seven years Compound Interest, that is, multiplied by 1.2749, will produce £15.38358; and £15.245, at age 82½, paid *eighteen* half years in the Table (125.): So that, because A died aged 57, B will have Stock sufficient to support him till he is near the age of 73, which is 9 years longer than his own Stock could

Q q have

have supplied ; which advantage is gained by hazarding the chance of *receiving it*, or of *dying himself* first, and leaving the *same benefit* to A, in case he should have been the survivor.

CXXVII. Perhaps these cases of Single and Double Annuitants, simply considered, may assist the imagination in obtaining an idea of a large Society, consisting of one or two thousand Members or more, where *all ages* are intermixed ; and where, for every one that *dies* without receiving *any* benefit, another may *live* to upwards of *Ninety* years of age ; or for every one that *dies* at *Fifty-seven*, another *lives* to *Seventy-three*, and in *like* proportion for *all ages* : So that there would be neither Loss or Gain to the general Stock, chance being *here* nearly *equal* (125.) ; and as surely as some would *live beyond* the *Equal (a)* Chance of Life, as surely some would *die before*, and leave a *sufficiency* to support the *Aged* ; for by the Table (116.), if a Society consisted of 294 Members aged 40, *one half* would have died in the proportions there seen in 17½ years, and they would leave a *sufficiency* in the Fund to *maintain the other half* during *life*, provided they also die according to the proportion given by that Table ; but if there should be some deviation from it, it would
cause

(a) What is meant by an *Equal* Chance of Life, has been explained (123.)

cause but a *small* difference among such a number (65.): whereas, of *two* Members *only* taken in consideration, the one must survive a certain time *beyond* the Equal Chance of Life, as much and no more than in proportion as the other died *before*; otherwise there would be a *deficiency*, or a *superfluity*, of the Stock.

But if a Single Annuitant, aged 50, unconnected with any one, lives upon his own Stock of £21.788, and takes from it 1 l. every half year, his whole support would be exhausted in less than 14 years, without the hope of being *benefited* from the *demise* of others; because he gave no chance of *others* being benefited by *him*. This may shew the advantage of association.

Having premised these Instances, and Tables being prepared for finding the value of an annuity for life, let the advantages made by *risking* the principal, be examined.

CXXVIII. Any age may preserve a Principal entire, and receive an interest during life, and leave it entire at his demise; but suppose the age *Forty* would choose an *immediate annuity*; that is, to *lose* the Principal at the time of *death*, to receive *greater Interest* during *life*; How many years purchase must he pay, and *what Interest* would be made of the money?

It is proved by the Table (117.), that £7283.474 would, according to the supposition of that Table, pay 294 persons, aged

Q q 2

Forty,

Forty, an annuity of 2 *l.* during the lives of all of them, and leave *no* Stock remaining after the *last* payment; which shews *that* sum to be neither too much or too little, but just sufficient; and therefore the *true* one. The share of *one* Member to pay towards it must, certainly, be the whole sum divided by the 294 persons who share in it ;

$$294) 7283.474 (24.7737$$

£24.7737 being for 2 *l.* annuity, the *half* of it, £12.38685, is for 1 *l.*; and the number of pounds that is paid for 1 *l.* annuity is the number of *years purchase* given, and the Answer to the first part of the Question.

CXXIX. To find the interest, it being known that £12.38685 yields 1 *l.*; then, by the Rule of Three, What will 100 *l.* give? 100 *l.* divided by £12.38685 will give the Answer (33.):

$$12.38685) 100.0 \text{ \&c. } (8.073077$$

which, when reduced, is £8 1 5½ Interest *per cent.* so that when the Age *Forty* preserves his principal, and receives 3½ *l.* interest *per annum*, he might, by sinking it, receive above 8 *l. per cent. during life.*

CXXX. How many years purchase must the Age *Sixty* give, to receive an *immediate* annuity; and what interest will be made of the sum paid?

How

How to answer this was shewn (110.), and an *easier* method was promised (111.) to be explained, which is this : Instead of looking into the Tables CVII, CXIII, CXVI, CXIX, which find the Capital Stock, look into the Proof Tables CXII, CXIV, CXVII, CXX, for the age required, and directly *opposite* the age is the *sum* that will pay that age ; and directly *above* the sum of money is the *number of persons* of that age which the sum will pay. — Thus, in the Table (117.) opposite the age *Sixty* is the sum of £2352.59, and immediately *above* that is 130, the number of persons living at that age ; and the Table itself proves, that the sum will pay the number of persons of that age an *immediate annuity during life* ; in the *same* manner as before was seen in the case of Age Forty (72.) : therefore the *sum divided* by the *number* of Members will give the share of *each* in the Quotient.

$$130) 2352.59 (18.0968$$

the *half* is 9.0484 for the sum to be paid for 1 l. annuity, and is also the number of *years purchase*. — The interest is found, as before (129.),

$$9.0484) 100.0, \text{ \&c. } (11.0516$$

which is above Nine years purchase, and more than £11 1 0 interest *per cent. per annum*, made of the money.

In

In the same manner the value of an *immediate* annuity may be found by any of the Tables, and for any age that the Tables contain; examples of which, for *every fifth* year, are exhibited in the following Table; in which the Reader may observe the different *values* produced both by different *ages*, and by different *Tables*; and the interest made of the purchase money in the last column, by Mr. *Simpson's* Table,

CXXXI. A Table to shew the value in *present* money, or the number of years purchase paid for 1 *l.* *immediate* annuity, during *life*, by *four* different Tables of Mortality; with the *interest* made of the money paid according to Mr. *Simpson's* Table.

Age.	London Bills for 43 years	Dr. Halley's at Breslau	Dr. Price's in London.	Mr. Simpson's London.	Interest per Cent. by Simpson's Table.
40				12.38685	£ 8 1 5
45				11.7185	£ 8 10 8
50	10.7459	11.6356	10.9045	10.894	£ 9 3 7
55	9.8658	10.5292	9.93	10.027	£ 9 19 5½
60	8.8514	9.1796	8.66135	9.0484	£ 11 1 0½
65	7.7873	7.6948	7.4355	7.9855	£ 12 11 3½
70	6.7023	6.0777	6.347	7.0714	£ 14 2 10
75	5.5717	4.7347	5.143	6.19156	£ 16 3 0½
80	4.4965	3.9255	4.1506	4.705	£ 21 5 1
85	3.154	3.0076	2.804	3.487	£ 28 13 6½

The values in present money were found by dividing the sums opposite each age, in the different Tables, by the number of persons immediately *above* each sum (130.);— and the interest was found by dividing 100 *l.* by the value (129.) of the life, in present money,

CXXXII.

CXXXII. This Table may furnish an argument, from common practice, to convince such as may not understand Figures, that the Calculations are *not* rated *too high*; for, at the general rate of $3\frac{1}{2}$ *per cent.* interest, the age *Forty* is allowed above 8 *l. per cent.*; the age *Sixty*, above 11 *l. per cent.*; and the age *Eighty*, above 20 *l. per cent.* interest, for their purchase money. Let this question be answered impartially: Where may these ages apply to have *such* immediate annuities granted for life, and *perfectly secure*?

It may be answered, that there is frequently more than 8 *per cent.* advertised in the Public Papers, and well-secured by Parish Rates, Tolls on Roads, Bridges, and Rivers, and very often *Landed Security*.—But let it be considered, that Tolls are not always *certain*; the money raised is generally to make a *new* Road, Bridge, or *new* Navigation; which, when completed, the Tolls thereon may *not* answer the expectations of the Borrowers or Lenders; and if Parishes give at any time *extraordinary* interest, it is on some emergency, such as Building or Repairing Churches, Work-houses, &c. and in such cases, the Annuities granted may, perhaps, be above the common Rate, for the sake of obtaining Ready-money to execute the Work immediately.—When advantageous Annuities are offered on Land Security, it may be presumed, that the Borrower may have some means of making larger Interest by Trade,
or

or otherwise; or that the exigency of his affairs requires immediate cash, for which he would give *more than its value*; rather than be still in want of it. And granting all these to be *perfectly secure*, yet they do not prove the Annuities by them allowed are at the *common* Rate of Interest, and that *every Individual* may obtain the *same* at any time. There are particular cases when money is borrowed at 5 *l. per cent.* with a *premium*; perhaps, given beside; but such do not prove 5 *l. per cent.* to be the *General Rate* of Interest. If it was, who would buy into the Public Funds to receive but 3½ *l.*? It is not needful to insist farther on this, as every one knows that most of the *advantageous* offers of *these kinds* fall into the hands of People who make a *Trade* of watching for and snatching them up, called *Usurers*, and that at *all times* *all* persons cannot obtain them; therefore they are but little to the present purpose.

CXXXIII. Or if interest really was 5 *l. per cent.* Mr. *Simpson's* Table of Mortality would allow full as much interest made by the purchase money, as, perhaps, by any of those *extraordinary* cases. To satisfy the Reader, if he should have scruples, let it be examined what Annuities might be granted, if Interest was 5 *l. per cent.*?

In Mr. *Simpson's* *Doctrine of Annuities*, page 38; and in Dr. *Price's* *Observations*, &c. page 328; there is a complete Table of
the

the value of 1 *l.* annuity, at 3 *l.* 4 *l.* and 5 *l.* per cent. for any age from Six to Seventy-five inclusive, calculated by Mr. *Simpson* from his own Table of Mortality, out of which the Ages and Values in the following Table are taken, and the *Interest* made of the purchase-money is added in the next column ; being found by dividing 100 *l.* by the value of each life (129.).

Age.	Years Purchase.	Interest per Cent.	
10	14.3	6.913	
15	13.9	7.194	
20	13.0	7.692	
25	12.3	8.13	
30	11.6	8.6207	Totals.
35	10.9	9.174	38.6297
40	10.3	9.7087	
45	9.8	10.204	
50	9.2	10.869	
55	8.5	11.7647	
60	7.9	12.658	51.7204
65	7.1	14.0845	
70	6.2	16.129	
75	5.2	19.23	

62.1015

Total of Interest is £152.4516 for £1400 borrowed.

At the rate of 5 *l.* per cent. The Age Twenty-five makes more than 8 *l.* per cent. interest of what money he gives for an im-

R r

mediate

mediate annuity ; the Age Sixty makes more than 12 l. ; and the Age Seventy-five makes more than 19 l. Surely this is as much as given in the most extraordinary case, scarcely excepting even these two following remarkable Advertisements, that appeared daily in the Papers :

“ Eleven per Cent. for Life given to all Ages.

“ Any Persons desirous of purchasing Annuities for their own, or any other Life or Lives, may have the same secured on a Freehold Estate, and regularly paid at a Banker’s in London. For farther Particulars apply to R. W. at Tom’s Coffee-house, Cornhill.”

“ Eleven per Cent. for Life given to all Ages.

“ Annuity Office, Numb. 8. Pope’s-Head-Alley, Cornhill.

“ Persons inclined to purchase Annuities for their own Lives, or the Lives of others, of any Age, may have the same well secured, and regularly paid half-yearly, at a Banker’s in London. Further Particulars may be had by applying personally, or by Letter, to this Office.”

The total of the Interests in the last Table is £152.4516, to be paid for 1400 l. borrowed ; for if Age Ten pays 100 l. he would have 6.993 annual interest ; and Age Seventy-five would have £19.23 for 100 l. paid down ; and each *intermediate* age would have for 100 l. the yearly interest or annuity that is inserted opposite to it. If, then, 152.4516 be divided by 14, the quotient will be the equal annuity for every age to receive alike, without regarding any difference between old and young.

14) 152.4516 (£10.8894

It would be *indifferent* to the *Borrower*, whether he paid every *Annuitant* an annuity according to the *Age*, as set down in the *Table*, or whether he paid £10.8894 to *each alike*; for the *Total* sum paid would be the *same*; therefore these *Calculations* at 5 *l. per cent.* interest, and *Mr. Simpson's Table of Mortality* will *very nearly* pay 11 *l. per cent.* to *all ages*, and certainly then they cannot be thought to be estimated at *too high* a rate for purchasing.

CXXXIV. But though it be *indifferent* to the *Borrower*, whether the £152.4516 interest be paid in *equal* portions, or in such as each age should receive; yet it certainly is *not so* with respect to the *Annuitants*. Can it be *equitable*, that when the *Age Seventy-five* should receive £19.23, he must take only £10.8894; that £10.8894 may also be paid to the *Age of Ten*, who ought to receive only £6.993; and the *same* also of others? This is such a glaring absurdity, that it might be thought there could be found no precedent of the like; but it is very true, that a *Society in Serjeants-Inn* has, for a long time, assured all lives within the *Age of Twelve and Forty-five* on *equal Terms*; but a bare inspection in the last *Table*, must shew how preposterous such a procedure must be.

The Reader, *Fifty-five* years old, if he regards the Table, will scarcely accept of 11*l.* *per cent.* when he should have more; and the *Advertisers*, if they regard the Table, will be backward in admitting any age *under Fifty*. It is to be again noted, that this is on consideration of 5*l.*; which is an *extraordinary* rate *per cent. interest* at this time; and to be *perfectly secure*; therefore the *Land Security* promised by the *Advertisers* should be *well* examined.

CXXXV. Suppose the Age *Forty*, instead of receiving an *immediate* annuity, would chuse to *commence* Annuitant at the end of *one year*; What is the value in *present money* of such an annuity *for life*?

The Age would become an *immediate* Annuitant when *Forty-one* years old; and in the Table (117.) £6962.568 will pay 284 Members of that age *then* living: but as the money which Age *Forty* pays now would gain a year's interest, and as there are supposed now 294 living, that *sum* must be multiplied by a year's *Reversion*, and the *product*, divided by 294, will give the Answer for 2*l.* annuity.

$6962.568 \times 965898 = 6725.130506064 \div 294 = 22.874593$, or £11.4372956, for 1*l.* annuity.

CXXXVI. It may be proper to observe here,

here, that the sum in the Table of Reversion being always a part of an Unit too little or too much, does not give the product *quite* exact, though the difference is but little: as in the present case; when the Reversionary (47.) Sum was found, the remainder, being *more* than *half* the divisor, was made an Unit; whereby the present product is *too much*; and if the remainder be not made an Unit, but entirely omitted, then the product would be *too little*: as,

$$6962.568 \times 965897 = 6725.123543496$$

This is nearly the same as the other, but neither of them is *perfectly* right; an *exacter* sum is found in the last column of the Table (116.), opposite the Age *Forty-one and a half*, when the Annuitant would become Recipient, and is £6725.1297655, and already multiplied by the proper Reversions; which being divided by 294 gives £22.874591 for 2 l. or £11.4372955 for Answer for 1 l. annuity.

This may assure the Reader, that though the products are not perfectly exact, yet they are not so far from the truth as to make any *material* difference in the quotient, since this Answer is the same as the other to *six* places of decimals.

CXXXVII. What is the value in *present* money of 1 l. annuity *during life*, for the Age *Forty*, to *commence* at the expiration of *Seven* years?

Any

Any Reversion for the *particular* Age of *Forty*, is found truer and easier by Table (116.) than by the other (117.); for the Totals of all the values that the Age has a chance of receiving, is opposite the Age when he will become *Recipient* (108.). Thus opposite the Age *Forty-seven and a half* is the total 4090.817809 (116.); which being divided by 294 persons living when aged *Forty*, gives £13.914346 for 2 l. and £6.957173, for Answer to the Question.

CXXXVIII. What is the value, in *present* money, of 1 l. annuity *during* *Life*, for the Age *Forty*, to *commence* when aged *Fifty*?

Opposite the Age *Fifty and a half* (116.), which is the time that Age *Forty* by the Question will become *Recipient*, is the sum 3141.713905, multiplied into proper Reversion, and

$$294) 3141.713905 (10.686102$$

or £5.343051 for 1 l. annuity, and Answer to the Question.

CXXXIX. In this manner the Reversion of every year from *One* to *Ten* has been found for the Age *Forty*, to shew the *Degrees* of the *Purchase*, and the *Interest* made of the money ; as in this Table :

The Age <i>Forty</i> becomes <i>Recipient</i> when		
Aged	Value in present Money.	The Interest made per Cent.
40½	12.38685 (128.)	£ 8.0736 (129.)
41½	11.4372955 (135.)	£ 8.7433
42½	10.5521325	£ 9.4767
43½	9.7280568	£ 10.2795
44½	8.95972402	£ 11.16106
45½	8.2435555	£ 12.1306
46½	7.57688598	£ 13.1983
47½	6.957173 (137.)	£ 14.3736
48½	6.3800445	£ 15.6738
49½	5.842684	£ 17.1154
50½	5.343051 (138.)	£ 18.715

By this Table it appears, that the Age *Forty*, may make £18 14 3½ per cent. interest of what money is given for the purchase of any annuity for life, to commence after *Ten* years ; and but £8 1 5½ if the annuity is immediate.

CXL. The Author, among other strange things, has heard it said, (he does not mean by a person who understood any thing of the subject) that Dr. Price in his Calculations does not allow to Annuitants much more than even common interest for their money.

Let

Let it be examined here what *common* interest a person aged *Forty* could make of 100 *l.* in Ten years. The Table (42*.) shews the interest to be £41.4778 by *Compound Rate*: and his *interest* and *principal* together would then be £141.4778, which at $3\frac{1}{2}$ *l. per cent.* would supply him with nearly 5 *l.* a year; or, if he *sunk* the money, when aged *Fifty*, for an *immediate* annuity, What should he receive *yearly*?

CXLI. To find what annuity any given sum will purchase, divide the sum by the value of 1 *l.* for the age, and the quotient will be the Answer: Thus the value of 1 *l. immediate* annuity for the age of *Fifty* is (131.) £10.894; therefore (129.)

$$10.894) 141.4778 (12.987$$

If the age of *Forty* preserves his principal and interest entire, he will at *Fifty* years of age have £141.4778 principal, and near 5 *l. per annum* for ever; if the Age *Forty* *sinks* the principal at *Forty*, to receive an annuity after 10 years, he will have *per* } £18.714
annum for Life only ——— }
 Or, for £141.4778 *sunk* when 50, 12.987.

Difference of the *annual* income £ 5.727

Whence it is seen that the Age *Forty* will receive £5.727 a year *for life* from *others*,
 merely

merely for risking the chance of leaving 100 *l.* by his death to the benefit of others) more than *possibly* can be made by preserving the principal till aged *Fifty*, though he were *then to sink it*.

CXLI*. What is the value in *present* money of 1 *l.* annuity *during life*, for the Age *Ten*, to *commence* when aged *Fifty*?

At the Age *Fifty*, in the Table (117.) is the sum of £4444.828, which is there proved to be sufficient for the paying 204 Members, that would be survivors, 2 *l.* *immediate* annuity *during life*; but, by the question, Age *Ten* will be *Forty* years before he becomes an annuitant, and also there are 524 living when aged *Ten*, therefore the sum £4444.828 must be multiplied by *Eighty half-years* reversion, which is .249601, and the product 1109.433513628 must be divided by 524, and the quotient 2.117239529 is for 2 *l.* annuity, and the *half* £1.0586197645 is the Answer to the Question.—For proof, 524 Members, aged *Ten*, pay in *present* money £2.117239529, each to receive 2 *l.* annuity *during life*, when aged *Fifty*. The sum multiplied by the number 524 (which sum was the *quotient*, and which number was the *divisor* in finding the *last* Answer) must give the same as the dividend, if the omitted fractions be taken in (17.), which dividend was 1109.433513628, and which being divided by .249601, which is 40 years Re-

S f

tion,

tion, must give £4444.828 for Compound Interest for 40 years, which is seen in the Table to pay 2 *l.* annuity to 204, which are all the Survivors at *Fifty*, out of 524, aged *Ten*.

CXLII. What is the value in *present* money of 1 *l.* annuity *during life*, for the Age *Twenty*, to *commence* when aged *Fifty*?

Answer.

$$£4444.828 \times .35313 = 1569.60211164 \div \div 462 = 3.39740716 \div 2 = 1.69870358.$$

Which, in words at length, may be read thus: The Stock that will pay the Survivors at 56 is multiplied by 60 *half*-years reversion, and the product is divided by the 462 living, when aged *Thirty*, and the quotient being for 2 *l.* is divided by 2 for an Answer to the Question: It may be proved in the same manner as the last.

CXLII*. What is the value in *present* money of 1 *l.* annuity *during life*, for the Age *Twenty*, to *commence* when aged *Forty-eight* (a)?

Opposite the Age *Forty-eight*, in the Table (117.) is £4951.67, which, multiplied by .378505, that is, 56 *half* years reversion, produces

(a) What is the value in *present* money of 1 *l.* *immediate* annuity, for the Age *Twenty*, to *cease* when aged *Forty-eight*?

By

produces £1874.23185335; which product being divided by 462 living, aged *Twenty*; the quotient is £4.0567789 for 2 *l.* annuity; the half of which is £2.02838945, for 1 *l.* annuity.

CXLIII. What is the value in *present* money of 1 *l.* annuity *for life*, for the age of *Thirty*, to *commence* when aged *Fifty*?

The Answer is found in the same manner as for the Ages *Ten* and *Twenty*; that is,

$$\begin{aligned} &£4444.828 \times 499601 = 2220.640513628 \div \\ &\div 385 = 5.767897438 \div 2 = 2.883948719 \end{aligned}$$

which is the same sum multiplied by *Forty half* years reversion, and the product divided by 385, the number living aged *Thirty*, which gives the quotient for 2 *l.* the half of which is for 1 *l.* and is the Answer.

S f 2

CXLIV.

By this Question, the Age would not receive the *four last* payments in the Table (93.); therefore,

From the total of the <i>Sixty</i> payments	13225.9990865
Deduct the total of the <i>Four</i> last ditto	304.632484
	£12921.3666025

Let the remainder be divided by the 462 persons living aged *Twenty*, and the quotient will be £27.9683259 for 2 *l.* annuity, which will be £13.984 6295 for 1 *l.* annuity.

CXLIV. What is the value in *present* money of 1*l.* annuity *during life*, for the Age *Sixty*, to *commence* when *Seventy* years old ?

In the Table (117.), opposite the Age *Seventy*, is £975.855, the sum that will pay an immediate annuity of 2*l.* to 69 Survivors at that age ; therefore

$$975.855 \times .706824 = 689.75773452 \div 130 = \\ = 5.305828727 \div 2 = 2.6529143635 ;$$

that is, the sum opposite the Age *Seventy* is multiplied by 20 *half-years* Reversion, and the product is divided by 130 living aged *Sixty* ; and the quotient being for 2*l.* the half of it is the value for 1*l.*

CXLV. What is the value in *present* money of 1*l.* annuity *during life*, for the Age *Sixty*, to *commence* when aged *Sixty-seven* ?

Answer.

$$1312.39 \times .784364 = 1029.39146996 \div \\ \div 130 = 7.91839592 \div 2 = 3.95919796 ;$$

that is, the sum opposite the Age *Sixty-seven* in the Table, multiplied by 14 *half-years* Reversion, and the product divided by 130 living when aged *Sixty*, and the quotient being for 2*l.* the half of it is for 1*l.* annuity,

CXLVI.

CXLVI. What is the value in *present* money of 1 *l.* annuity *during life*, for the age of *Twenty*, to *commence* at the expiration of 27 years?

Answer.

$$5215.452 \times .391869 = 2043.773959788 \div \\ \div 462 = 4.423753159 \div 2 = 2.2118765795;$$

that is, the sum opposite the Age *Forty-seven* in the Table (117.) is multiplied by 54 *half-years* reversion, and the product is divided by 462 living aged *Twenty*; and the quotient being for 2 *l.* the *half* of it is the value of 1 *l.* annuity.

CXLVII. What is the value in *present* money of 1 *l.* annuity *during life*, for the Age *Thirty*, to *commence* when aged *Forty*?

Answer.

$$7283.474 \times .706824 = 5148.134226576 \div \\ \div 385 = 13.371777212 \div 2 = 6.685888606;$$

that is, the sum that will pay the Age *Forty* an immediate annuity, is multiplied by 20 *half-years* reversion, and the product divided by 385 living at the age of *Thirty*; and the quotient being for 2 *l.* annuity, the *half* is the value of 1 *l.*

If the same Question be solved in another manner, the two Answers may prove the truth of each other. The value of 1 *l.* annuity

annuity for the Age *Thirty*, to receive from the Age *Forty* till *Fifty*, and then to cease, was found (100.); and also the value of 1 *l.* for the same age, during life, to commence when *Fifty* (143.). These two values added together, give the Answer to the present question.

Age <i>Thirty</i> to receive when <i>Forty</i> until aged <i>Fifty</i>	}	— 3.80195
Age <i>Thirty</i> during life to commence when <i>Fifty</i>	}	— 2.88394
The Answer (<i>as before</i>)		£ 6.68589

This last method of finding the Answer merits some attention, for it may point out the way to obtain the value of immediate annuities for the Ages *Ten*, *Twenty*, *Thirty*, or any intermediate ones; and also how to obtain the value of *half-yearly* payments from one age till the Life attains another: Thus,

CXLVIII. What is the value of an *immediate* annuity of 1 *l.* during *Life*, for the Age *Thirty*?

The value of 1 *l.* for that Age, to receive from *Thirty* to *Fifty*, was found (98.), and the value of 1 *l.* to commence when *Fifty*, was also found (143.); therefore by the preceding Answers, the two values added together are the solution required.

The

The Age <i>Thirty</i> , to receive } from <i>Thirty</i> to <i>Fifty</i> }	11.205349
The Age <i>Thirty</i> , from <i>Fifty</i> } during Life ——— }	2.288394
<i>Immediate Annuity</i> , during } <i>Life</i> ——— }	£13.493743

CXLIX. What is the value in *present* money of 1 *l.* annuity, for the Age *Thirty*, to *commence immediately*, and to *cease* when aged *Fifty*?

This Question has been answered before (98.), by a particular Table, and is here repeated, that the Reader may observe that the value of 1 *l.* immediate annuity till *Fifty*, and then to *cease*, must always be the value of 1 *l.* annuity *during life*, to *commence* when *Fifty*, subtracted from the value of an *immediate annuity during life*, for the same Age; and this Rule is *universal* for any Age.

The value of an <i>immediate an-</i> nuity; <i>during life</i> , for the } Age <i>Thirty</i> is, (as above) }	13.493743
The value of an annuity to } commence when <i>Fifty</i> }	2.288394

Immediate ann. till aged Fifty £11.205349(98.)

CL. Because the Tables do not shew the *Immediate annuity* for so low an age as *Thirty*, the value found above may be called *supposition*, and not be admitted for sufficient proof:

proof: Let the Age *Forty* therefore be examined. The value of an *immediate* annuity *during life*, for the Age *Forty*, was found (116.) in a particular manner; as was the value of an *immediate* annuity for the *same* age, to *cease* when *Fifty* (71.), by a particular Table; as also the value of 1 *l.* annuity for the *same* age, to *commence* when *Fifty* (138.). Let it be seen how these values prove each other.

(128.) Value of an <i>immediate</i> annuity <i>during life</i> }	12.38685
(138.) Ditto of Annuity to <i>commence</i> when <i>Fifty</i> }	5.34305
(71.) Ditto to <i>cease</i> when <i>aged Fifty</i> }	7.04380

If the Reader will take the trouble to turn back to these three separate Cases, he will find them agree in this manner; and if afterward there should remain a doubt, let him look into Dr. *Price's Observations on Reversionary Payments*, pages 19 and 109.

CLI. Let this be the more *particularly* considered, because it affords *another* (132.) argument to convince such Gentlemen in the management of the Societies for the Benefit of Age, as are *unwilling* to believe their plans *deficient*, and their promises *mere delusion*, that the values here found are not beyond *their own* estimation in *some* cases.

The value, in *present* money, of 1 *l.* *immediate* annuity for the Age *Twenty*, to *cease* when

when aged *Fifty*, being (93.) £14.3138518,
let the Annuity, instead of 1*l.* be £2.2
then the value of 1*l.* multiplied by the £2.2
produces the *Total present value* of that An-
nuity. As,

$$£14.3138518 \times £2.2 = £31.79047396$$

The value, in *present money*, of 1*l.* an-
nuity for the *same age during life*, to commence
when aged *Fifty*, was found (142.) to be

	£1.69870358
Let this also be multi-	
plied by	2.2
	339740716
	339740716
The value of £2.2 an-	
nuity <i>during life</i> , to	
commence when aged	
<i>Fifty</i> , is	£3.737147876
Add the immediate annu-	
ity to cease when aged	
<i>Fifty</i>	£31.79047396
<i>Present value</i> of £2.2	
<i>immediate annuity</i> , to	
continue <i>during life</i>	£35.527621836(a)

(a) If from the value of an *immediate annuity during life*,
the value of an annuity to commence when *Fifty* be *subtracted*,
the remainder will be the value of an *immediate annuity to cease*
when aged *Fifty* (150). — Or, if from the *same* the value of
an *immediate annuity to cease* when aged *Fifty* be *subtracted*, the
remainder will be the value of an annuity *for life*, to commence
when aged *Fifty* (149).

Multiplying £14.3138518 by £2.2 produced the value of *all the payments* or annuities in *present money* :—therefore *dividing* the *present money* paid by 14.3138518, gives the value of the *yearly payments* or annuities (17.). As,

<i>Value of 1 l. immediate annuity till aged Fifty.</i>	<i>Present money,</i>	<i>Yearly payments,</i>
---	-----------------------	-------------------------

14.3138518) 31.79047396 (2.2

Here the *present money*, being the *dividend*, is divided by the value of 1 l. *immediate annuity* till aged *Fifty*; and the *quotient* is the value of each *yearly payment* to be made *during that term*, in lieu of *ready money*.—It is also known, that multiplying the *yearly payments* by the value of 1 l. *immediate annuity* till *Fifty*, gives the product equal to the value of the *whole* in *present money*; that is, multiplying the *quotient* by the *divisor*, or the *divisor* by the *quotient*, the product will be equal to the *dividend*.

If, instead of £2.2 *immediate annuity*, to *cease* when aged *Fifty*, the same annuity for the same age was to be *during life*, to *commence* when aged *Fifty*, the value of it, in *present money*, would be (*as before*),

$$1.69870358 \times 2.2 = 3.737147876$$

or, instead of *present* money, the yearly payments to be made till aged *Fifty*, would be

Value of 1 l. immediate annuity till *Fifty*. *Present Money*. Yearly Payments.

14.3138518 : 3.737147876 (.261

which, when reduced, is about £0 5 24

The age of *Twenty* is to pay 60 half-yearly sums of 1 l. 2 s. each, which is *decimally* £2.2 per annum for 30 years, in the *Laudable Society*; and the age, if it survive the term, will have paid 66 l. for which, in *present* money, 38 l. 1 s. is required.—These Calculations have just shewn the true value of the said £2.2 annuity, to be worth no more than £31.79; nay, if the said annuity was to continue for the *whole life* of Age *Twenty*, the *present* value in a single payment is but £35.5276.—If this is not rated too high, neither can the following be so (a).

Let the annuity for life, to commence when *Fifty*, be 44 l. for the same Age.—The value of it, in *present* money, must be

$$1.69870358 \times 44 = \underline{\underline{£74.74295752}} \quad (164.)$$

T t 2

or,

(a) It is not known what the other Societies require for Age *Twenty*, but they will be found equally absurd at the Age *Fifty*.

or, in lieu of *present money*, the *yearly* payments to be made till aged *Fifty*, would be

<i>Value of 1 l. immediate annuity till aged Fifty.</i>	<i>Present Money.</i>	<i>Yearly Payments.</i>
14.3138518	) 74.74295752	(5.222

which, reduced, is nearly	—	6.	s.	d.
or, <i>half</i> -yearly	—	5	4	5½
		2	12	2½

By the same reasoning, the value of 1 l. *immediate annuity*, till *Fifty*, for the Age *Forty*, being, as just seen (150.), £7.0438; and the value of 1 l. annuity *for life*, to *commence* when *Fifty*, being £5.34305; let the annuity be £6.7 for each case.

The value of the first was	}	£47.193527
found (75.) to be		
Ditto of the second is	}	£35.798435
5.34305 × 6.7 =		

The *present* value of *both* is £82.991962

which is, *nearly*, the same as the value of 1 l. *immediate annuity for life* for the Age *Forty*, found before (128.), multiplied by 6.7, *viz.*

$$12.38685 \times 6.7 = \underline{\underline{£82.991895}}$$

The value of the *yearly* payments, in lieu of £47.193527 *present money*, for £6.7 annuity,

nuity, to be received till aged *Fifty*, can be no other than the same as the annuity to be received.

$$7.0438) 47.193527 (6.7$$

(17.)

But the yearly payments, instead of £35.798435 in present money, for £6.7 annuity for life, to commence when aged *Fifty*, is,

$$7.0438) 35.798435 (5.082$$

which is about 5*l.* 1*s.* 7*d.*—which sum, if multiplied by the divisor, must be equal to the dividend, or present money.

Or, if the annuity for life, to commence when aged *Fifty*, was to be 44*l.* instead of £6.7, multiply the value of 1*l.* annuity, on the same conditions, by the 44*l.*

$$5.34305 \times 44 = 235.0942(164.) \left. \begin{array}{l} \text{Mr. Simp-} \\ \text{son's Table.} \end{array} \right\}$$

And if, instead of paying present money, yearly payments be made; let the present money be divided by the value of 1*l.* immediate annuity, to cease when aged *Fifty*.

$$7.0438) 235.0942 (33.376$$

which,

which, by half yearly payments, }
 in each ----- } £16 13 9(a)

CLII.

(a) A says to B, who is aged *Forty*. If you will pay me an annuity of £48.61476, for only *Eight* years and a *Half*, I will bind myself and *Heirs* to pay to you an annuity of 50 £ as long as you shall live after that time. — Perhaps there are few persons who would readily propose the same as A does here, yet these Calculations prove the values to be equal; for opposite the Age *Forty-eight and a Half*, is the sum £4822.324; which, being multiplied by .744586; that is, 17 half years reversion; is reduced to £3590.634937864; and this sum, divided by 294 persons living aged *Forty*, gives £12.213 for 2 £. which is £6.1065 for 1 £. annuity. — The value of 1 £. immediate annuity during Life, for the Age *Forty*, is (131.) £12.387
 From which deduct the value of 1 £. annuity } £6.1065
 during life, to commence when aged 48½ (150.)

The value of 1 £. immediate annuity till aged 48½ £6.2805

$$\begin{aligned} £6.2805 \times 48.61476 &= £305.325 \\ £6.1065 \times 50 £ &= £305.325 \end{aligned}$$

That is, the values of the 1 £. annuities, multiplied by the annuity, produce the worth of the annuity in one single payment present money, which are here equal.

Again, — A says to C, a person aged *Forty-six*, Pay me; if you live, an annuity of £49.426897, for *eight* years only, and I will oblige myself and *Heirs*, &c. to pay to you, during your life, an annuity of 50 £ from the expiration of the said *Eight* years.

The Age *Forty-six*, at the expiration of eight years, would be aged *Fifty-four*; opposite which, in the Table (117.) is £3521.082; which, being multiplied by .757616, which is *Sixteen* half-years reversion, produces £2667.628060512; and this product divided by the 237 persons living when aged *Forty-six*, gives £11.298 for 2 £. or £5.649 for 1 £. annuity, to commence when aged *Fifty-four*.

Opposite

CLII. To know at what value the Societies compute the half-yearly sums they receive from their Members ; the Author stated to them the following case, to which he received the *Answers* as here given.

“ SIR,

“ A Person, aged *Forty*, is desirous of
 “ paying *one single* sum, in *ready* money,
 “ for the *Twenty* payments of £3 7 0,
 “ each (*a*), required to be made till aged
 “ *Fifty*.—Please to send immediate answer
 “ how much he should pay in lieu of the 67 £
 “ expected to be received ?”

THE

Opposite the Age *Forty-six*, in the Table (117.), is £5486.319, which, divided by 237, then living, is £22.727 for 2l. or £11.3635 for 1l. *immediate annuity, during life, for the said Age.*

From the *immediate annuity during life*, £11.3635
 Deduct the annuity to commence at Age *Fifty-four*, £5.649 (150)

Remains, *the immediate annuity till aged Fifty-four*, £5.7145.

$$\begin{array}{rcl} £5.7145 \times £49.426897 & = & £282.45 \\ £5.649 \times £50 & = & £282.45 \end{array}$$

The Author wishes to know, *How many years less than Eight* will an annuity be accepted for, on condition to grant the same annuity during the Purchaser's life, by a person who can properly secure the Payment?

(a) Or the Sum required by each Society.

THE LAUDABLE SOCIETY.

Twenty half-yearly payments } £67
 of 3 l. 7 s. each, is
 Required, *present* money, in } £54 7 0*
 lieu of them,
 The *true* value has been seen } £47 3 10†
 to be — — —

THE AMICABLE SOCIETY.

Forty quarterly payments of } £60
 1 l. 10 s. each, is
 Required, *present* money, in } £51 *
 lieu of them
 The true value is $7.0438 \times 6l. = £42$ 5 3

THE PROVIDENT SOCIETY.

Twenty half-yearly payments } £84 0 0
 of 4 l. 4 s. each, —
 Required, *present* money, in } £71 10 11*
 lieu of them, —
 The true value is, $7.0438 \times £8.4 = £59$ 3 4

THE

SOCIETY OF LONDON ANNUITANTS.

Twenty half-yearly payments }
of 7 l. 7 s. each — } £147

*"The Directors can have no objection to receive
"present money, (but are not impowered to allow
"any discount."*

The true value is $7.0438 \times 14.7 = £103 \ 10 \ 10\frac{1}{2}$

THE EQUITABLE SOCIETY.

Twenty half-yearly payments }
of 6 l. 10 s. each, — } £130

Required, *present* money, in }
lieu of them, — } £104 *

The true value is $7.0438 \times 13 = £91 \ 11 \ 4\frac{1}{2}$

THE WESTMINSTER UNION SOCIETY.

Forty quarterly payments of }
2 l. 10 s. each, — } £80

No Answer.

The true value is $7.0438 \times 10 = £70 \ 8 \ 9$

THE LONDON UNION SOCIETY.

Twenty half-yearly payments }
of 8 l. 1 s. 4 d. each } £161. 6 8

Required, *present* money, in }
lieu of them — } £150 0 10*

In true value is $7.0438 \times 16.1 = £113 \ 12 \ 9$

U. n

THE

* See the following Note.

THE PUBLIC ANNUITANT SOCIETY.

Forty quarterly payments of } £140
 6 l. 10 s. each (8th class)

No Answer.

The true value is $7.0438 \times 14 = \text{£}98 \ 12 \ 3$

THE RATIONAL SOCIETY.

Twenty *half*-yearly payments } £88
 of 4 l. 8 s. ————

No Answer.

The true value is $7.0438 \times 8.8 = \text{£}61 \ 19 \ 8\frac{1}{2}$

Thus

* It is difficult to say by what Rule these Societies adjust the Discount: for, except the LAUDABLE, none of them agree exactly with the following.—In Dr. Price's *Observations*, &c. page 312, is a Table to shew the present value of 1 l. annuity *certain*, for any time within 100 years, at several rates of interest.

An annuity of 1 l. for 10 years, at 3 l. per cent. interest, is 8.5502
 Ditto ————— at 4 l. ————— 8.1108

By which values multiply the yearly sums paid by the Members—

The *Laudable Society* $8.1108 \times 6.7 = 54.34236$
Amicable ——— $8.5302 \times 6 = 51.1812$
Provident ——— $8.5302 \times 6.4 = 71.65368$
Equitable ——— $8.1108 \times 13 = 105.4404$
London Union — Deducts about 7 l. per cent. only.
Consolidated ——— Did allow 15 l. per cent. Discount.

N. B. *Annuities certain do not allow for Chance of Life.*

Thus it appears, that by their *own* estimation, the Gentlemen rate the value of an *immediate* annuity from the Age *Forty* to *cease* when *Fifty*, at a *higher* value than *these Calculations*.

The value of an annuity for *this* term, and that for an annuity *for life*, to commence when aged *Fifty*, have been here found by the *same* Table of Mortality, by the *same* rate of Interest and Reversion, and by the *same Method*; it is left then to the Gentlemen's reflection to account how these values can be too *low* in the one case, and at the same time too *high* in the other.

CLIII. For further illustration add the value of (141*.) 1*l.* annuity for Age *Ten*, *during life*, to *commence* when *Fifty*; to the Values of the several decades found before (88.).

(141*.) Annuity, <i>during life</i> , to <i>commence</i>	
when 50	1.0586197
At Age 40, to <i>cease</i> when 50,	1.39553
30, to <i>cease</i> when 40,	2.7175872
20, to <i>cease</i> when 30,	4.8071063
10, to <i>cease</i> when 20,	7.8713125

Value in *present* money of 1*l.* }
immediate ann. during life } £17.8501557.

Not only the value in present money of an immediate annuity for the Age Ten, is here seen, but also the several gradations in the different decades produced by the *chance of life* and Table of *Reversion*, being blended together in their due proportion.

CLIV. It must now be extremely obvious, that if the Age Ten was to pay 1 *l.* yearly to any Society, by yearly payments, or 10 *s.* half-yearly, until he attained the Age of *Fifty*, the present value of all the payments, which would amount to 40 *l.* is only £16.791936; for the annuity of 1 *l.* (88.) is worth no more, and even *for life* is worth but £17.85; and surely the value must be the *same* whether it be *given* or *taken*.

CLV. Thus is obtained the worth in *present money* of the *half-yearly* payments till aged *Fifty*; and also *what yearly* payments should be made, equivalent to *any present* sum; by which it is very immaterial, whether the Age pays any admission money, if it pay *half-yearly proportionably* to the Age; or whether it pays any half-yearly sums, provided it pays the value of them in a single payment in *present money*, for which the discount would be for these three ages,

Age *Twenty* to pay 30 payments } £66
of each £2.2 — — }

Ditto, pays *present money* in lieu = £31.7904

Discount £34.2096
Age

Age <i>Forty</i> to pay 10 payments	}	£67
of each £6.7		
In lieu of them pays in <i>present</i>	}	£47.1935
money		

Discount £19.8065

Age <i>Sixty</i> to pay 10 payments	}	£67
of each £6.7		
In lieu of them pays in <i>present</i>	}	£42.849
money		

Discount £24.151

In like manner the Discount *per Cent.* may be known (a).

The

(a) It is known (92.) that the *present* value of 1 l. annuity for the Age *Twenty* to receive *Thirty* years, is £14.3138518— say, by the *Rule of Three*,

As 30 l. : 14.3138518 :: 100 l. : £47.7128393

Deduct from	_____	100 l.
The proportion last found,	_____	47.7128393
The discount <i>per cent.</i> for the Age <i>Twenty</i>		£51.2871607

Likewise

The Age *Twenty* has many years reversion and chances of not living that makes the *discount* large ; even more than *one-half* of 66 *l.* that the *half*-yearly payments would amount to: the Ages of *Forty* and *Sixty* have *equal* years of reversion ; but their *chance of life* being *different*, makes a proportionable difference in the *discount*. In a number of the Age *Forty*, the Societies may expect to receive, from one with another, a sum equal in present money to more than 47 *l.* each ; but from the Age *Sixty*, not quite 43 *l.* each member ; although the ages pay *equal* half-yearly sums each, and the sum total of the payments of each Age would be 67 *l.* but there would not

Likewise, the *present* value of 1 *l.* annuity for the Age *Forty*, to receive, or pay, *Ten* years, is £70.4381.

$$\text{As } 10 \text{ l.} : £70.4381 :: 100 \text{ l.} : £70.4381$$

Deduct from	_____	100 <i>l.</i>
The proportion last found,	_____	70.4381
The discount <i>per cent.</i> for the Age <i>Forty</i>		£29.5619 (a)

Also, the *present* value of 1 *l.* annuity for the Age *Sixty*, to receive, or pay, *Ten* years, is £6.3955.

$$\text{As } 10 \text{ l.} : £6.3955 :: 100 \text{ l.} : £63.955$$

Deduct from	_____	100 <i>l.</i>
The proportion last found,	_____	63.955
The discount <i>per cent.</i> for the Age <i>Sixty</i> ,		£36.045

(a) The Consolidated Society allowed but 15 *l.* for *same*,

not live so many in proportion to their number from the Age *Sixty* to *Seventy*, as from *Forty* to *Fifty*.

CLVI. To endeavour to make it still more unquestionable that 47 *l. present* money, for the Age *Forty* to pay, is *equivalent* to the 67 *l.* which the same age is to pay by *half*-yearly payments; let it be supposed that a Society hath *already existed* the 10 years *past*, instead of supposing, as hitherto, 10 years *to come*; and that the Members have actually died, as given by the Table (62.): then, if the Members paid 1 *l. half*-yearly, the total of the sums paid, with the Compound Interest of each payment, would be as in the following Table:

Age	Living	Il. with (42*) Comp. Interest.	Product.	Total:
40	294			
40½	289	X 1.3904	=£401.8256	
41	284	X 1.366	=£387.944	
41½	279	X 1.343	=£374.697	
42	274	X 1.3199	=£361.6526	
42½	269	X 1.297227	=£348.95406	
43	264	X 1.2749	=£336.5736	1875.07326
43½	259½	X 1.2529	=£325.1275	
44	255	X 1.231439	=£314.0169	
44½	250½	X 1.210259	=£303.1698	
45	246	X 1.18944449	=£292.6033	
45½	241½	X 1.168987	=£282.31036	1571.4911
46	237	X 1.1482	=£272.2656	
46½	232½	X 1.129122	=£262.5208	
47	228	X 1.1097	=£253.0116	
47½	224	X 1.090616	=£244.29798	
48	220	X 1.071859	=£235.809	1314.40634
48½	216	X 1.0534	=£227.5344	
49	212	X 1.0353	=£219.4896	
49½	208	X 1.0175	=£211.64	
50	204	X 1.	=£204.	
Total 4893 £. paid.				£1098.473

Total of all the Sums paid with Interest £5859.4437

The last payment being made *now* just at the expiration of *Ten* years, has had *no* time to gain interest; therefore what the 204 Survivors pay is worth but £204: likewise whatever money the Age *Forty* paid, was, *at that time, present* money, and not to be included *here* in the *reversionary* payments; so that, upon the whole, it is found that £4893 have been paid, but at such different periods of time *ago*, as to have acquired

Com-

Compound Interest to make that sum equivalent to £5859.4437.

If, instead of paying at several times, and risking the chance of life, the Society and Members had agreed to take and give, for the 20 l. (which each Member was to pay, if he lived) the sum of £14.0876203 each, down in *present money*, then the Society would have had that sum multiplied by the 294 living at *Forty*, who were to pay that sum each at that age.

$$£14.0876203 \times 294 = 4141.7603682$$

therefore, instead of £4893, which the Society might have *expected* to receive, there would have been paid but £4141.76, which is a *seeming* loss to the Society.

But it will be found *no loss*, when it is considered, that this sum was paid down in *present money*, and that there was *no risk* of losing part of it, but that the *whole* has been accumulating *Ten years Compound Interest*; for let it be seen what the amount of that Stock is *now* at this time.

Ten years Compound Interest is 1414.778 (42*), by which sum if the Stock be multiplied it will produce £5859.6714502012596, which is the same number of *Pounds* as before in the *last Table*.

Had there been *Two Classes*, consisting of

X x

294

294 persons aged *Forty* in each class, and had the one paid by *half-yearly* payments, and had the other paid £14.0876 each, down in *present* money, the Society would now find that it had gained or lost no more by the one than by the other.

CLVII. The annuity of 2 *l.* to receive from the Age *Forty* till *Fifty*; (or, which is the same thing, the *half-yearly* payment of 1 *l.* for the *same Age*,) for the *same Term*, has been now so *particularly* (156,) so *often* (71, 172.) and in such *different ways* explained (150.), that tis hoped every Reader understands that £14.0876 is the just value of it in *present money*, which is £7.0438 for 1 *l.* annuity. And, if this be true for the Age *Forty*, the values for the *other Ages* must be true also; they being found by the *same method*.

CLVIII. What is the value of 1 *l.* annuity *for life*, for the Age *Ten*, to *commence* when aged *Forty*?

By the Table (117.) £7283.474 will pay 2 *l.* *immediate* annuity to 294 persons aged *Forty*; but as the Age *Ten* will be *Thirty* years before he becomes an *immediate* Annuitant, that sum must be multiplied by *Thirty* years reversion, and the product must be *divided* by 524 living at the Age of *Ten*; the quotient is for 2 *l.* and the half of it is £2.45421104 for 1 *l.* annuity; thus,

£7283.474

$$\begin{aligned} & \pounds 7283.474 \times .35313 = 2572.01317362 \div \\ & \div 524 = 4.90842208 \div 2 = \pounds 2.45421104 \end{aligned}$$

Or, add the two values already found (153.) and the total will be very *nearly* the same.

$$\begin{aligned} (141^*.) \text{ 1 l. Annuity during life, } & \left. \begin{array}{l} \text{to commence when Fifty} \\ (84.) \text{ Ditto, to commence when} \end{array} \right\} \pounds 1.0586197 \\ \text{Forty to cease when Fifty} & \left. \right\} \pounds 1.39553 \end{aligned}$$

The Answer $\pounds 2.4541497$

CLIX. What is the value of 1 l. annuity for life, for the Age *Ten*, to commence when aged *Thirty*?

Add the three following sums together (153.).

$$\begin{aligned} 1 \text{ l. Annuity during life, to } & \left. \begin{array}{l} \text{commence when Fifty} \\ \text{Ditto, to commence when} \\ \text{Forty, to cease when Fifty} \end{array} \right\} \pounds 1.0586197 \\ \text{Ditto, to commence when } & \left. \begin{array}{l} \text{Forty, to cease when Fifty} \\ \text{Ditto, to commence when} \\ \text{Thirty, to cease when Forty} \end{array} \right\} \pounds 1.39553 \\ \text{Thirty, to cease when Forty} & \left. \right\} \pounds 2.7175872 \end{aligned}$$

The Answer $\pounds 5.1717369$

CLX. What is the value of 1 l. annuity for life, for the Age *Twenty*, to commence when aged *Forty*?

X x 2

The

The sum that will pay the *immediate* annuities, at the Age *Forty*, must be multiplied by 20 years reversion, and the product be *divided* by 462 living aged *Twenty*; and *half* the quotient is £3.938128673: Thus,

$$7283.474 \times .499601 = 3638.830893874 \\ \div 462 = 7.876257346 \div 2 = £3.938128673$$

Or, add to the value of the } £1.69870358
 life, to commence at *Fifty* }
 (131.)

(96.) The value to commence }
 when *Forty*, to cease when } £2.239421
Fifty }
 The Answer, (as before) £3.93812458

CLXI. What is the value of 1*l.* annuity for life, for the Age *Twenty*, to commence when aged *Thirty*?

$$6098.5627565 \div 462 = 132003522 \div 2 = \\ £6.6001761 \quad (97.)$$

The total of the last 40 payments in the Table (93.) is divided by 462, and the quotient also divided by 2 gives £6.6001761

To which add the value of }
 the annuity *during life*, }
 to commence when *Fifty*, } £1.6987035
 (142.)

The Answer is £8.2988796
 Having

Having thus answered several Questions in particular, and by several methods, 'tis hoped that the one hath so explained and confirmed the other, that they are clear and satisfactory to the Reader : And, that he may have a distinct idea of the values given by *different Tables* and by *different years of reversion*, here follows a Table from which he may obtain it by *Inspection* only.

CLXII. A Table to shew the value in present money, of 1*l.* annuity, to commence when *Fifty*, or after *Ten* years when the Age is more than *Forty*; according to four different Tables of Mortality (50.).

Age.	43 years. London Bill.	Dr. Hall's	Mr. Simpson's	Dr. Price's	(129.) Intereſt per Cent.
4	0.68146333	0.89041636	0.60037244	0.55517121	160.56
5	0.871835	1.15413078	0.804077	0.79345923	124.36
10	1.1142144	1.5202334	1.05861976	1.07527538	94.46
15	1.36374526	1.90324145	1.34100423	1.37344805	74.567
20	1.68061467	2.37738129	1.69870355	1.74607988	58.868
25	2.159009	2.9823636	2.19125846	2.25487938	45.635
30	2.838449	3.78786235	2.88394872	2.95481088	34.674
35	3.7936641	4.88244102	3.8813061	3.944358	25.744
40	5.229612	6.3946488	5.343051	5.3618154	18.716
45	6.6152914	5.4739486	4.753305	4.7223106	21.036
50	3.96332528	5.381012	4.0755479	4.0176	24.535
55	3.24687943	5.7626438	3.3710764	3.1679075	29.639
60	2.5058905	2.52072416	2.657014	2.319534	37.694
65	1.8003009	1.5338541	1.9792405	1.57198	50.2702
70	1.1596164	0.80113771	1.3577342	0.9650565	71.544
75	0.60375286	0.406208	0.7008966	0.545418	130.103
80	0.17393106	0.11876367	0.25673991	0.171726	389.468

These values were found by taking the sum opposite the Age when the life is to become Annuitant (117.). and multiplying it into as many years reversion as the *present*

Age

Age now wants of that future one; and dividing the product by the number living of the present Age (62.), for 2 l. annuity, and again dividing that quotient by Two for 1 l. annuity (130.).

The Interest in the 6th Column is for the Purchase money, by Mr. Simpson's Table, and was found as before (129.).

CLXIII. The value of 10 l. 100 l. or 1000 l. annuity, may be known by inspection of the last Table, because 'tis only supposing the dot removed one, two, or three places (15.); but to make it still easier to such as do not readily apprehend, the values by Mr. Simpson's Table are here given for such annuities: As,

Age	10 l.	100 l.	1000 l.	(129.) Interest per 1000 L.
2	0.003724	60.03724	600.37244	1665.6
5	8.04077	80.4077	804.077	1243.6
10	10.586197	105.861976	1058.61976	944.6
15	13.410642	134.106423	1341.064233	745.67
20	16.987035	169.87035	1698.703551	588.68
25	21.912584	219.125846	2191.258465	456.35
30	28.830487	288.304872	2883.04872	346.74
35	38.843061	388.43061	3884.3061	257.44
40	53.43051	534.3051	5343.051	187.16
45	47.537505	475.37505	4753.7505	210.56
50	40.7565459	407.565459	4075.65459	243.35
55	33.738765	337.387654	3373.87654	296.39
60	26.52914	265.2914	2652.914	376.94
65	19.899249	198.924905	1989.24905	502.702
70	13.977342	139.77342	1397.7342	715.44
75	7.66849	76.684906	766.84906	1304.93
80	2.567599	25.675991	256.75991	3894.68

If a person pays 1000 *l.* *present* money, he may, when *Fifty*, or after *Ten* years, if the Age is more than *Forty*, receive *yearly*, for *life*, the sum that is *opposite* the *Age*, in the *Fifth* Column of the last Table.

CLXIV. The value of 44 *l.* annuity, according to the Table (162.), is as follows :

Age	London Bills.	Dr. Halley's.	Mr. Simpson's.	Dr. Price's.
2	29 19 $8\frac{1}{4}$	39 3 $6\frac{1}{2}$	26 8 3	24 8 $6\frac{1}{2}$
5	38 10 $8\frac{1}{4}$	50 15 $7\frac{1}{2}$	35 7 7	34 18 3
10	49 0 6	66 17 $9\frac{1}{2}$	46 11 7	47 6 3
15	60 0 1	83 14 10	59 0 $11\frac{1}{2}$	60 8 $7\frac{1}{2}$
20	73 18 $11\frac{1}{4}$	104 12 $1\frac{1}{2}$	74 14 $10\frac{1}{4}$	76 16 $6\frac{1}{2}$
25	95 0 0	131 4 $5\frac{1}{2}$	96 8 $3\frac{1}{2}$	99 4 $3\frac{1}{2}$
30	124 17 10	166 13 $3\frac{1}{4}$	126 17 $10\frac{1}{2}$	130 0 $2\frac{1}{4}$
35	166 18 5	214 16 $6\frac{1}{2}$	170 18 $2\frac{1}{4}$	173 11 $0\frac{1}{2}$
40	230 1 9	281 7 $3\frac{1}{2}$	235 1 $10\frac{1}{4}$	235 18 $4\frac{1}{2}$
45	203 1 $5\frac{1}{4}$	240 17 1	209 3 $3\frac{1}{2}$	207 15 $7\frac{1}{2}$
50	174 7 $8\frac{1}{2}$	199 13 $6\frac{1}{4}$	179 6 7	176 15 6
55	142 17 3	157 7 $1\frac{1}{4}$	148 9 0	140 5 $4\frac{1}{2}$
60	110 5 $2\frac{1}{4}$	110 18 $2\frac{1}{2}$	116 14 $6\frac{1}{4}$	102 1 $2\frac{1}{4}$
65	79 4 3	67 9 $9\frac{1}{2}$	87 10 $6\frac{1}{4}$	69 3 4
70	51 0 $5\frac{1}{2}$	35 5 0	61 10 0	42 9 3
75	26 11 $3\frac{1}{2}$	18 1 $4\frac{1}{2}$	33 14 10	19 19 8
80	7 13 $0\frac{1}{4}$	5 4 6	11 5 $11\frac{1}{2}$	4 9 $6\frac{1}{2}$

These values are found by multiplying each sum in the Table (162.) by 44 *l.* (75.), and reducing the fractions as directed (29.).

CLXV. The Table (163.) is calculated for the use of persons in *affluent* circumstances; and if such should approve the plan, and form a Society of themselves, perhaps the most agreeable part of the Scheme would

would be to provide an income for their *Children*, when they should become of *Age*.

Let it be inquired, then, what sum must be paid in *present* money for a child, when under *one, two, three, four, or five* years of age; to receive 100 *l.* or 1000 *l.* yearly, during *Life*, and to commence Annuitant at the Age of *Twenty-one* ?

The total of the Table for Age *Twenty* (93.) is £13225.9990865 ; but as the Age *Twenty-one*, by the Question, is not to receive the *two first* payments in that Table, they must be *deducted*.

The total of Age *Twenty* £13225.9990865
Deduct the *first two* half } £ 890.0978485
years

£12335.901238

Also, every one of the remaining sums, in the Table, are multiplied into *one year's reversion too much* for the Age *Twenty-one*, to receive an *immediate* annuity ; but either *dividing* the total by one year's *reversion*, or *multiplying* it by one year's *Compound interest*; will restore it (92.) to its just value ;
as

.965898) 12335.901238 (12771

or,

12335.901238 x 1.03530625 = 12771.4356510841375

Y y

The

The *quotient*, or the *product*, is the sum that would pay 455, aged *Twenty-one*, an immediate annuity of 2 *l.* till aged *Fifty*; but as the Age *One* will be *Twenty* years before he commences Annuitant, the sum must be multiplied by *Twenty* years reversion, and the product be divided by the 870 living aged *One*; and the quotient being for 2 *l.* must be divided by 2 for the value of 1 *l.* annuity; thus,

$$\begin{aligned} & \pounds 12771.435651 \times .499601 = 6380.622022 \div \\ & \div 870 = 7.334048 \div 2 = \underline{\underline{\pounds 3.667024}} \end{aligned}$$

Thus it is known *What* the Age *One* should pay for 1 *l.* annuity, to *commence* when aged *Twenty-one*, to *cease* when aged *Fifty*; and, by proceeding in the same manner,

For the Age *Two*, it will be

$$\begin{aligned} & \text{19 years reversion.} \\ & 12771.435651 \times .51724 = 6605.897386, \&c. \\ & \text{living aged Two.} \\ & \div 700 = 9.43699626 \div 2 = \underline{\underline{\pounds 4.71849813}} \end{aligned}$$

The Age *Three*.

$$\begin{aligned} & \text{18 years reversion.} \\ & 12771.435651 \times .535502 = 6839.12933398, \&c. \\ & \text{living aged Three.} \\ & \div 635 = 10.77028241 \div 2 = \underline{\underline{\pounds 5.3851412}} \end{aligned}$$

For

For the Age *Four* it will be

$$\begin{aligned}
 & \text{17 years reversion.} \\
 & 12771.435651 \times .554408 = 7080.5860963, \&c. \\
 & \text{Aged Four.} \\
 & \div 600 = 11.800976827 \div 2 = \underline{\underline{\pounds 5.90048841}}
 \end{aligned}$$

For the Age *Five*,

$$\begin{aligned}
 & \text{16 years reversion.} \\
 & 12771.435651 \times .573982 = 7330.5741778, \&c. \\
 & \text{Aged Five.} \\
 & \div 580 = 12.638920996 \div 2 = \underline{\underline{\pounds 6.319460498}}
 \end{aligned}$$

CLXVI. Therefore the sums to be paid for an annuity till aged *Fifty*, to commence when *Twenty-one*, of 10 *l.* 100 *l.* or 1000 *l.* are, according to this Table,

Under Age	Ann. of 1 <i>l.</i>	10 <i>l.</i>	100 <i>l.</i>	1000 <i>l.</i>
1	3.667024	36.67024	366.7024	3667.024
2	4.71849813	47.18498	471.8498	4718.49813
3	5.3851412	53.851412	538.51412	5385.1412
4	5.90048841	59.00488	590.04884	5900.4884
5	6.3194604	63.194604	631.94604	6319.4604

Or if 100 *l.* or 1000 *l.* be paid for either of the ages, the annuity till *Fifty*, to commence when aged *Twenty-one*, is under the

Y y 2

sum

sum, and opposite the age in the following Table, which was found, as the other (129.), by dividing 100 *l.* by the value of 1 *l.* annuity.

	<i>Paid 100 l.</i>	<i>1000 l.</i>
Under the Age of 1 year	27.27006	272.7006
2 —	21.1932	211.932
3 —	18.5696	185.696
4 —	16.9477	169.477
5 —	15.8241	158.241

It remains now to know what is the value of 1 *l.* annuity *during life*, for the same ages, to commence when aged *Fifty*?

The value for the Age *Two*, and for the Age *Five*, have been already found (162.).

For the Age *One*.

$$\begin{aligned} &£4444.828 \times .182653 = 811.861168684 \div \\ &\div 870 = 0.933173757 \div 2 = \underline{\underline{£0.4665868785}} \end{aligned}$$

That is, the sum that will pay Age *Fifty* an *immediate* annuity is multiplied by *Forty-nine years* reversion, and the product is *divided* by 870 living aged *One*; and *half* the quotient is the value of 1 *l.* annuity.

For

For the Age *Three*.

$$\begin{aligned} \pounds 4444.828 \times .195778 &= 870.199536184 \div \\ \div 635 &= 1.37039297 \div 2 = \underline{\underline{\pounds 0.685196485}} \end{aligned}$$

That is, the sum that will pay Age *Fifty* is multiplied by *Forty-seven* years reversion, and the product is *divided* by 635 living aged *Three*; and *half* the quotient is the Answer,

For the Age *Four*.

$$\begin{aligned} \pounds 4444.828 \times 20269 &= 900.92218732 \div \\ 600 &= 1.501536978 \div 2 = 0.750768489 \end{aligned}$$

That is, the sum that will pay Age *Fifty* is multiplied by *Forty-six* years reversion, and the product is divided by 600 living aged *Four*; and half the quotient is the Answer.

CLXVII. Whence the values of 1 *l.* 10 *l.* 100 *l.* or 1000 *l.* *per annum* during life, to commence when aged *Fifty*, are as in the following Table.

<i>Under Age</i>	1 <i>l.</i>	10 <i>l.</i>	100 <i>l.</i>	1000 <i>l.</i>
1	0.4665868	4.6658687	46.658687	466.58687
2	0.6003724	6.0037244	60.037244	600.37244
3	0.6851964	6.8519648	68.5196485	685.196485
4	0.7507684	7.5076848	75.076848	750.76848
5	0.804077	8.04077	80.4077	804.077

Or

Or, the following annuities on paying *present* money.

	<i>Paid 100 l.</i>	<i>1000 l.</i>
Under Age of 1 year	214.32239	2143.2239
2 —	166.5632	1665.632
3 —	145.9435	1459.435
4 —	133.1968	1331.968
5 —	124.36619	1243.6619

It has been observed by several instances, particularly by that of Age *Ten* (153.), that the *immediate* annuity to *cease* when aged *Fifty*, added to that for *life*, to *commence* when *Fifty*, gives the value of an *immediate* annuity *during life*?

CLXVIII. Therefore adding the separate values already found (166, 167.) gives the following values for an annuity *during life*, to *commence* when aged *Twenty-one*.

<i>Age</i>	<i>1 l.</i>	<i>10 l.</i>	<i>100 l.</i>	<i>1000 l.</i>
1	4.1336108	41.336108	413.36108	4133.6108
2	5.31887053	53.188705	531.88705	5318.8705
3	6.0703376	60.703376	607.03376	6070.3376
4	6.65125681	66.512568	665.12568	6651.2568
5	7.1235374	71.235374	712.35374	7123.5374

Or, by paying 100 *l.* or 1000 *l.* *present* money, the Ages may receive, *during life*,
the

the following annuities, to *commence* when aged *Twenty-one*.

	<i>Paid 100 l.</i>	<i>1000 l.</i>
Under the Age of 1 year	24.1919	241.919
2 —	18.80098	188.0098
3 —	16.4702	164.702
4 —	15.03475	150.3475
5 —	14.3797	143.797

CLXIX. From which it appears that parents, by paying for their children, when *under One year* old, the following sums, do provide for them the following annuities when of age, if they are so happy as to survive *Twenty-one* years.

Paying 3667l. to receive 1000l.	{	<i>per ann. till</i> <i>aged Fifty.</i>
4133l. ————— 1000l.		
1000l. ————— 272l.	{	<i>ditto, during</i> <i>life.</i>
1000l. — nearly 242l.		
	{	<i>to cease when</i> <i>aged Fifty.</i>
	{	<i>per ann. du-</i> <i>ring life.</i>

It may be said that it is a very *great* chance whether a child, *not aged One*, may live *Twenty* years: Please to consider that it
is

is *that very great chance* that would enable the Society to grant *such* annuities.

Such Questions as will be found applicable to the several Societies, having been now *particularly* answered, it will be convenient to have them collected in one view; and the Reader may, if he pleases, ask himself, Whether the Answers have been satisfactory to him? If they have *not*, let such be *re-considered*; but if they have, let them be henceforth *indisputable*.

CLXX. What is the value of an *immediate* annuity of 1 *l.* to *cease* when the Age attains to *Fifty* (154.)?

For the Age

10?	Answer	£16.7915434	(88, 153.)
20?	_____	£14.3138518	(92.)
30?	_____	£11.205349	(98, 149.)
40?	_____	£ 7.04381	(74, 156.)
60?	{ to cease after } 10 years	£6.3955015 (101.)	

CLXXI. What is the value of 1 *l.* *immediate* annuity, to *cease* at the expiration of *Ten* years?

For the Age

10?	Answer	£7.87131252	(80.)
20?	_____	£7.713675	(94.)
30?	_____	£7.4034	(99.)
60?	to cease after 7 years	£5.08921525 (102.)	

CLXXII.

CLXXII. What is the value of 1 *l.* immediate annuity, to *cease* at the expiration of *Twenty* years ?

For the Age 10 ? Answer £12.67841884 (81.)
 20 ? ——— £12.07443055 (95.)

CLXXIII. What is the value of 1 *l.* immediate annuity to *commence* after *Ten* years, and to *cease* when aged *Fifty* ?

For the Age 10 ? Answer £8.92023 (86.)
 20 ? ——— £6.6001768 (97.)
 30 ? ——— £3.80195 (100.)

CLXXIV. What is the value of 1 *l.* annuity *during life*, to *commence* when aged *Fifty* ?

For the Age
 10 ? Answer £1.0586197645 (141*.)
 20 ? ——— £1.69870358 (142.)
 30 ? ——— £2.883948719 (143.)
 40 ? ——— £5.343051 (138.)
 60 ? { to commence } £2.6529143635 (144.)
 { when 70. }

CLXXV. What is the value of 1 *l.* annuity, *during life*, for the Ages, and to *commence* at the Times, here given ?

Z z

For

For the Age of *Ten*, to commence after the expiration of

30 years? Answer £2.45421104 (158.)

20 years? ————— £5.1717369 (159.)

For the Age of *Twenty*, to commence at the expiration of

10 years? Answer £8.298879734 (161.)

20 years? ————— £3.93812458 (160.)

CLXXVI. What is the value of 1 *l.* annuity *during life*, for the Age *Thirty*, to commence when aged *Forty*?

Answer £6.68589 (147.)

CLXXVII. What is the value of 1 *l.* annuity, *during life*, to commence when aged *Forty-seven*?

For the Age

20? Answer £2.2118765795 (146.)

40? ————— £6.957173 (137.)

60? } to commence } £3.95919796 (145.)
 } when 67. }

The value of these for 30 *l.* annuity is (75.),

For the Age 20 ————— £ 66.356295

40 ————— £208.71519

60 ————— £118.7759388

CLXXVIII.

CLXXVIII. What is the value of 1 *l.* annuity for *Three* years;

For Age

20 to comm. when *Forty-seven* to cease at *Fifty*.

Answer .5131737 (104.)

40 ————— *Forty-seven* ————— *Fifty*.

Answer 1.614122 (105.)

60 ————— *Sixty-seven* ————— *Seventy*.

Answer 1.306286 (103.)

Or, the same years annuities of 30 *l.* value, is (75.)

For the Age 20 ————— £15.395211

40 ————— £48.42366

60 ————— £39.18858

CLXXIX. Thus the principal Calculations, intended to be applied hereafter, have been made; and 'tis hoped, that whoever assents to the justness of them now, will be neither doubtful nor disgusted, when the same are applied to find the true State of the Society, of which he himself may be a Member.—And the Reader should observe, that these Calculations have not only been rated at the *lowest* possible by the Table of Mortality and of Interest; but also some particulars have been purposely omitted *hitherto*, lest the Enquiry should become too complicated

Z z z

complicated by taking in every consideration :
but it is fitting *now* to make these

MEMORANDUMS.

First. No provision is made for *any expences* which a Society must necessarily be at in the management of it.

Secondly. Of 294 living at *Forty* years old, only 289 are supposed to live Six months to receive *any* annuity ; but of the *five deceased* Members, one might have had *one* month's annuity due ; another, *two* ; another, *three* ; another, *four* ; another, *five* ; and the like at other Ages : for which no Fund is provided.

Thirdly. Mr. *Simpson's* Table is much *lower* rated than Dr. *Halley's* for the *most* part, and may prove *deficient* in practice (66.).

Fourthly. Though it is a common remark, that the young and healthy are snatched out of this life as suddenly as those who are more aged and infirm ;—yet such are *particular* cases that engage the attention, and rather surprize by the *improbability* of the event :— For generally *all Tables* at *all* Times shew, that more, in proportion to their number, die between *Fifty* and *Sixty*, than between *Thirty* and *Forty* and
there

there can be but little doubt that fewer will die of 1000 *chosen* Persons than of 1000 *promiscuously* taken.—If this be admitted, a Society will probably consist, for the most part, of the *healthiest*, (a) especially the *Nominees*, if any; and this will occasion a *Deficiency* that ought to be provided for (65.).

Another cause of deficiency will be explained in the *Second* Article of the ADDENDA.

CLXXIX*. To *explain* the application of these Calculations to the several Societies, and to avoid continual repetitions of the *same* remarks, let *Two imaginary* Societies be *supposed* now going to be *instituted*; the *first* (founded on *Honesty* and *just Principles* of Calculation) may be called,

THE IMPARTIAL SOCIETY ;
and let the name of the *second* be,
THE POLITIC GOTHAM SOCIETY.

Ani-

(a) At *Christmas*, 1771, the LAUDABLE SOCIETY had been instituted *Five* years; the number of the Members have been gradually increasing till they were *then* 1364 of *all* ages,—and only *Thirty-seven Deaths* were then reckoned among the Members of that Society, within the said *Five* years (64.).

Animadversions may be *freely* made on both of *these*; and the *Praise* or *Censure* that is due to either, may be bestowed without *Prejudice* or *Restraint*, since no one can be an *interested* party in a Society that has no *real* existence. As the writer would wish to give *no offence* to any Society whatever that *doth exist*, but to preserve impartiality to *all* of them, 'tis hoped they will not apply *more* of what follows to themselves, than what *they certainly know* to be *apt* and *true*.

CLXXX. The *General Enquiries*, intended to be made, are these *Six*:

First. What do Members pay in *present* money? that is, the *Total* of the *Value* of the money paid down, and of the *half-yearly* payments *reduced* to *present* money?

Secondly. What annuity *for life* ought to be received, when aged *Fifty*, for the money paid, and according to the age?

Thirdly. Whether the several *Ages* pay in due proportion to each other, for annuities of *equal* value? and, What is the difference?

Fourthly. Whether the Tables, *with* Admission-Fine, are *equal* to those *without* Ad-

Admission-Fine? and, What is the difference?

Fifthly. What annuity each Society can afford to pay?

Sixthly. In what proportion to those who are Members, or to the annuity that the Capital Stock can afford, must a person now pay to become a Member; according to Proposals delivered out in December, 1771?

Every Society will be examined by the same method, excepting when some difference among them renders *particular* enquiries necessary.

CLXXXI. It would be laborious and voluminous to examine *every* particular Age; therefore, to save trouble, and to avoid swelling this Essay to an unnecessary bulk, let the Ages of *Twenty*, *Forty*, and *Sixty*, only be examined; and these *three* are selected for no other reason, than as being *young*, *middle-aged*, and *old*: It is supposed the *intermediate* ages will *partake* of any difference that may be found between *these three*; that is, there will be a proportionable difference between the Ages *Twenty-one*, *Forty-one*, and *Sixty-one*, or between *Ten*, *Thirty*, and *Fifty*; or whatever three Ages that may be chosen in *like* manner.

CLXXXII.

(186)

CLXXXII. The PROPOSALS of the

FIRST IMAGINARY SOCIETY,

(Which has been called)

THE IMPARTIAL SOCIETY

OF

A N N U I T A N T S,

Are, according to this Table,

To receive 50 l. Annuity *during Life*,

WITHOUT DEDUCTIONS,

To commence when Aged *Fifty*.

The Age of	To pay half-yearly	& {	Admis- sion- Fine.	Half-yearly without Admis. Fine.	Half-yearly, and the First Payment in Present Money.
20	£ 1.1		52.345	2.9669	£ 2.8667
40	£ 3.35	2	16.6065	18.9635	£ 17.70657
60	£ 3.35	86.44515	10.37		£ 9.61823

N. B. The Age *Sixty* to commence Annuitant when *Seventy*.

CLXXXIII. What doth each Member pay in value of *present money with* Admission-Fine ?

An annuity of 1 l. to commence *immediately*, and to *cease* when the Age attains to *Fifty* ; or, after

after *Ten* years for the Age *Sixty*, was seen (92, 157, and 101.) to be worth, in ready money,

For the Age 20	_____	£14.3138518
40	_____	£ 7.04381
60	_____	£ 6.3955

And either of these values, being multiplied by the *yearly* sum paid, gives the value of *all* the payments in *present* money; and it has been seen (77.) that it is the *same* thing, whether an annuity be *received* or *paid*: for the *half* yearly payment may be considered as an *annuity*, which the Members are obliged to pay, or to *redeem* by paying an *equivalent* sum in *present* money.

The Age 20 pays *yearly* £2.2 by *this* Scheme.

40	_____	£6.7
60	_____	£6.7

The *former* sums, multiplied by *these last*, are the values of all the *half* yearly payments in *present* money.

£14.3138518	×	2.2	=	31.49047396
£ 7.04381	×	6.7	=	47.193527
£ 6.3955	×	6.7	=	42.84985 (a)

A a a

The

(a) If the sums paid in *present* money were *divided* by the value of 1 l. annuity till *Fifty*, the quotients would be the *yearly* payments, *viz* £2.2, and £6.7.

The Age *Twenty* pays,

Admission-fine, by the Table of	}	52.345
the Scheme — — —		
(a) The first <i>half</i> -year's payment		1.1
<i>Sixty</i> ditto worth in <i>present</i> money		31.49

£84.935

The Age *Forty* pays,

Admission — — —	216.6065
First <i>half</i> -year's payment	3.35
<i>Twenty</i> ditto worth in <i>pres.</i> money	47.1935

£267.15 (5.)

The Age *Sixty* pays,

Admission — — —	86.44515
The first <i>half</i> -year's payment	3.35
<i>Twenty</i> ditto worth in <i>pres.</i> money	42.84985

£132.645 (5.)

CLXXXIV.

(a) The first half-yearly payment in *present* money is, *properly*,
so much added to the *Admission-Fine*.

CLXXXIV. What annuity *for life* ought to be received when aged *Fifty*, or after *Ten* years for the Age *Sixty*, for the *present money* paid (141.)?

The value of 1 *l.* annuity on these terms, for the three ages, was found (174.); therefore, by the Rule of Three, if *those* sums would purchase 1 *l.* annuity, what will *these above* sums purchase?

Answers.

	<i>Value in present money of 1 l. ann.</i>		<i>Money paid.</i>	<i>Annuity.</i>
Age of 20	As 1.6987	: 1 ::	84.935	: 50 <i>l.</i>
40	As 5.343	: 1 ::	267.15	: 50 <i>l.</i>
60	As 2.6529	: 1 ::	132.645	: 50 <i>l.</i>

Observe, that the value of 1 *l.* annuity, multiplied by the annuity (which *here* is 50 *l.*), gives the *worth* of the annuity in *present money*; — as the *total sum* paid in present money, divided by the value of 1 *l.* annuity, gives the *annuity* which ought to be received for the *sum* paid (17.).

CLXXXV. Do the Ages pay in *due proportion* to each other, or what is the difference?

Mere *inspection* shews that the annuities are *here* perfectly equal, and in just proportion to the ages and sums paid; therefore there is not the least difference.

CLXXXVI. Is the Table *with* Admission-Fine equal to that *without* Admission-Fine; and what is the difference?

A a a 2

If

If the Question, by *without Admission-Fine*, means without *any present money paid*, not even the *first half-year* in advance, but at the end of *six months*; then the sums in the *fourth column* of the Scheme (182.) are the *half-yearly* sums to be paid; and the *yearly* sums, being *double the half-yearly*, are these:

Age

20 to pay yearly $5.9338 \times 14.3138518 = 84.935$

40 Ditto — $37.927 \times 7.04381 = 267.15$

60 Ditto — $20.7404 \times 6.3955 = 132.645$

The *yearly* sums, being multiplied by the value of *1 l. till Fifty*, produce the *same* money as before, which purchased *50 l. annuity* for each age; therefore there is *no difference* between the two parts of the Table; it being perfectly *equal* whether the Age *Twenty* pays $\pounds 52.345$ Admission-Fine, and $\pounds 1.1$ half-yearly till aged *Fifty*, or pays *no Admission-Fine*; but, in lieu of it, $\pounds 2.9669$ half-yearly till aged *Fifty*; and the same with the *other Ages*.

CLXXXVII. But if the Question, tho' it says *without Admission-Fine*, does nevertheless require the *first half-yearly* payment to be paid down in advance, and to have *20 half-yearly* payments in reversion besides; it is plain this *immediate* money would be *.5* for *1 l. annuity*, or $\pounds 462$ which *so many* Members (93.) must pay at the Age *Twenty*; — for this

this *present* money is not lessened by reversion nor chance of life : Therefore the *total* sum in that Table would be £13687.9990865, which is 462 *l.* added to £13225.9990865 ; and the said Total, divided by the 462 living at the age of *Twenty*, gives for each to pay £29.6277036 for 2 *l.* annuity till aged *Fifty* ; which is 1 *l.* to each Purchaser more than before, and is £14.8138518 for 1 *l.* annuity ; or .5 more than before (170.) for a *half-year's* payment (a).

CLXXXVIII. The same considerations being applied to the Ages *Forty* and *Sixty*, the value of the *half-yearly* payments will be found by dividing the *whole* value in *present* money of 50 *l.* annuity *during* life, by the value of 1 *l.* (170.) annuity till *Fifty*, or for *Ten* years, at Age *Sixty*, increased by .5.

As Age 20 —	14.8138518	: 1 ::	84.935047396	: 5.7334
40 —	7.54381	: 1 ::	267.15	: 35.41314
60 —	6.8955	: 1 ::	132.645	: 19.23646

The *half* of those *yearly* payments is to be paid in *advance* ; that is,

For Age 20	—	2.8667
40	—	17.70657
60	—	9.61823

the

(a) Dr. Price, in his *Observations*, &c. page 19, treating of *Yearly* Payments, says, " If the first payment is made immediately,

(192)

the same as in the Scheme (182.); and the *present* value of all the *same* sums, paid *half-yearly*, is found as before (183.):

$$\begin{aligned} \text{Age 20} & - 14.31385 \times 5.7334 = 82.07 \\ & 7.04381 \times 35.41314 = 249.4434 \\ & 6.3955 \times 19.23646 = 123.02677 \end{aligned}$$

The Age *Twenty* pays,

First <i>half-year's</i> payment,	—	2.8667
60 ditto worth in <i>present</i> money		82.07
Total in <i>present</i> money		<u>£84.9367</u>

The Age *Forty* pays,

First <i>half-year's</i> payment	—	17.70657
20 ditto worth in <i>present</i> money		249.44344
Total in <i>present</i> money		<u>£267.15001</u>

The Age *Sixty* pays,

First <i>half-year's</i> payment		9.61823
20 ditto worth in <i>present</i> money		123.02677
Total in <i>present</i> money		<u>£132.645 (5.)</u>

Which

“ ately, the value will be the single payment divided by the
 “ value of the Life for the given term increased by Unity: that
 “ is, in the present case, 222 l. divided by 10.53 (instead of
 “ 9.53); or £21.08.”
 Dr, Price increases by Unity for yearly payments; which is
 the same as .5 for half-yearly ones,

Which Totals being so nearly the same sums as before, are worth the *same annuities*, and differ in nearly nothing from the *other values*: so that it is *equal* whether the ages pay by *either* of the *three* modes here considered, or pay down in *present* money the *whole* value of the Annuity in *one single* payment, by which the half-yearly payments would be *redeemed*.

.. CLXXXIX. What Annuity can this Society afford to pay?

It is supposed, in *this* Society, that every Member has paid the *full value* for the purchase of his annuity; by which the Society is enabled to *perform* its promise *faithfully*, and to pay 50*l.* annuity to *each* of its Members.

CXC. In what proportion, to those who *are* Members, or to the *annuity* that can be afforded by the *Joint Capital Stock*, must a person *now* pay, to *become* a Member?

It has already been declared, that *both* Societies are purely *Ideal*;—and *this* is supposed to consist of perfectly *honest, worthy, and just* men; *such* would be far above the *meanest* of deriving any other advantage from the Institution of the Society, than the *mutual* benefit of *Association* (127.). Every Member would then pay, from the *beginning*, by the *same Table*; and the *last* and the *first* would contribute

contribute in *equal* proportion for the 50 l. annuity expected.

Such a Society, having proceeded on a *regular plan*, and upon *fair and true principles*, could never become *despicable* by making *false promises*, *deluding* with false hopes, or by *defrauding* under false pretences;— but would rather become *respectable* for the uprightness of its conduct; *durable*, from the solidity of its foundation; and act with *universal Justice*, from the true symmetry of all its parts; whereby it fulfils the Intention of its Institution, and has answered the *Queries* to entire satisfaction. — Oppose to this, in which such harmony prevails, the *most direct contrast*; let the *Second Imaginary Society* be a perfect CHAOS, in which ignorance and craft, absurdity and injustice, are so blended, that the mass of confusion hurries on the *inevitable destruction* of

THE POLITIC GOTHAM SOCIETY.

CXCI. An idea of *Perfect Beauty* has been obtained from Nature, not given by her in *one single object*, but diffused in a vast variety of parts; which, being attentively collected by the Imagination, enabled the Artist to form a Figure that at once astonished and delighted the Beholders. — So selecting the *separate Imperfections* of *Many*, and representing them *united*, may excite such a strong idea of *Deformity*, as may inspire the most contempt-

contemptible disgust, and an abhorrence to the least connection.

It is with the view of making the *Politie Gotham Society* such an object, that the Author has collected not only the *real imperfections* of all the Societies in actual being, but has also *SUPPOSED* some, which, it is hoped, cannot be found belonging to *any one* of them; that themselves may not only confess *this Imaginary Society* to be truly *despicable*, but also may be ashamed of, and *immediately reform*, the least resemblance to it.

This *non-existing* Society is supposed (and, after what has been said, it is hoped it may be supposed, *without offence* to any Society that doth exist) to be instituted on no rational plan, or just principle; by *such* as were so totally ignorant of the *first rudiments* of Calculation, as not, perhaps, to understand *even decimals*; having no idea of the subject, but merely *guessing* that the Age *Forty* should pay something more than a *younger* Age, because he would become a Recipient sooner; yet not able to penetrate *so deeply* as to understand that there can be any difference between the Age of *Forty* and of *Forty-five*.

For a Specimen of Cunning, Ignorance, and Dishonesty, let the Institutors be imagined to reason in this manner:—— As *we* are the *Founders* of this Society, it is very reasonable that *ourselves* should pay but *lit-*

B b b

tle,

tle, if any thing, towards it; for we will contrive means *some-how* to make it succeed. And, to make it more inviting to such as do not understand figures *so well as us*, let the *nominal* value be double its *real* value; for we can make a *deduction* out of the annuity *each payment*, to make it the same thing.—If we promise 100 *l.* annuity, we may deduct 25 *l.* every half-yearly payment; and so the *real* annuity would be but 50 *l.* and all those who should not perceive the deception, would be *fairly* taken in.—Now, as to *what sum* should be paid for an annuity, to *commence* when aged *Fifty*; That nobody, for it is *impossible*, can tell; but we may *guess* pretty nearly, perhaps, if we consider, that whatever we pay now will increase in a few years to a *vast prodigious* sum indeed! and consider too, what an *amazing* number will die in the time, and leave whatever they should have paid for our Benefit.—But the *best* of the scheme is, that a great many may become *poor*, and unable to pay their *half-yearly* dividends. Let all such *forfeit* what has been paid to our use: for, by this, *'tis hoped*, we shall get very great sums indeed.—On the whole, if the Age Forty pays 3 *l.* every half year, till Aged Fifty, to receive 50 *l.* annuity for life after that time, *surely* it will be full sufficient for *us*, who have been at the trouble of calculating in *this manner*; and if we immediately

mediately raise the payments to *double or treble* of what we pay, it is but *common JUSTICE*; for those who partake of the *Benefit* of our Scheme should pay us *handsomely*; and let them pay an *Admission-fine* besides, for that will make it *better still for us*.

For these reasons then, suppose we fix our payments for the Age Forty at 3 *l.* half-yearly; then it will be as plain as *any thing in our whole Scheme*, that the Age Twenty should pay but 1 *l.* for he will be just *three times* as long as the Age Forty, before he becomes Recipient, and so will pay for *Thirty* years, 2 *l.* each year; which is 60 *l.* in all; and Age Forty pays 6 *l.* yearly for *Ten* years, which will be 60 *l.* too: So we may defy *Euclid* to give a *plainer demonstration*. But a grander proof of *our skill* in Calculation, is the finding out what Age Sixty should pay; because he is to pay the same term of years as Age Forty. *Every body* would think, and *so did we once*, that he ought to pay *equally* with the Age Forty; but having laid *all our heads together*, we found out at last, that the Age Sixty was half as old again as the Age Forty, and therefore not quite so likely to live as the Age Forty; so we have set him down at 2 *l.* half-yearly payments. And we can *prove* this to be very right; for the *half-yearly* payments increase in value by the *same* proportions as the *Ages* do: for as 2 *l.* is *twice*

1 l. and 3 l. is *three* times 1 l. so the Age Forty is *twice* the years of Age Twenty; and Sixty is *three* times the Age Twenty.
Q. E. D.

It is agreed then, that all present pay now in the said proportions, and then proceed to *raise* the payments *immediately*.

Nemine Contradicente.

(*Mr. Gull'em speaks.*)

NOW, Gentlemen, having settled this affair in an *equitable* manner among ourselves, a motion is made by Mr. *Santhonesté*, which is seconded by Mr. *Cheataill*; That the half-yearly payments be immediately raised to *double* the value we have just paid; and also that an *Admission-Fine* be required in these proportions:

Age 20 to pay Admission-Fine 20 l.

40 ————— 60 l.

60 ————— 40 l.

So many of you as think this motion is LAUDABLE, PROVIDENT, EQUITABLE, RATIONAL, AMICABLE, PRUDENTIAL, and FRIENDLY, please to signify it by holding up your hands,

Gentlemen, as you are all so perfectly *unanimous*, there is no need of putting the contrary opinion to the vote.

Motion

Motion for Thanks to the Managers for their Care and Uprightness, made, seconded, and carried,—Nem. Con.

(Court adjourned.)

This Society is supposed to have been instituted *before* the publication of Mr. *Simpson's Doctrine of Annuities*, or Mr. *De Moivre's Annuities*, or Dr. *Halley's Tables*; when the subject was a real mystery, and the Managers had not these guides to conduct them through the perplexing labyrinth: for surely there are no persons *now* so ridiculously presumptuous, as to pretend to institute any Society of the kind, without *first* consulting these able masters on the subject, and whose assistance is so *nigh at hand* (a).

The Institutors of this Society are supposed to be, *in every thing*, the *reverse* of the other; their hearts as imperfect as their heads; so that, in defiance of Calculation, they feel something *within* that dictates to them to contribute in such small portion *themselves* to the *Common Stock*, and to seduce others to join them, and to contribute *much more*, by pro-

(a) How inexcusable must any Society *now* be to persist obstinately in publishing abstracts calculated to *deceive* and *defraud*, after the publication of Dr. *Price's Observations*; which has not only *enlightened* the subject, but has also made *particular Calculations* for *their use*, at page 109, and elsewhere.

promising far *greater annuities* than, as even themselves must know, could be paid by any other means than (a) "*by Impositions on the Public.*" And being "*encouraged by Credulity and Folly,*" as "*their Gain*" could be no other than so much "*Plunder taken from the Public,*" every wile that Craftiness can suggest may be expected to be practised by the mean Impostors: so that, with a conceitedness that Ignorance is oft possessed of, they imagine *to dupe* the Public by an equivocal Promise of perhaps 1000 *l.* or any annuity; and suppose others not sufficiently sagacious to find out that 478 *l.* deducted *half-yearly* therefrom would leave but 44 *l.* remaining; intending to catch the unwary eye, and beguile the unsuspecting (b).

CXCII.

(a) See Dr. Price's *Reversionary Payments*, page 112, and *Introduction*, page xi.

(b) Should such conduct plead, in vindication of its promising *much*, on condition of deducting a *part*, the example of the most *respectable* Assembly in the nation,—the reply may be this:—The Public Welfare undoubtedly requires very great sums for its support, and Government alone hath the lawful authority to impose and levy the necessary Taxes. GAMING is a species of folly that, being incorporated with Covetousness, begets Knavery, and becomes a Vice; and one which hitherto has been found *irreclaimable*: on which account it is, perhaps, the most *proper* object of Taxation that may be chosen; and such a Tax would certainly be levied in the most *equitable* manner by *Lotteries*, since every one may freely choose whether he will pay the Tax or not; and what is paid, is *always* paid in the exact proportion to the *circumstances*, *Covetousness* and *Folly* (taken together) of the party; and thus a Private Vice is made a Public Benefit.—But does it follow, that a
Private

CXCII. Let it be supposed farther, that the Directors, Trustees, and Auditors, should procure each other that have been *out* of the management to be ballotted in again, and by continual rotation maintain these offices among a *select few* (a).

CXCIII. Let them procure votes to pass, that no Accompts or Books should be inspected by any Member out of the Management, but upon the *General Court* days; when it would be impossible, on account of the numbers assembled and the business to be transacted, for any Member to examine them with accuracy, or any degree of satisfaction; and this might be done on no other but the weak pretence, lest such Books might be worn or torn; so *embezzlements* might be made at pleasure: for what Member could inform himself whether the Stock, that the Society is declared in possession of, be the *Whole* that should be, or not; if he can have no better access to the *books to examine*

Private Society may, with Honesty, assume to itself the power of levying contributions from *Ideotism*? for surely the promising 1000 or 50 l. instead of 44 l. only meant to be actually paid, could answer no purpose whatever but decoying of such as humanity would *Pity*, but not *Punish* nor *Defraud*.

(a) If these Offices are supposed to have not the least *visible* emolument affixed to them, might it not be asked, What could be the motive and intention of these men in seeking to be *re chosen*?

amine the accounts? Let these Managers adopt a notion that the Members, by appointing them Directors, Trustees, Auditors, &c. had given totally out of their own power *all claim* to inspect whether the business was *faithfully* executed or not; then it would be proceeding but *one* degree farther for them to adopt the notion likewise, that the Society, by so doing, had also transferred their Capital Stock to them, which they might *share among themselves*, without being accountable to any.—And *so ends* (independent of Calculation) a Society, begun by *such people*, and conducted in *such a manner* (a).

Let this Society be examined, and perhaps the Reader will be surprised to find what a contrast it affords to the harmony of the *first* imaginary one.

CXCIV. By the *Second* abstract the Members of

THE POLITIC GOTHAM SOCIETY

Are

(a) By these cautionary marks the Members of any Society, without being *ever* jealous, may suspect some *unfair* proceeding; when they ought to ring the alarm-bell immediately by advertisements to collect such of their Brethren as may be supposed to have no connections with such iniquitous practices, to oppose such votes from passing as may be prejudicial to the good of the *Whole*.

Are to pay,

Age	Half-yearly.	Admission.	Half-yearly without Admission.
20	£2.	£20.	£2.5
40	£6.	£60.	£7.5
60	£4.	£40.	£5

CXCV. What do the several Ages pay in value of *present* money by the Second Abstract (183.)?

The Age *Twenty* pays,

Admission-Fine	—	20.
First <i>half</i> -year's payment		2.
60 ditto in reversion, worth in		} 57.2554
<i>present</i> money,	—	
Total		£79.2554

£14.3138518 is the value of 1*l.* annuity till *Fifty* for this Age (93.), which is multiplied by 4*l.* the *yearly* sum paid by this Age; and the produce is, as above, 57.2554.

The Age *Forty* pays,

Admission-Fine	—	60.
First <i>half</i> -year's payment		6.
20 ditto in reversion, worth in		} 84.52572
<i>present</i> money,	—	
		£150.52572
C c c		that

viz. $7.04381 \times 12 = 84.52572$, that is, the value of 1 *l.* annuity (74.) multiplied by £12. which is the yearly payment, gives £84.52572.

The Age *Sixty* pays,

Admission-Fine	—	40.
First half-year's payment		4.
20 ditto in reversion, worth in } present money, — }		51.164
		£ 95.164

$6.3955 \times 8 = 51.164$; that is, the value of 1 *l.* annuity (101.), multiplied by the yearly payment, gives £51.164.

CXCVI. What annuity doth *each* Age purchase *separately* (184.) ?

Age	Value of 1 <i>l.</i> ann.	Present Money paid.	Annuity.
20 as 1.6987	: 1 ::	79.2554	: 46.656
40 as 5.343	: 1 ::	150.52572	: 28.172
60 as 2.6529	: 1 ::	95.164	: 35.871

that is, if 1.6987 gives 1 *l.* annuity (184.), for the Age *Twenty during life*, and to commence when *Fifty*; then, What will £79.2554 give (which is the sum *that* Age has been found above to pay in *present* money), and the Answer is £46.656; that is, the sum paid

paid is *divided* by the value of 1 l. annuity, thus :

$$1.6987) 79.2554 (46.656$$

for the Unit makes no alteration in the Rule of Three, as a multiplier, but leaves the sum 79.2554, as before, to be divided by 1.6987 (37.).

It is the same by Age *Forty* and *Sixty*, taking the sum that each of them paid in *present* money, and dividing it by the value of 1 l. annuity (184.), which gives the annuity that is opposite the respective ages.

CXCVII. Do the several ages pay in due proportion to each other ?

Mere inspection shews these annuities to be in very *unequal* proportions:

CXCVIII. Is the Table *with* Admission-Fine equal to that *without* Admission-Fine (186.) ?

The Age *Twenty* pays *without* Admission-Fine,

The first *half-year's* payment (194.) 2.5
60 ditto in reversion, worth in }
present money, ——— } 71.569

—————
£ 74.069

(206)

that is, $14.3138518 \times 5 = 71.569259$

The Age Forty pays *without* Admission-Fine,

First <i>half</i> -year's payment	7.5
20 ditto in reversion, worth	} 105.65715
in <i>present</i> money,	
	£ 113.15715

that is, $7.04381 \times 15 = 105.65715$

The Age Sixty pays *without* Admission-Fine,

First <i>half</i> -year's payment	5.
20 ditto, worth in <i>present</i> money,	63.955
	£ 68.955

that is, $6.3955 \times 10 = 63.955$

The Annuity that each Age purchases *without* Admission-Fine,

Age.	Value of 1 l. Ann.	Pres. Money paid.	Annuity.
20 as	1.6987 : 1 ::	74.069259 :	43.603
40 as	5.343 : 1 ::	113.15715 :	21.178
60 as	2.6529 : 1 ::	68.955 :	25.992

The

The value of the annuity is found, as before (196.), by dividing the sums paid by the value of 1 *l.* annuity.

The *present* Money paid.

	Age 20	Age 40	Age 60
<i>With</i> Admis- sion-Fine	79.2554	150.5257	95.164
<i>Without</i> ditto	74.069	113.1571	68.955
<i>Difference</i>	5.1864	37.3686	26.209

The Age Forty pays, by the Table *with* Admission, 37 *l.* more than the same Age pays *without* Admission-Fine.

The Annuities purchased are,

	Age 20	Age 40	Age 60
<i>With</i> Admis- sion-Fine	46.656	28.172	35.871
<i>Without</i> ditto	43.603	21.178	25.992
<i>Difference annually</i>	3.053	6.994	9.879

The Age *Sixty* purchases, by the Table, *with* Admission-Fine, an annuity of near 10 *l.* more than the same Age does *without* Admission-Fine; therefore the *two* Tables are very *unequal*.

CXCIX. What annuity can this Society afford to pay (189.) ?

Supposing the *same number* of Members to have been admitted on paying by the Table *with* Admission-Fine, as on paying *without* Admission-Fine ; then adding the *two* annuities together, and *dividing* the *Total* by *two*, will give the *Medium*.

	Age 20	Age 40	Age 60
<i>With</i> Admis- sion-Fine, }	46.656	28.172	35.871
<i>Without</i> ditto }	43.603	21.178	25.992
Divide the Totals by 2 }	90.259	49.35	61.863
The <i>Medium</i>	45.1795	24.675	30.9315

Thus the two Tables will pay the *Medium*, instead of the Annuity which each Table purchased separately ; and the one Annuitant will *lose half*, and the other will *gain half* of the difference between the said Annuities.— So that Age *Sixty*, that bought *with* Admission-Fine, will receive *less* than the annuity he purchased, which was £35.871 ; and the *same* Age who bought *without* Admission-Fine will receive *more* than the annuity he purchased, which was £25.992 ; for each will

will receive an *equal* dividend of £30.9315 annuity, instead of their *actual* purchase.

The *Medium* of the two Tables being gained, the medium of the *Ages* must be also gained, to tell what the annuity must be by *equal* dividend; which the Society is supposed to pay, instead of the annuity which each Age *actually* purchased.

The Age 20 purchased at a medium	£45.1795
40	24.675
60	30.9315

Divide the Total by 3)100.786

The Medium	£ 33.595
------------	----------

CXCIX*. The three Ages might receive £33.595 annuity, each by *equal dividend*, provided the Society hath an *equal number* of *each age* admitted; for the equal annuity will be different, according to the *number* that have actually purchased *high* or *low* annuities.

Suppose the difference in the *numbers* should be so *great*, that there is found Six of the Age *Twenty*, to One aged *Forty*, and to One aged *Sixty*.

(210)

Six Ann. for Age 20 is $45.1795 \times 6 = 271.077$
 One _____ 40 _____ 24.675
 One _____ 60 _____ 30.595

Divide the Total by the *Eight* }
 Annuitants _____ 326.347

The *equal* Dividend for each £ 40.793

In this case the Society would be *richer* than by the last, and afford to pay £40.793 to each Member.

Again : Suppose the difference in the numbers to be Six of the Age *Sixty*, to One of *Forty*, and One of *Twenty* years old.

Six Ann. for Age 60 is $30.9315 \times 6 = 185.589$
 One _____ 40 _____ 24.675
 One _____ 20 _____ 45.1795

Divide the Total by the *Eight* }
 Annuitants _____ 255.4435

The *equal* Annuity for each £ 31.9304

Again : Suppose the difference in the number to be Six of the Age *Forty*, to One aged *Twenty*, and to one aged *Sixty*.

Six

Six Ann. for Age 40 is	$24.675 \times 6 =$	148.05
One	20	45.1795
One	60	30.9315

Divide the Total by the *Eight* }
 Annuitants } 224.161

The *equal* Annuity for each is £ 28.0201

These four Examples shew how greatly the knowledge of what annuity the Society can afford to pay, depends upon knowing *how many* Members there are of *each Age*: but this must be understood of a Society like the POLITIC GOTHAM, whose Members purchase *different* Annuities; for when the purchased annuities are equal at *all Ages*, the *number of each Age* is then immaterial.

CC. When the purchased annuities are *different*, those only who know the number at *each Age*, can tell *exactly* what annuity can be paid; and the most *impartial Computation* that can be made by a person who does *not* know the *Number* at each Age is, by *supposing* the number at *all Ages* to be *equal*.

By this supposition the *Politic Gotham Society* might pay £33.595 annuity, and no more (121.), though they should promise £44, or £50, or whatever other annuity above £33.595.

CCl. But even *this* annuity cannot be afforded, unless the *whole* Society had paid as
 D d d much

much as *this* Abstract requires; whereas it is *bere* known, that the *first* Members paid by a very *different* Abstract, and the value of those payments must be found.

The Age *Twenty* paid,

The first *half*-year's payment 1.
60 ditto, worth in *present* money, 28.6277

£29.6277

that is, $14.3138518 \times 2 = 28.6277036$ (183.)

The Age *Forty* paid,

The first *half*-year's payment 3.
20 ditto, worth in *present* money, 42.2628

£45.2628

that is, $7.04381 \times 6 = 42.26286$

The Age *Sixty* paid,

The first *half*-year's payment 2.
20 ditto, worth in *present* money, 25.582

£27.582

that is, $6.3955 \times 4 = 25.582$

The

The annuities these sums would purchase for these Ages, are (196.),

Age 20 as 1.6987 : 1 :: 29.6277 :	17.441
40 as 5.343 : 1 :: 45.2628 :	8.471
60 as 2.6529 : 1 :: 27.582 :	10.997

Divide the Total by <i>three</i>	<u>36.309</u>
----------------------------------	---------------

The <i>equal</i> Dividend for each is	<u>£12.103</u>
---------------------------------------	----------------

The Answer to the Question is *now* known to be, That the Members who paid by the *first* Abstract could not afford to pay more than £12.103, if the number of Members at *all* Ages were *equal*:—And that when *as many* were admitted by the *second* Abstract as were by the *first*,

The annuity by the <i>first</i> Abstract,	12.103
Added to the annuity by the <i>second</i> ,	<u>33.595</u>

And the Total divided by <i>two</i> ,	<u>45.698</u>
---------------------------------------	---------------

Gives the <i>equal</i> annuity to be paid,	} £22.849
by which the 33.595 annuity	
the last Members should receive,	
is now reduced to	

CCII. If the numbers by the *second* Abstract should increase to one hundred times the number admitted by the *first*, the insuffi-

ciencies of the first Annuitants payments would be less, in proportion, with respect to the General Fund; and the annuities would be proportionably nearer to *that* purchased by the Members admitted by the *second* Abstract.

The <i>equal</i> annuity of <i>one</i> Mem- ber, by the <i>first</i> Abstract,	} 12.103
Ditto of 100 Members, by the <i>second</i> Abstract,	
	} 3359.5.

Divide the Total of the annu- ities for 101 Members by 101,	} 3371.603

The <i>equal</i> annuity for each Member is,	} £33.3822

for $3371.603 \div 101 = 33.3822$; which is only .2128 less than a Member actually purchases by the *second* Abstract: so that in *such a case*, if the Founders of the Society had paid *nothing*, the deficiency would be but *small* in the General Fund, when the number of them should become in proportion of *one* to a *hundred*.

CCIII. This observation is made, because the Author, not always knowing the real numbers admitted by each Abstract, nor the real numbers of each Age, will in such cases always *suppose* the numbers to be *equal*;

equal (198.). If such a supposition is found by any of the *existing* Societies to give a more *unfavourable* calculation than the reality of theirs, such may publish their *true* state. This Essay will not pretend to give the *exact* state of each *particular* Society; but only, from a supposition that the *numbers* at all *Ages* and by each *Abstract* are *equal*, deduces a *General* Calculation, which is as *probable* to give *more* than the Annuities can be afforded to be, as it is to give *less*.

The Probable *General* Answer to the Question remains then still to be £22.849; and it must be left to the Society to shew how nearly they approach to Case (202.).

CCIV. In what proportion to those who are Members, or to the annuity that can be afforded by the Society, must a person pay to become a Member?

The *first three* Members who entered by the *second* Abstract, with Admission-Fine,

Paid at the	Age 20	Age 40	Age 60
In <i>present</i> money	79.2554	150.5257	95.164
or <i>without</i> Admission	74.069	113.1571	68.955
Total of Both	153.3244	263.6828	164.119
Median between	76.6622	131.8414	82.0595
The <i>first</i> Members paid (101)	29.6277	45.2628	27.582
The Difference is	47.0345	86.5786	54.4775

What

What the Members pay by the *second Abstract more than by the first*, is £47.0345 at the Age *Twenty*, and £86.5786 at the Age *Forty*, and £54.4775 at the Age *Sixty*.

Or if the number of Members by the *second Abstract* are *equal* to those by the *first*,

	Age 20	Age 40	Age 60
To the Medium of the 2d Abstract	76.662	131.841	82.059
Add the 1st Abstract —	29.627	45.262	27.582
Divide the Total by — 2)	106.289	177.103	109.641
The Medium of the two Abstracts	53.144	88.551	54.82
To pay more than any one collectively	23.518	43.29	27.239

If the number of Members by *both Abstracts* are *equal*, a person, by the *second Abstract*, must pay *more* than any one Member, *collectively*, has paid at the

Age 20	—	£23.518
40	—	£42.29
60	—	£27.189

CCV. Suppose A to have no Stock himself, and to take B as a partner on paying 20 *l.* their whole Stock would be 20 *l.* which is 10 *l.* each taken collectively. Here B loses or gives 10 *l.* to A, and it makes no difference in the Stock itself whether they had paid

paid 10 *l.* each, or B had paid 20 *l.* and A *nothing*.— Suppose C taken into partnership on paying 30 *l.* the whole Stock would be 50 *l.* among *three*; which is £16.66 each person, and C would have given £13.33, and B would have given £3.33 to A, who having no Stock at first, has now £16.66, the same as the others collectively.

Again: Suppose A to pay 0

B — 20

C — 30

D — 40

E — 50

F — 60

G — 70

The whole Stock is 270 *l.*

Divide by the Seven Partners, and } £38.57
the Share for each is

The share of each Partner being £38.57, if H be admitted on paying 80 *l.* it is plain that he not only pays 10 *l.* more than G, but also that he pays £41.43 more than any one Partner has paid, when taken collectively, or that the Stock is worth; for whether they had paid £38.57 each, or had paid in the different proportions as before given, the
Stock

Stock would be worth no more than 276*l.* consequently H would pay £41.43 more than any of his Partners, or so much more than the Stock was worth.

CCVI. In the same manner a Member *Aged Twenty* must pay 79*l.* with Admission-Fine, or 74*l.* without Admission-Fine, or rather at a *Medium* £76.66 to be admitted into this Society, the Members of which had paid, one with another *collectively*, no more than £53.144, even supposing as many Members had already been admitted by the *second* Abstract, as had been by the *first* (204.).

Therefore, supposing the numbers admitted by both Abstracts to be *equal*, Members would pay in these proportions :

	<i>Age 20</i>	<i>Age 40</i>	<i>Age 60</i>
A person to become a member, pays at a medium	76.662	131.841	82.059
Each member collectively has paid	53.144	88.351	54.82
The Difference is (as before)	23.518	43.29	27.239

The *Age Party*, either *knowingly* gives away, or is *unknowingly defrauded*, of, £43.29, even on the supposition that the number of Members admitted by *both* Abstracts are *equal*

equal. So likewise the *annuities* purchased by

	Age 20	Age 40	Age 60
The <i>second</i> Abstract (199.)	45.1795	24.675	30.9315
The <i>first</i> and <i>second</i> ditto equally	31.285	16.573	20.664
<i>Annual Loss</i>	13.8945	8.102	10.2675

For the annuity that each Age in the Society has purchased *collectively*, and on supposition that the numbers admitted by both Abstracts are *equal*, are (204.),

	Value of 1 l. ann.	Sums paid.	Annuity.
Age 20 As 1.6987 : 1 ::		53.144	31.285
40 As 5.343 : 1 ::		88.551	16.573
60 As 2.6529 : 1 ::		54.82	20.664

The money that Age *Twenty* is required to pay by the *second* Abstract, should yield him an *annuity* of £45.1795, instead of which he could receive not £31.285; for *part* even of *that* must supply the *deficiencies* of the *other* Ages.

Thus Age 20 purchased			31.285
40	—	—	16.573
60	—	—	20.664

Divide the Total by 3) 68.522

The ann^y. for each by *equal* dividend £22.84

E c c

which

which is, as before (201.); and Age *Twenty* could receive by *equal* dividend but £22.84 annuity, though he had *actually* purchased £45.1795; so that he would lose £22.3395 *annually*, or very near *half* his purchase-money (a.).

CCVII. The *Politie Gotham Society* being thus examined, it is found that its Members pay in very *unequal* proportions by *with* Admission-Money and *without*; and also by the *different ages*: That it is very *unable* to pay *half* the promised annuity of 50 *l.* and that those who are admitted by the *last* Abstract pay, at the Age of *Twenty*, near *half* the money to make good the *deficiencies* of those admitted by the *first*; and if such a Society proceeded upon the Principles imputed to it (101, &c.), all *just* men (even if *themselves* were Members of it) would agree in opinion, that the *Founders* could not be too much EXPOSED, RIDICULED, and DESPISED.

CCVIII. Though all will acknowledge, that if the imputations are just, the *Founders* of the *last Imaginary Society* cannot be too severely reprehended; yet the Author, being now upon the very verge of examining *real* existence,

(a) The Reader is desired to perfect himself in the clear knowledge of the *method* used in the examination of *these two Societies*, as the *same* will be followed in the enquiries into the state of *others*.

existence, feels uneasy apprehensions lest any thing that has been said of it may prove

“ So pat to all the Tribe,
“ Each cries, *That was levelled at Me.*”

In such a case, his well-meant zealous endeavours to be of *service* to the SOCIETIES, by explaining to them the *true state* of their Schemes, would not receive the approbation from them which he is anxiously solicitous of deserving.—They will then consider the *second Imaginary Society*, as being formed *on purpose* to apply it to *their's*.—They will not remember or regard that there are *Two Imaginary Societies*, and that *both* were *supposed* in being with the sole intent of exhibiting a *forcible contrast* to each other; a contrast of Perfection and Imperfection, of Order and Confusion, of Uprightness and Knavery. The Author's sincere hope and wish is, that the Schemes of the *existing Societies* may resemble the *first Imaginary Society* much more than the *last*; yet if it should unfortunately prove otherwise, there is the greater necessity for *immediate REFORMATION*.

CCIX. *Whatever defects* may on examination be found in any of the *existing Schemes*, they are all CURABLE; and the Author begs leave to prescribe a remedy that will not fail doing it; *provided the*
E c c 2
cause

cause of them was *no other* than a *want* of a *due knowledge* of the subject; and if the Managers will please to recommend and apply the prescription, they may not only entirely free themselves from *every other* imputation; but may also make their Society as *perfect* and *durable* as the *first Imaginary* one. In a word, it must be by this plain and *honest* method :

Find what every Member hath paid *separately* in worth of *Ready Money* (*a*) ; and let *that annuity only* which the sum has *purchased* for the *Age* be paid to *each Annuitant*. Then these three *Ages* (201.) will not receive an *equal* annuity of £12.103, but each will receive his *own* annuity *separately*, and of the *just* value for his *Age* and *Money* paid.

Thus Age 20 will receive to himself	17.441
(201.) 40 _____	8.471
60 _____	10.397

and the others, that paid by a *second* Abstract, will receive their *just* due in *these* proportions :

Age

(*a*) If the Author's assistance should be thought serviceable, it may be had by application to the Publisher.

Age 20 separately to himself	46.656
(196.) 40 _____	28.172
60 _____	35.871

by which *fair* JUSTICE is done to *All*, and no one can *reasonably* complain of any thing but *disappointment* in what they were made to expect by *equal dividend*.

Another method has been prescribed by Dr. Price (see *Observations*, &c. page 116.), that every Member “*shall make compensation for what they have hitherto paid too little;*” that is, every one to pay up to the *full value* of 44 *l.* or whatever the annuity may be (a). but as this may not be in *every* Member’s power, it is the *same* thing, if they are each allowed an annuity *equivalent* to the *Age* and *Money* paid.

CCX. If all the Societies could be convinced of their present Errors, and could be induced to make the *same* reformation, there could be no reason (but the *particular* Interest of a few) why they should not *all unite* in one *General Impartial Society* (b), for the benefit of an *extensive* association (127.),
and

(a) Dr. Price, in his Supplement, page 32, published with the Second Edition, adds further,—“*or consented to such deductions from their Annuities as shall be proportioned to the deficiencies in their payments.*”

(b) See the first Article of *Addenda*.

and greatly *reducing* the necessary expences that each Society is now at ; for 'tis supposed, that *one large* Society may be conducted at one quarter of the expences that Eight or Ten *small* Societies are at together.

But such an event will not probably ever come to pass, and the consequence may not be of very great importance ; especially if the Societies have a sufficient number of Members to reduce the *expences* of them to a *light burden*, and *chance* to a near *equality* (127.).

CCXI. It may be proper to take notice of a very prevailing notion ; which is, *that* whether *the Societies* can *pay the promised Annuities* or not, depends on an *accession of new Members* continually to the Society.—This is very true in the POLITIC GOTHAM SOCIETY, whose Annuitants having spent their *own* provision (121.), must depend for subsistence on the accumulation of *new* Members, *whose* Stock will preserve them from *immediately* perishing (a).—But in the first *Imaginary Society*, the continual accession of *new* Members doth nothing more than lengthen the *existence* of the Society, without increasing, or in the least affecting, the annuity. For suppose, as in the Table (117.),

204

(a) See Note page 108.

204 Members Aged *Fifty* have £4444.828, they might go on receiving 2 *l.* yearly during Life ; and though, at the same time, 294 new Members Aged *Forty* enter the Society, no benefit or increase could derive from it to the annuity of those Aged *Fifty*; for the *Stock* paid by the new Members, whether mixed with or kept separately from the *Stock* of the old Members, would be equally preserved untouched for the use of the owners, who were the new Members, when they should become Aged *Fifty*, to receive 2 *l.* annuity, as the old Members did before, and the survivors continue to do now. All the difference these make, *excepting the advantage of association* (127.), is only, that the *Society* by the accession of these new Members will *exist* 54 $\frac{1}{2}$ years ; whereas the *Society* having no Member younger than *Fifty*, would have *existed* but 44 $\frac{1}{2}$ years, because every Member is supposed to be *dead* before the Age of 95. In a word: the Members in this Society do not *profit* by any one *LIVING*, but by such as are *DEAD* ; and it must be the same in any other such Society.

The *existence* of a Society, like the race of Beings, may be preserved as long as the loss of a part is supplied by a fresh accession of new Members ; but whether that length of existence of the Society is preserved or not, does not much import the present Members :

so that if a Society consist of a number sufficient for the purpose of association (127.), *and to defray expences*, it might, in respect to themselves, as well be closed or shut up as kept open for the admission of new Members. Thus 204 Members (117.) Aged *Fifty*, and all aged above, would receive their annuity the *same*, whether *new* Members *were* admitted or *not*.

CCXII. And when a Society has no accession of Members, and all are become Recipients, the Capital Stock may continually *decrease*, as in the Tables (114, 117, 120.) without the danger of any *deficiency* in the end, as in the *first Imaginary Society*; but when it *decreases* in the POLITIC GOTHAM SOCIETY, as in the Table (121.) ruin approaches with huge strides; and all the hopes of a short existence longer, is in the *accession* of *new* Members, whose small Stock they would almost instantaneously devour (*Page 102. note*).

CCXIII. The Writer has no need to conceal, that he is a Member of one of the Societies for the Benefit of old Age, and so early in the Institution as to be numbered 412. The particular Society he belongs to, and perhaps all the others, know his Name;—it is not announced to the Public, lest it should be deemed Vanity in him to publish it as an Author.

He

He was 42 years old the April before he entered as a Member in the July following. He will become an Annuitant in his 52½ year, opposite to which is the sum £3852.691 in the Table (117.), which being multiplied by .706824, that is 20 half-years reversion, produces £2723.174463314, which being divided by 269 living Aged 42½, the quotient gives £10.123325142 for 2 l. annuity; which is £5.061662571 for the value of 1 l. annuity to be received during Life, when Aged 52½ (141.*).

The value of an *immediate* annuity for the Age 42½, is £6497.892 opposite the Age in Table (117.), divided by the number 269 above the sum, and who are living at that Age (62.), which gives £24.15573 for 2 l. annuity, the *half* of which is £12.07786 for 1 l. during life.

(149.)

From the value of the <i>immediate</i>	}	12.07786
annuity during life —		
Deduct the value of the annuity	}	5.06166
to commence when aged 52½		

The value of 1 l. *immediate* annu- } £7.0162
ity till aged Fifty-two and half }

He was to pay 3 l. 7 s. half-yearly, which is decimally £6.7 yearly, therefore the value of the sums in a *single* payment, *ready-money*, is

$$7.0162 \times 6.7 = \underline{\underline{£47.03854}}$$

He paid Admission-Fine	12.6
The <i>First half-year's</i> payment	3.35
20 ditto worth in <i>present</i> money	47.00854

Total paid in value of *ready* money £62.95834

The annuity this sum will purchase for the Age 42½ ; as,

$$5.06166 : 1 :: 62.95854 : 12.438 \text{ (184.)}$$

The annuity that Age 42½ should receive during Life, when it has attained the Age of 52½, is 12 l. 8 s. 9 d. for £62.95854 *ready* money paid, or for 82 l. 19 s. expected *to be paid*. And notwithstanding the managers *generously* promise to pay 44 l. yet the Author will very *willingly* accept of £12.438, if he survives to become a Recipient of the said Annuity, for he is *now* perfectly convinced that he has purchased no more, nor can receive more without *injuring others*.

CCXIV. And every Member in the Society will, no doubt, be to the full as willing to accept of such an annuity only as is his just due, as soon as he is convinced of the rectitude and necessity of the proposed regulation.

But most especially *all* the *Managers* and *Institutors*, who have been the cause (it is hoped innocently) of involving some thousands

sands in a scheme that will be *ever* stigmatized by the Candid and Upright with such opprobrious epithets as Honesty, from its strong antipathy and natural impulse, gives to villainy.

These Gentlemen, who have *fair* reputations, will be alarmed when they understand how much they will injure it by continuing to publish either Advertisements or Abstracts, full of deceit and falshood, to *impose* upon and *defraud* such as confide in them. If they were the *first* to *mislead*, they will be *more solicitous* to give an *early* example of correcting past errors, and doing JUSTICE to all men, —and will recommend with earnestness the *Reformation* of their plan to the consideration of a *speedy* GENERAL COURT.

THE Author finding himself thus insensibly lead into a particular Address to the particular Society he belongs to, begs leave to take the opportunity to inform the Members of his own thoughts and conduct, respecting the Society from his first connection with it, in

A

L E T T E R

TO HIS

FELLOW MEMBERS.

GENTLEMEN,

THOUGH I have, *for the most part*, been a silent auditor at our General Courts, yet *this publication* may, perhaps, testify that I have not been altogether an inactive Member: but it will probably be thought, that, as a *Fellow* of the SOCIETY, it would have been but a common kindness to have communicated the substance of it *privately* to our Managers.——The truth is, I have not withheld my mite, and should have contributed much more liberally towards the good of the Society, had I met with the *least* encouragement; or rather, *if I had not met with a*
DIS-

DISCOURAGEMENT *to do so*.—A few plain facts will thoroughly explain my meaning.

Having met in the country with a Manuscript Abstract of the terms of Admission into a Society formed for Granting Annuities for the Benefit of Old Age, I wrote a Letter to the *Secretary* that enquired into the particulars of it; to which I received an answer that occasioned the following.

S I R,

June 1768.

I HAVE wrote to a friend to pay the Admission Sum for me on the 21st; but as I have not heard from him, I suppose he is out of town; in which case it cannot be done before the 5th of July, as I understand the Managers meet at no time between.—In the interim, I shall make free to mention a few particulars to you that rather surprize me in yours.—In the first place, your Proposals say, and so does your letter in one place, that persons from 39 to 45 pay £12 12 0 Admission; yet in another place you say, that the first payment of a person of those years will be £15 19 0, which I suppose is a half year's payment required at the same time as the Admission: but then, together, the *Admission* will certainly be £15 19 0 instead of £12 12 0 as one is led to believe from the proposals; for your division of it into *two* articles makes not the least difference to the Subscriber; who, when admitted, must pay £15 19 0, call it *for what you will*.—Again, of the like kind is the Annuity itself, which, the Proposals say, is to be *fifty* pounds per ann. But your letter says, that a Member, when he is entitled to the Annuity, is still to pay a *half* yearly sum of 3*l*. Certainly then his Annuity will *in reality* be no more than £44 (a).—All agreements should be as clearly

(a) The 9th Article of the Deed of Settlement says expressly, in the first part, "The Annuitants are to receive the CLEAR as-
" *nnal*

clearly understood as possible ; and as your "*Society has no need of assurances to excite persons to enter into it,*" if your Proposals were to promise but 44 *l.* Annuity, and require £15 19 0 Admission, it would be a fair, ingenious proceeding, and much more to the credit of any Society than PREVARICATIONS.—No doubt there are thousands, of whom I am one, that would, from motives of prudence, be glad to secure a maintenance, on easy terms, for the latter part of life : and the chance on lives seems to be as fit a scheme to answer that purpose as any ; and I conceive two ways of carrying it into execution.

The first is, supposing some Gentlemen of property to establish a Fund, and, having made the necessary calculations, find the terms they may propose to the Public, and they oblige themselves to abide by their Proposals ; and whether there be profit or loss, contrary to their estimate, themselves *only* are to suffer, or be benefited by it ; for which reason, their care would be to allow themselves in their calculations the probable chance of sufficient gain ; and every individual will examine whether the terms are advantageous to himself before he subscribes.—By this Scheme a *certain* Annuity may be proposed which the Subscribers in due time may be certain of receiving ; provided the security is good, without hope of receiving *more*, or fear of receiving *less*.—Another way may be for the Subscribers themselves to stand the chance of their own Stock, when there will be a necessity for appointing Trustees, Managers, Secretaries, &c. who, as they must be persons of character, should have genteel allowances from the Common Stock ; but have their power *granted* and *limited* by the GENERAL MEETINGS of the Proprietors, and to whom they should be accountable.—This Scheme is more difficult to be put in execution, for want of persons to undertake it ; and therefore, those who do undertake to bring it to success, should have some degree of preference.—On this plan also, a proposal of a certain

^a *and sum of 50 *l.**"—And in the latter part, "The Managers
^b *may deduct and retain in their hands out of the said Annuities*
^c *respectively, the sum of six pounds yearly,*"

Annuity may be made from a proper calculation; but there will be a *deficiency* or *overplus*, more or less to be accounted to, and settled by, the GENERAL MEETING, and the Annuity of consequence must often vary.—If your plan be either of these, and I suppose it something like the last, it certainly is a very fair one, to which no one desirous of entering into a Scheme of that nature could make the least reasonable objection, provided it was plainly understood by him at the time of his subscribing. I therefore intend becoming a Member thereof on the fifth of July, if I can get the business transacted for me (*notwithstanding you would not favour me with the name of any one TRUSTEE*, nor have in a year and half issued any public printed Proposals, by which one is rather in the dark as to SECURITY, which is a main essential in the Scheme).—You intimate that the Admission may be raised on the 5th of July (a), I hope to ~~all~~ after that day, and not to those who shall subscribe on that day, and signify ~~not~~ their intention of so doing.—Though you say “*the Society is in no want of money, nor will be for many years*,” yet no doubt you are well aware that the demands will become heavy after the expiration of ~~ten~~ years; whether the FUND will afford to answer them, is not at present in my power to satisfy myself of, for want of yearly Bills of Mortality from which I might calculate a probability: but in confidence that the TRUST will be managed with *Fidelity* and *Judgement*, I shall, if I become a Member, be content with what Annuity shall appear to the majority of the then Proprietors to be *EQUITABLE*.—Supposing your Society to be established on the above plan, I hope one point is *unalterably* settled; that is to say, that the Proprietors shall *certainly* receive their Annuity at the expiration of *ten* years, if they are *fifty* or upwards: otherwise, if all matters are determinable by a General Meeting; the younger part, for in-

(a) The Reader will please to allow for this being written before the Author had seen any *printed* Abstract; and when he was *totally* ignorant of the subject.—On maturer consideration, this hope was not of the *honestest* kind.—See the *Friendly Society*, *note* (a)

stance, those of thirty, may, on pretence of the insufficiency of the Fund, propose and carry to have the Fund shut up *five, seven, or ten years* longer; whereby the old ones, *notwithstanding they have paid proportionably*, may be excluded sharing any Benefit from it; as very likely most part would be retired to their graves *at those years*, before it was opened. Therefore the Fund ought never to be shut up; but every one entitled should share the Annuities *as far as they will allow*.—I take the liberty to suggest this hint, because it seems to me as if your plan was not yet entirely settled; and, if not thought on before, may be useful.—The favour of a line in answer to any thing that herein may seem to merit it, will much oblige,

S I R,

Your humble Servant,

.. ..

N. B. *No answer, nor any notice whatever taken of this afterwards.*

I was admitted a Member at the time intended; and, being then very busily employed other ways, all thoughts of examining into the sufficiency of the Fund to answer the Proposals died away, till they were revived as related in the following, which is the PURPORT of the *beginning* of a letter left for the perusal of our Managers, about *Michaelmas 1770*.

GENTLEMEN,

I HAD not an opportunity of being present at any of the General Meetings before July last (a); till when I had not the least doubt but that sufficient calculations had been made by the Institutors of the Society, so as to

ascertain *nearly* the value of the Annuities on the proposed terms; or at least that the Succeeding Managers would make, or cause to be made, such Calculations.—I did not imagine that Gentlemen would choose to go on *so far*, groping *blindfold* and stumbling in the *dark*, when it was as *easy*, to walk *safely* in *broad day-light*.—For my part I had such *implicit* faith, (which the unsuspecting are apt to have in professions, till there is *cause* to believe them to be doubtful) that I never attempted to satisfy myself of the TRUTH till since the said Meeting, though your Proposals always appeared (glaringly to me) to be greatly *too advantageous* for the Subscribers.—Indeed I thought this might have been meant, at the beginning, to encourage the *first* Subscribers to the undertaking, and as a compensation for the risk they ran, and the trouble they were at, in the ESTABLISHMENT.—But when I found on the above day that such a structure had been carrying on *three years and a half*, without the least inspection whether the Foundation was a Rock or Sand, who could help being amazed, that what should have been *first* done, was *as yet* to do?—Who would not be apprehensive of the great *disappointment* to All the Subscribers?—Could any one have possibly expected to hear on *that* day the *first* Motion for a Calculation to be made? or have thought, that when the motion was carried, and the calculation recommended to be made with the *utmost expedition*, that at the *next* General Court held afterward, no other mention was made of the so necessary work, but a hint, that a Subscriber would be *five years receiving back his own money*?—As to conjectures on the reason of this conduct, I will not make any; but rather (as every Member ought) endeavour to lend a helping hand to remedy a defect, which All seem now sensible of.

The Letter was too long for the *whole* to be inserted here.—Its principal contents were these four Questions and Answers:

Ques. I. What doth a Member of *Forty* contribute, in the whole, to the Society in *present money*?

• F f f

Answer.

Answer.

The Admission-Fine	—	105	0	0
First half-year's payment	—	3	7	0
Twenty ditto in Reversion, worth in } present money	—	46	14	10 ⁵⁴ / ₁₀₀
		£155 1 10 ⁵⁴ / ₁₀₀		

Quæst. II. If 313 Persons (Forty years old) pay £164 9 2½ (a) each, the total being to accumulate Compound Interest, How long will the Fund pay each surviving Member 44 l. per annum, the first payment to be made at the expiration of Eleven years?

Answer. Twelve Payments and near a half.

Quæst. III. What sum will pay 44 l. per annum, during life, to each member Aged 40?

Ans. £226 19 5½ (b) present money for each to pay.

Quæst. IV. What sum will pay the same Annuity to those Aged 45?

Ans. £199 15 1½

These last Two Answers were recommended to the consideration of that particular Gentleman, who, when I spoke for each particular Age to be calculated, replied, that the Ages of Forty and Forty-five were as the same Age, or at least generally reputed so in all Calculations.

(a) The sum by Reversion only (47,) without allowing for Chance of Mortality (68.).

(b) These Calculations were by the London Bills from 1728 to 1768, both inclusive, by YEARLY Decrements, Interest and Payments.

Ans

An explanation, how the Answer to the *first* Question was obtained, was at that time given, in much the same manner by *yearly*, as in the Table (71.) by *half-yearly* interest and payments ;—*and it was added :*

That the proof of the Answers to the other quæries were far plainer and easier to be comprehended, and would exhibit the harmony there was in the due proportion of every Age.—But as it required the transcribing a multitude of figures, I was sorry that I could not afford the leisure to send it, *till I knew how that would be received.*—(*It was also added :*) —You will observe, that in the Reversion of half-yearly payments, you calculate *too high*, and in that of Admission *too low*.—It seems to me more rational and just, that a Calculation should be made and laid down in such a plain manner, for the information of those intending to subscribe, that (supposing them to understand common Arithmetic) they might be convinced they were going to exchange one value for another, in the same manner as giving a guinea for one-and-twenty shillings ; and it might be of use to the present subscribers, to demonstrate the IMPOSSIBILITY of their receiving the PROPOSED Annuity *many years* without *entirely breaking* the Society, and thereby *injuring their successors* in the Annuities ; and that therefore they *must* content themselves with such a *proportion*, as a *fair* Calculation shall prove to be EQUITABLE.

We often hear it asserted, that our Fund will be fully SUFFICIENT ; and all proposals made to provide some security for succeeding Annuitants end in procrastination.—But we never hear from our Managers any intimation that they have received the least information *to the contrary*, though doubtless they have had some beside the Letter just quoted ;

nor could any one suppose by their present proceedings, that the said Letter was dated *eighteen* months ago.—I believe it is not easy to find any *honest* reason why they should wish to suppress any *fair* Enquiry into the present state of our Society; but I may be mistaken; yet whether they do discourage such enquiry or not, and what encouragement they have given the Author let the following fact determine;

Having leisure, and being urged to study by what passed at the General Court in *July*, 1770, I procured the 4to, Volume mentioned before (50.), and having finished and delivered the Letter just mentioned, I thought *I had done* with the subject;—but at that time several General Meetings of the Society were held on various business, at each of which something was agitated that still awakened an attention to the same subject; and also meeting with *Mr. Thomas's Banker's Sure Guide*, I was brought acquainted with *Mr. Simpson's* and *Mr. Demouire's Annuities*, by whose Tables I was assured I had pursued the right *Method*; which was an encouragement much wanted; for however plain it appeared to me, I was not entirely without some diffidence, till superior judgments gave me more confidence.—I then began to prepare, though in a leisurely way, a more complete Calculation for the Information of the Managers, and the use of our Society; and, to reduce

8b-

objections as much as possible, I formed it as nearly to the State of the Society as I could, *viz.* by *half-yearly* interest and payments, nor did I spare the trouble of examining the subject by different Tables of Mortality. In the interim *Dr. Price's admirable Observations, &c.* were published, which I supposed would render useless any thing that I could offer on the subject that had now caught my whole attention, but for a while was laid aside, till I found that not only *our* Society, but likewise *all* the others, still persisted in the course, which Reason could not *convert*, nor Censure *shame* them from; and, to cover their audacity with some shew of modesty, they still affect to treat his Work as written in an *unknown* language, which they *acknowledge not to understand*. As the method I had adopted *seemed to me* to be more on a level with their understandings, I resumed my Calculation; and with an intention of adapting it as correctly as possible to the State of *our* Society, I wrote some time in *July* last to our Secretary, desiring to know

WHAT Number of Members the Society consisted of, at each of the several times the Admission Money had been raised? And also, What Capital Stock the Society was possessed of?

The Answer was,

Midsummer 1770	Capital £28450 — Members 1162
Midsummer 1771	Capital £41400 — Members 1332

About

About a Month afterwards I wrote the following to him :

SIR,

I RECEIVED, in due time, your Answers to the Questions relating to the *Capital Stock* ;—but my desire was to know also the *Number* of Members that the Society consisted of at each time the Admission-fine has been raised: For Example; I know the Admission-fine for the Age of *Forty* has been raised from *nothing* to the following sums :

		Admission.	No. of Memb.
By the First Abstract	_____	£ 0 0 0	
Second	_____	12 12 0	
Third	_____	21 0 0	
Fourth	_____	31 10 0	
Fifth	_____	52 10 0	
Sixth	_____	73 10 0	
Seventh	_____	105 0 0	

And there may have been Abstracts published that I know nothing of; but the *number* of Members at the time *each* of them took place are the Answers desired.—I trouble you again, because my questions were either misunderstood, or it was intended to evade giving *direct* answers to them: if the first, I hope the Gentlemen will now understand them rightly, and oblige me with the numbers desired; but if the last, I mean to be no further troublesome than to beg a line of you to acquaint me so. From,

SIR, Your humble Servant.

The Answer.

SIR,

LAST night I laid your Letter before the Committee of Managers; they don't think it any way necessary to give any Answer than the former to you.

I am, SIR,

Your most humble Servant,

Aug. 22. 1771.

***, Secretary.

Exr

Exclusive of the Calculation I had obtruded on them, I conceived that, as a Member, I had a RIGHT to ask, and *to be informed* of, the State of the Society; but it seems by this that our Managers do not admit the privilege.—I did not choose to dispute the point with *them*, because, *from that time*, I resolved to appeal THUS PUBLICLY to you, my Fellow-Members, particularly to those who have been remiss in their attendance at the General Meetings, who may be ignorant how easily *any* motion passes among persons who are neighbours and acquainted, who are the majority that attend, and among whom our affairs may have been previously settled. To what else can be imputed the present power of the Managers, who nominate the *List* out of which the succeeding ones are to be chosen, allowing no Member to nominate more than three himself; who say it has been *customary* (*a*), and therefore nominate the TRUSTEES (*b*)? And by a motion made and carried by one who *had been*, and is now *re-chosen*, a manager, they are to have the nomination of the INSPECTORS; whence, *as it seems to me*, the FORM of balloting may be dispensed with, and the Managers ap-

(*a*) Five years time are ordinarily sufficient for PRESCRIPTION; whereas for CUSTOM, there are required a Hundred.

(*b*) The Fourteenth Article of the Deed of Settlement says *expressly*, that the MEMBERS shall choose the Trustees.

point each other at a *Word*, since it cannot possibly make any other difference than that ballotting *amuses* the Members with a *mock* appearance of their having some CHOICE.

My idea of conducting a Society, such as ours, will be fully explained in the first Article of the ADDENDA: where it will be recommended as a prudent precaution; not only for the security of the Stock, but likewise to prevent suspicions, to endeavour to check all engrossments of power in those we elect our *servants*, and who no sooner become so than they immediately aspire to be our MASTERS.—Reflect then, Gentlemen, that whenever it shall so happen, it must be by *your own* remissness and permission, in not attending at the Meetings, and in not opposing every attempt to render the State of the Society a SECRET; and in not *censuring*, instead of *returning Thanks* for, the conduct of our Managers, when it becomes MYSTERIOUS.

Much of the kind might be added; but lest what has been already said may be too officiously concluded as meant to compare our Society with the POLITIC GOTHAM; I humbly beg leave, out of respect due to the characters of Gentlemen of reputation, to declare for myself, that not only such an allusion is not intended, but that, *notwithstanding appearances*, I have yet some *Hope* that they are ALL HONEST MEN; and for some
palliation

palliation of their criminal proceedings, I further beg leave to add, that though the Calculations of the Society will be proved to the Public to be very *erroneous*, yet it is not known how expensive they might have been to the FIRST PROJECTORS of the Scheme: and the Meetings, Secretary, &c. must have been a *heavy* burden while the Members were but *few* (a); and, in case the Scheme had not succeeded, so as to collect a great number into the Society, the charges had been not only very great, but merely thrown away, and would have been a proportionable loss to each individual who first engaged in it.

For these reasons it was very natural for Gentlemen to provide some *compensation* for the chance they ran of *Loss*. This might induce them to pay at a low rate themselves

(a) That future Members ought to contribute, *in some degree*, more than the present Members to the Stock, may be proved reasonable from this consideration: The expences of a Secretary, House-room to assemble to conduct business in, Fire, Candle, &c. may be the same when the Society consisted of *Fifty*, as when it should consist of *One Thousand*; and suppose the sum for *annual* charges to be 200*l.* then, when there were but *Fifty* Members, the expence was 4*l.* each; but when the Society should consist of *One Thousand* Members, they would contribute but *Four Shillings* each towards the same expences.—It could be scarcely deemed *injustice* then, if the latter Members were charged something more to supply the deficiencies that would be in the Fund, occasioned by these expences: which were *considerable* while the Members were but *few*, but which *few* were the CAUSE of the MANY,

* * F f f

(rather

(rather too *low* indeed), especially as they are not yet convinced that the value of the annuity, on the promised conditions, is any thing near of so great value as it has been herein found to be; and likewise they might flatter themselves with the HOPE, and perhaps confide too much in it, that the Society would increase in such proportion as in Case 202; so that the *many*, who paid much, would reduce the deficiencies of the *few*, who paid little, to nearly nothing. —Nor is it known how the Plan might have succeeded if the Society had had an *exclusive privilege*; but *without* it the consequence has been, that before they could raise the Premiums and obtain Members to a *sufficient* degree, OTHERS caught the hint, and instead of paying a high Premium to be admitted into this Society, they instituted *another* themselves, at *less* value than at that Time this was advanced to; and still upon the same principle, which is for *future* Members to supply the deficiencies of the *first*. —

To one Society succeeded another, till at length the Town swarms with them; and all established with the same views, and upon the avowed principle, that as the number of Members increases, so shall the payments of the last Member: by which they have not only defeated the design of this Society, but have neither hope or possibility of compleating the same themselves, being
equally

equally as incapable of attaining to the aforementioned case (202.); therefore, to do JUSTICE, they must every one alter their schemes to the other plan (209.), by which had one of them proceeded at first, it would have escaped both disgrace and disappointment, and also would have precluded the pretence for the establishment of any future Society of the kind.

Beside thus much in vindication of the Institutors of our Society, it may be added, that they have, and, it may be, still do confide in some Calculator, who has most piteously imposed upon them; such an One (*if not the same*) as him who declared at one of our latter Meetings, " That finding the
 " Members were much alarmed on account
 " of Dr. PRICE's Book, he could assure
 " them that Dr. PRICE had the most confused Ideas, and was perfectly unintelligible, for that he, *himself*, could not comprehend one word he had written on the
 " subject; but he could *again* assure them,
 " upon his *own knowledge*, that they need
 " not be under the least apprehension; for,
 " by Calculations which himself had very
 " carefully made (a), he was *certain* they

(a) Which said Calculations, 'tis hoped, the Gentleman will publish for the Satisfaction of the Members, and Information of the Public.

" were

" were fully *able to pay* the Annuity which
 " was promised (b)."

I have taken the liberty to say thus much in behalf of the Gentlemen concerned (to which no doubt, *they* can add much more) to shew, that what has been said before did not proceed from resentment for any supposed ill treatment, for I would as willingly (indeed *much* more willingly) exculpate as censure.—— My principal meaning is to excite, to rouse them to a reflection on the critical situation of their reputations, which cannot but suffer, if through ignorance and obstinacy they have *established*, and still *per-severe* in, a Scheme that must prove ruinous to so many.—— If they will be Managers, why will they not apply themselves to attain a knowledge of the nature of the business which they undertake ? or, if they have not leisure for that, why will not they listen to the wholesome admonitions repeatedly given by Dr. PRICE, the Reviewers, and even the Daily Papers ?—— But I beg pardon.—— They are the best judges of, and apologizers for, their own conduct.—— I shall therefore conclude with the most earnest wishes, that there may be found among us, at our

(b) It may be expected, that the Author, being present, and knowing the falsity of part of the assertion, made some reply. The Truth is that, beside being but poorly qualified for public speaking, he was at that instant struck dumb with astonishment.

next Meeting, a MAJORITY *at least*, of Men who can perceive, acknowledge, and blush, for our past errors; and who will endeavour to be the FIRST to bespeak the indulgence of the Public, to admit what has been offered as some Apology for presuming to conduct an important affair without Abilities, by reforming our Plan immediately according to such Calculations as perfectly agree with the methods laid down, and recommended, by the able Mathematicians so often mentioned in this Essay.—And with this hope

I am, GENTLEMEN,

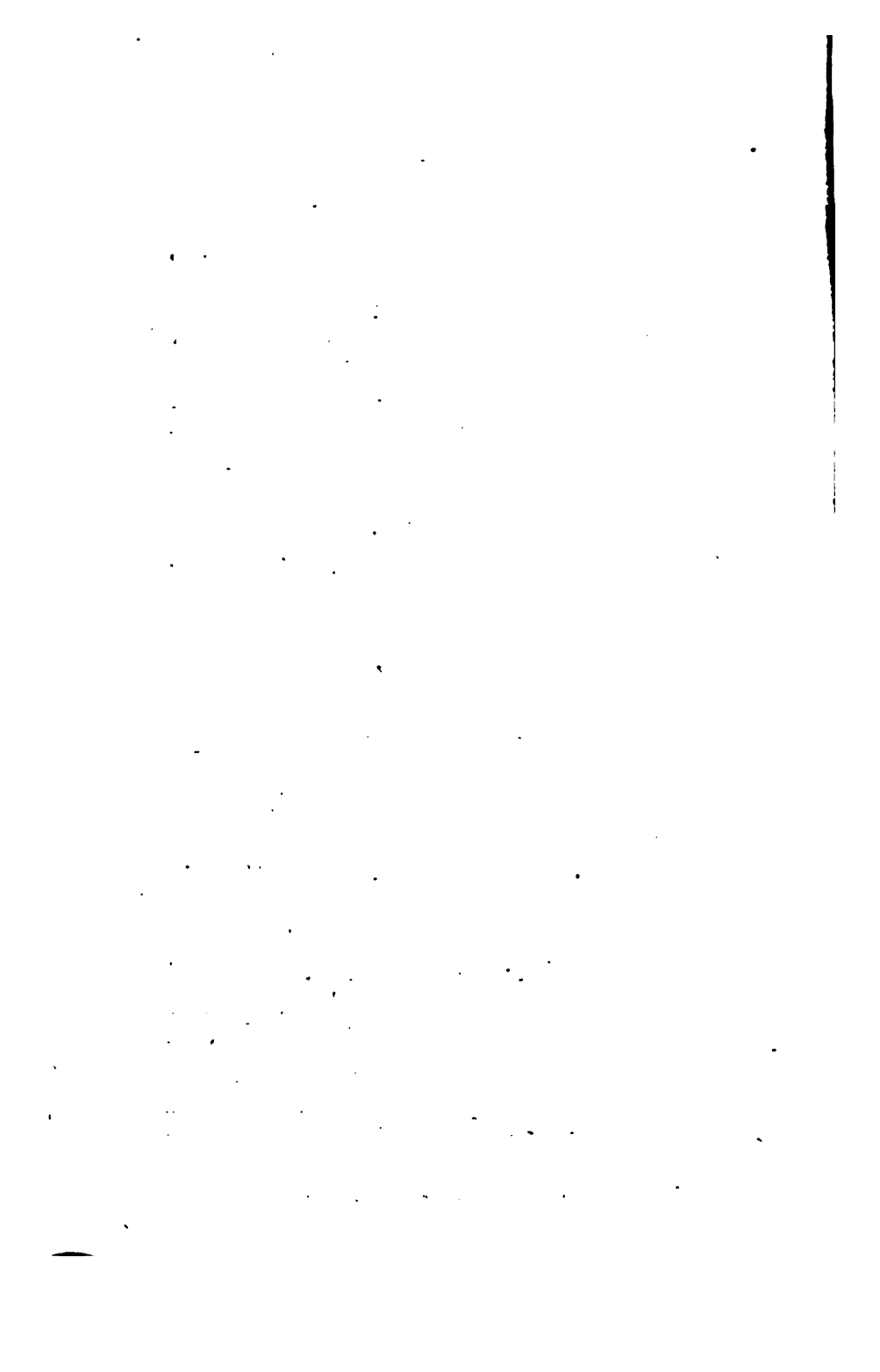
Your most obedient,

Humble Servant,

*Portland Row,
March 22, 1772.*

W. D.

P. S. Consider well the Case 205, and compare it with the Admission-Fines paid, as in Page 240, and when you shall understand our Inability to pay HALF of what is *promised*, 'tis impossible to suffer our present Advertisements and Abstracts to be published without tacitly acknowledging *ourselves* to be a Set of PUBLIC CHEATS, &c.



THE LAUDABLE SOCIETY.

(182.)

MEMBERS, when 50 years of age, are entitled to 50*£*. per annum during life—*when entitled to the annuity, to pay the half yearly sum of 3*£*.*—Such as become members at above forty years of age, must continue members ten years before they are entitled to the annuity.—The terms of admission are, for the ages of twenty, forty, and sixty (181.)

	Age.	Admission	Half yearly.	Half yearly without Admission.
To pay	20	11 11 0	1 2 0	1 15 0
	40	105 0 0	3 7 0	11 0 0
	60	0 0 0	3 7 0	3 7 0

What doth each member, of the said ages, pay in value of *present* money? (183.)

THE LAUDABLE SOCIETY.

The age of twenty.

Admission money	—— (27.) ——	11.55
The first half year's payment	——	1.1
60 ditto worth in <i>present</i> money	——	31.49
		£. 44.14

The age of forty.

Admission money	——	105.
First half year's payment	——	3.35
20 ditto worth in <i>present</i> money	}	47.194 +
(157.)		
		£. 155.544

The age of sixty.

Admission money	——	0.
First half year's payment	——	3.35
20 ditto worth in <i>present</i> money	——	42.85
		£. 46.2

What annuity ought to be *received* according to the value of the *present* money *paid*; and the terms of the society? (184.)

Answers.

Age of 20, as 1.6987	: 1 ::	44.14	: 25.985
40, as 5.343	: 1 ::	155.544	: 29.112
60, as 2.6529	: 1 ::	46.2	: 17.415
			£. 72.512
			In

THE LAUDABLE SOCIETY.

In what proportion doth each age gain or lose by receiving *equal* annuities, instead of *that* which each age *purchases*? (185.)

Answers.

Divide the total sum of the three annuities by 3.

$$72.512 \div 3 = 24.171$$

Age 20 purchased an annuity of — 25.985
Receives by *equal* dividend only — 24.171

Annual loss £. 1.814

Age 40 purchased an annuity of 29.112
Receives by an *equal* dividend only 24.171

Annual loss £. 4.941

Age 60 purchased an annuity of — 17.415
Gains, the annual loss of age 20, — 1.814
Gains, the annual loss of age 40, — 4.941

Receives by *equal* dividend — £. 24.17

What do members pay, when admitted *without* admission money? (186.)

3 G 2

Age

THE LAUDABLE SOCIETY.

The age of twenty.

First half year's payment ——— 1.75
 60 ditto worth in *present* money ——— 50.1
£. 51.85

The age of forty.

First half year's payment ——— 11.
 20 ditto worth in *present* money — 154.964
£. 165.964

The age of sixty.

First half year's payment ——— 3.35
 20 ditto worth in *present* money ——— 42.85
Total as before £. 46.2

	Age 20	Age 40.	Age 60.
Paid <i>without</i> admission money	51.85	165.964	46.2
Paid <i>with</i> admission money	44.14	155.541	46.2
Difference	7.71	10.41	0.0

The annuity that each age purchases, *without* paying admission money, is

Age 20, as 1.6987 : 1 : : 51.85 : 30.523
 40, as 5.343 : 1 : : 165.964 : 31.062
 60, as 2.6529 : 1 : : 46.2 : 17.414

Divide the total by ——— 3)78.999

The *equal ann'* for each member £. 26.333

THE LAUDABLE SOCIETY.

Age 20 purchases annuity ——— 30.523
 Receives by an *equal* dividend ——— 26.333

Annual loss £. 4.19

Age 40 purchases annuity ——— 31.062
 Receives by an *equal* dividend , ——— 26.333

Annual loss £. 4.729

Age 60 purchases annuity ——— 17.414
 Gains, the loss of age 20, ——— 4.19
 Gains, the loss of age 40, ——— 4.729

Receives by an *equal* dividend — £. 26.333

Or, the ages 20 and 40 jointly, contribute annually £. 8.919 for the deficiency of age *sixty*.

The annuity by equal dividend and }
without admission money, is } 26.333

To which add the same *with* admission 24.171

Divide the total sum by ——— 2) 50.504

The medium of the 2 annuities is £. 25.252

What annuity can the society afford to pay? (189.)

Supposing the number of members to be

THE LAUDABLE SOCIETY.

be *equal* at *all* ages, and that *as many* have been admitted *without* paying admission money, as those who *have paid* admission money: then the society might pay to each member, at a medium, an annuity of £. 25.252, (179.) *provided ALL have paid by the present abstract.*

In what *proportion*, to what those who *are* members, or, to the annuity the society can afford to pay, must a person pay *now* to *become* a member? (190.)

It has been answered, to the question, What annuity doth each age purchase? and also, What annuity can *most probably* be paid? by comparing which, the answer to this question is known, on supposition that every member had paid by the *last* published abstract.

But is a fact, well known to the society, that *all* have *not* paid *alike*, but in very different proportions: the *half yearly payments* are believed to have *been* constantly the same; but most, if not all, the *admission money* has been

By

THE LAUDABLE SOCIETY.

	Age 20.	Age 40.
By <i>wisdom</i> , or the First, Abstract	0 0 0	0 0 0
Second ditto	2 2 0	12 12 0
Third ditto	2 2 0	21 0 0
Fourth ditto	2 2 0	31 10 0
Fifth ditto	2 2 0	52 10 0
Sixth ditto	2 2 0	73 10 0
Seventh ditto	11 11 0	105 0 0
Divide the totals by 7)	22 1 0	296 2 0
The medium.	3 3 0	42 6 0

The admission money has been raised from time to time, as the society found it necessary, without regard to any *determinate* number of members admitted by each, as in some societies established *since*; so that it is uncertain to the author, how many have been admitted by each abstract: there may have been 400 by the *two first* abstracts, and but 200 by the *two last*; or, on the contrary, perhaps 200 by the *two first*, and 400 by the *two last*; which would make a *proportionable difference*; but in case the number is not *certainly* known, and there is no circumstance to incline to one side more than the other, the most *impartial* computation is, to suppose the number admitted by *each* abstract to be *equal*; which is done here, and will be hereafter in every *similar* case (199*.)

The

THE LAUDABLE SOCIETY.

The seven abstracts have therefore been added together, and the total divided by seven to find the *medium*; by which it appears, that the whole society, taken *collectively*, have paid each, (and persons to be admitted by the present abstract *with* admission fine, are to pay each) as follows.

	Age 20.			Age 40.		
Persons to pay admission	11	11	0	105	0	0
Members have paid ditto	3	3	0	42	6	0
Difference	8	8	0	52	14	0

The age of 40 gives *now* above 50 guineas *more* than the society have paid, one member taken with another. (205.)

N. B. *The age of 60 pays no admission fine, not even now.*

Again, the *total* of the *present* money that has been paid by each member *collectively* is

The age 20, admission	_____	3.15.
First half year's payment	_____	1.1
60 ditto in reversion, worth in } <i>present money</i>	_____	31.49
		<u>£. 35.74</u>

The

THE LAUDABLE SOCIETY.

The age 40 admiffion	————	42.3
First half year's payment	————	3.35
20 ditto in reversion, worth in } present money		47.194
		£.92.844

The annuities that have been purchased *collectively*.

Age 20, as 1.6987 : 1 :: 35 74	: 21.04
40, as 5.343 : 1 :: 92.844	: 17.376
60, as <i>before</i> —————	17.415

Divide by 3) 55.831

Equal dividend £. 18.61

It was found, by computing by the *last* published abstract *only*, that the age *forty* purchased the *greatest* annuity, and consequently *lost the most* by an *equal* dividend; but, taking *all* together, it is found the age *forty* has bought the *least* annuity, and the loss and gain of each age is

Age 20 purchased annuity	————	21.04
Receives by equal dividend	————	18.61
		Annual loss 2.43

THE LAUDABLE SOCIETY.

Age 40 received by equal dividend	18.61
Purchased	<u>17.376</u>
Gained, part of the loss of 20.	<u>1.234</u>
Age 60 received by equal dividend	18.61
Purchased	<u>17.415</u>
Gained, part of the loss of 20.	<u>1.205</u>

From a *general* computation the society may afford to pay an *equal* annuity of £.18.61 (179.)—A *particular* calculation will not find it *exactly* the same; but it is as *probable* to be *less*, as it is to be *more* (121.)

By the *last published* abstract, members should receive £.24.171 *equal* annuity: but taking *all* members *collectively*, they can receive only £.18.61, excepting the purchasers by *without* admission fine; who, having bought *dear*, would *increase* the general stock (205.)

THE
AMICABLE SOCIETY
OF
ANNUITANTS,

(Established in Westminster, Christmas, 1769.)

(182.)

EVERY annuitant will be entitled to 26*£*. per annum during life, *without any deduction whatever*, when 50 years old, if payments are made *ten* years, or more, according to the terms annexed. But the *annuity may be MORE*, as the INCREASE of the fund of the society may admit. The terms annexed, for the ages *twenty, forty, and sixty*, are

Age.	Admission money.	To pay quarterly.	To pay quarterly without admission.
20	1 0 0	0 6 9	0 8 0
40	31 10 0	1 10 0	3 5 0
60	1 1 0	0 18 0	0 19 0

THE AMICABLE SOCIETY.

Each person on admiffion pays 5s. for register money, &c.

What doth each member, of the said ages, pay in value of present money? 183.)

Let *two quarterly* payments be considered as equal to *one half yearly* payment; although, in fact, they are not *exactly* so, any more than two half yearly payments are equal to one yearly payment (30. 42.) and let the same consideration be with any other society that pays quarterly.

The age of twenty.

Admiffion money	_____	1	0	0
First quarter's payment and re- gister money	} _____	11	9	
60 half yearly pay- ments of, each,	} — 0 13 6 — 19 6 5 $\frac{1}{4}$			

Total in value of *present* money £. 20 18 2 $\frac{1}{4}$

The age of forty.

Admiffion money	_____	31	10	0
First quarter's payment and re- gister money	} _____	1	15	0
20 half yearly payments each	} _____	42	5	3
3£. (157.)				

Total in value of *present* money £. 75 10 3

The

THE AMICABLE SOCIETY.

The age of sixty.

Admission money	_____	1 1 0
First quarter's payment and re- gifter money	} _____	1 3 0
20 half yearly payments each	1 16 0 — 23 0 6	
Total in value of <i>present</i> money	<u>£. 25 4 6</u>	

Which sums, when reduced to decimals,
(28.) are,

The age of 20	_____	20.911458
40	_____	75.5125
60	_____	25.225

What annuities ought to be received
according to the value of the *present* money
paid, and the terms of the society? (184.)

Answers.

Age of 20, as	1.6987:1 :: 20.911458:12.31
40, as	5.343 :1 :: 75.5125 :14.133
60, as	2.6529:1 :: 25.225 : 9.508

In what proportion doth each age gain
or lose by receiving annuities *equally alike*,
instead of *that* which each age has pur-
chased?

Answer

THE AMICABLE SOCIETY.

Answer (185.)

The age of 20 purchased an annuity of	12.31	
40 ditto	14.133	
60 ditto	9.508	

Divide the total of the 3 annuities by 3) 35.951

An *equal* annuity, for each, is £. 11.984

The age of 20 purchased	12.31	
Receives by <i>equal</i> dividend	11.984	
		<u>Loses £. .326</u>

The age of 40 purchased	14.133	
Receives	11.984	
		<u>Loses £. 2.149</u>

The age of 60 purchased	9.508	
Gains from 20	.326	
Gains from 40	2.149	
		<u>Receives £. 11.983</u>

Or, the ages of 20 and 40 *jointly*, but very disproportionably, pay 2.475 annually for the deficiency of the age of 60.

The

THE AMICABLE SOCIETY.

The same ages considered as paying (186.)
without admission money.

	Age of 20	Age of 40	Age of 60
First payment and register	0.65	3.5	1.2
Value of the quarterly payments }	22.9	91.569	24.303
Total in present money	23.55	95.069	25.503
Paid <i>with</i> admission	20.912	75.5125	25.225
<i>Without</i> admission pays } more than <i>with</i>	2.638	19.5565	.278

The annuity that each age should receive in proportion to ready money paid, *without* admission money, is

The age of 20, as 1.6987:1 :: 23.55 : 13.864
 40, as 5.343 : 1 :: 95.069: 17.793
 60, as 2.6529:1 :: 25.503: 9.613

Divide the total of the 3 annuities by 3) 41.27

The *equal* annuity for *each* is — £. 13.7568

The age of 60 purchased an annuity of 9.613

Gains, the loss of the age of 20, *annually* .108

Gains, the loss of the age of 40, *annually* 4.037

Receives £. 13.758

Or, the ages of 20 and 40, in the said proportion, pay *annually* for the deficiency of sixty £. 4 2 10 $\frac{1}{4}$.

THE AMICABLE SOCIETY.

What annuity can the society afford to pay? (189.)

It has been observed that it is necessary to know the *number* and *ages* of the members to answer this question *exactly*: but if the *numbers* of all ages are *equal*, and *all* have paid according to the abstract under consideration, then, to the equal annuity *with* admission ———— 11.984
Add the same *without* admission money 13.7566

Divide the two annuities by — 2)25.7406

And the *mean* is for one annuity £.12.8703

Every member may receive an annuity of £.12.8703, provided that the number of those who *paid*, and of those who did *not* pay admission money be *also equal*. (121.)

If the numbers are *not equal*, the difference will be as the numbers *really are*; (199*.) but the *utmost* that is *possible* for the society to pay, is £.17.793 (179.) for that is supposing *every* member to have purchased an *equal* annuity with the age of *forty*, *without* admission money, which is the very *biggest* purchase that *any* in the society has made.

But see the answer to the next question.

In what proportion to what those who *are* members have paid, or to the annuity

THE AMICABLE SOCIETY.

nunity the society can afford to pay, must a person pay *now* to *become* a member? (190.)

It is not known to the *author* whether this society has published *more* than *three* different abstracts, or not: by the *three* in his possession the ages of *twenty* and *sixty* have paid the *same* as *now* from the *commencement*; but the age of *forty* has paid

	Admission	Quarterly	Value of the whole.
By the first Abstract	22.05	1.05	51.634
By a second, July 5, 1770	28.8	1.5	71.063
By a third, Dec. 6, 1770	33.25	1.5	75.513
Divide the values of the whole by ——— 3)			198.21
The members aged 40 have paid ———			66.07

Still, *supposing* the numbers entered by each abstract to be *equal*, for the *real* numbers are not publickly known, it then appears, that the members aged 40 have paid, *one with another*, taken *collectively*, no more than £. 66.07, because a third part have paid no more than £. 51.634, and another *third* part but £. 71.063, though the last *third* part have paid £. 75.513 (205.)

THE AMICABLE SOCIETY.

A person aged 40, to *become* a }
 member, pays _____ } 75.513
 But members of the *same* age have }
 paid only _____ } 66.07

Difference £. 9.443

£66.07 will purchase an annuity of 12.366
 only: therefore, *again, supposing equal*
numbers at all ages the purchases have been,
 Annuities

For the age of 20 *with* admission fine 12.31
 without ditto 13.864
 40 *with* admission fine 12.366
 without ditto 17.793
 60 *with* admission fine 9.508
 without ditto 9.613

Divide the total of the *fix* annui- }
 ties by _____ } 6)75.454

The annuity that each member }
 may receive is _____ } £. 12.576

(121.)

But the society, as well as purchaser,
 will please to take notice, that this is
 by a supposition, which, in all proba-
 bility, is *too favourable*; for it allows of
 an *equal number* of purchasers *without* ad-
 mission, as *with* admission fine; but, be-
 cause

THE AMICABLE SOCIETY.

cause the scheme *without* admission money was not published before December 6, 1770, it may very fairly be computed, that not more than one sixth of the society have purchased by it, and therefore the capital stock is poorer in proportion. (205.)

N.B. The *directors* of this society did promise at first 30*l.* annuity, and to deduct 4*l.* annually; but, since, they have found the *proper* name and *value* of the annuity meant, and have the *good sense* and *honesty* to call it *only* 26*l.*

A COPY, verbatim, (*excepting* the Notes)

OF A

L E T T E R

Left with the *Managers* of

THE PROVIDENT SOCIETY.

GENTLEMEN,

I Observed in Tuesday's Gazetteer your advertisement for a *Secretary*. I have, for some months, had the schemes of the *several* societies under consideration, and shall, *perhaps before Christmas*, publish some calculations which will prove in the plainest manner, that *not one* of them will be able to fulfill their engagements; that the *different ages* do not pay in *due proportion* to each other; nor are the terms of paying *admission* fine, and paying *none*, equal: and that a person, to be admitted member of *any one* of them must pay *now* considerably more than any one (*considered collectively*) has done; which, in some of the societies, is actually defrauding under false pretences; though yours partly escapes the censure, by having published what every hundred members have paid, and there-

To the PROVIDENT SOCIETY.

therefore the state of your society is not concealed.

If these facts can be proved in a clear and *undeniable* manner, and at the same time a plan proposed by which all shall pay, in *equal* proportion to their age, *time* before they become annuitants, and to the *annuity* to be received; and also be so *general*, that a person at his own option may receive an *immediate* annuity, or *after any term* of years, or any *sum* between ten and one hundred (a) for life; would not such a plan prevent thoughtful persons from throwing away *part* of their money, and being at mere *uncertainty* in any other society, and consequently *suppress* future addition to them?

This, in all probability, will soon be the case. It is therefore recommended to your consideration, whether it would be greatly *sacrificing* your interest, or whether it would not be for your *credit*, to advertise, that your society will admit no more members; for that, having reason to suspect you have hitherto gone by *mistaken* calculations, you are now *reforming* them in a manner which may render them *useful* and *satisfactory* to the *members*, and to the *public*.

If you should approve of a person in his 46th year of age; who lately lived with

(a) See Addenda.

To the PROVIDENT SOCIETY.

with a nobleman as house steward, and who has now declined all service, to be your secretary; I have leisure, if you think I have abilities, to engage in the employment.—If it be on a *respectable* footing, and if all terms should be agreed on, I engage to *show* and *explain* the afore-mentioned calculations, as far as yet finished, in manuscript, to any one, two, or three gentlemen, that you shall please to depute to call on me, *previous* to my entrance into office.

Mean time I send, for your perusal, the state of *your own* society, as it is *intended* to be *inserted* among others.

The Calculations have been deduced from *four* different tables of mortality; but, for sufficient reasons given, the two *data* which are chosen as a criterion to judge the societies by, are Mr. *Simpson's* Table of the Probability of Life, and $3\frac{1}{2}\%$ per cent per annum *interest* of money, and, for greater accuracy, by *half* yearly payments, and *half* yearly decrements or deaths. From this estimation, the value of 1% *immediate* annuity until 50 years old, and then to *cease*, is

For the age 20	_____	14.31385
40	_____	7.04381
60 for ten years	_____	6.3955

The

To the PROVIDENT SOCIETY.

The *half yearly payments* may be considered as an *annuity* paid by the member to the society; and the value of *all* the payments in *present* money is the sum opposite the age, *multiplied* by the *whole* year's payment. Thus, age 20, by your *last* abstract, is to pay 18s. *half* yearly, which is £. 1 16 0 per *annum*, and £. 1 16 0 is decimally 1.8

$$\text{And } 14.31385 \times 1.8 = 25.764930$$

That is, 14.31385 multiplied by 1.8 is equal to, or produces, £. 25.76493, which, reduced to shillings, &c. is about £. 25 15 3 $\frac{1}{2}$ for the value in *present* money of the 30 times £. 1 16 0, which age 20, if *he* lives, is to pay; and so likewise

$$\begin{array}{l} \text{For the age 40—} 7.04381 \times 8.4 = \text{£. } 59.168 \\ \quad \quad \quad 60—6.3955 \times 5.25 = \text{£. } 33.5764 \end{array}$$

In this manner what *each* age pays in different circumstances is known; and the only doubt with you will be, whether the sum opposite the age is *just* or *not*: to explain that, to satisfaction, takes up very many pages, which cannot be expected to be *transcribed*; and therefore you may, *at present*, rely on my word that it is so.

It will also be found hereafter, that the value of one pound annuity to *commence* when

To the PROVIDENT SOCIETY.

when 50, or when 70 for the age 60, is;
in *present* money, as follows:

For age 20	1.6987
40	5.343
60	2.6529

Therefore *whatever sum* any age pays in *present* money, being *divided* by the value of 1*£.* annuity above inserted, opposite the age, the *quotient* gives the *annuity* which the said sum purchases.

Thus age 20 pays by the *present* abstract, with admission fine, *£.* 27.9649 in value of *present* money.

Therefore as 1.6987 : 1 :: 27.9649 : 16.4624

That is, *by the Rule of Proportion*, as 1.6987 is to one, so is 27.9649 to 16.4624; *meaning*, that if *£.* 1.6987 will purchase 1*£.* annuity, then *£.* 27.9649 will purchase *£.* 16.4624.

The ages *twenty*, *forty*, and *sixty*, are chosen, as being *young*, *middle* aged, and *old*: it is supposed the *intermediate* ages will partake in *proportion* of the *difference* of these three. (181.)

P R O-

T H E PROVIDENT SOCIETY,

(Established, Feb. 21, 1770.)

(182.)

MEmbers of this society, having been so ten years, shall be entitled to an annuity of 25*l*. per annum for life, (*b*) when aged 50 or more; *together with such an INCREASE thereof*, as the fund of the society will admit of, NOT EXCEEDING 50*l*. in any one year.

By an abstract delivered Sept. 10, and dated Feb. 5, 1771, the terms (*c*) are the following: (181.)

	Age.	Admission	Half yearly.	Half yearly without Admission.
To pay	20	1.0 <i>g</i>	.9	1.
	40	31.5	4.2	6.4
	60	0.0	2.625	2.625

Every person on admission to pay 5*s*. *once for all*, as number money.

3 K

What

(*b*) The author believes this Society was *proper* guilty of the insufferable absurdity of *deducting* any part of the annuity.

(*c*) The terms continue the *same* in the abstract delivered December 5, 1771.

To the PROVIDENT SOCIETY.

What doth each member, of the said
ages, pay in value of *present* money? (183.)

Age 20	pays admiffion fine	————	1.05
	Number money	————	.25
	First half year's payment	—	.9
60	ditto worth in <i>present</i>	}	25.7649
	money		
			£. 27.9649

Age 40 pays admission fine	—	31.5
Number money	—	.25
First half year's payment	—	4.2
20 ditto worth in <i>present</i>	}	59.168
money (157.)		
		£. 95.118

Age 60 pays admiffion fine	—	0.
Number money	—	.25
First half year's payment	—	2.625
20 ditto worth in <i>present</i>	}	33.5764
money		
		<u>£. 36.4514</u>

What annuity ought to be *received* ac-
cording to the value of the *present* money
paid, and the terms of the society? (184.)

To the PROVIDENT SOCIETY.

Age 20, as 1.6987 : 1 :: 27.9649 : 16.4624

40, as 5.343 : 1 :: 95.118 : 17.802

60, as 2.6529 : 1 :: 36.4514 : 13.74

The total of the 3 annuities is £. 48.0044

In what proportion doth each age *gain* or *lose* by receiving *equal* dividends, *instead* of that annuity which each age *separately* purchases? (185.)

Divide the total of the three annuities by three.

$$48.0044 \div 3 = 16.0013$$

The age 20 purchases	_____	16.4624
Receives	_____	16.0013
		<u> </u>
Annual loss		£. .4611
		<u> </u>

The age 40 purchases	_____	17.802
Receives	_____	16.0013
		<u> </u>
Annual loss		£. 1.8007
		<u> </u>

The age 60 purchases	_____	13.74
Gains from the age of twenty	—	.4611
Gains from the age of forty	—	1.8007
		<u> </u>
Receives	_____	£. 16.0018
		<u> </u>

To the PROVIDENT SOCIETY.

The same ages considered as admitted
without admission fine. (186.)

	Age of 20	Age of 40	Age of 60
First payment	1.	6.4	2.625
Value of the half yearly ditto.	28.628	90.16	33.576
Number money	.25	.25	.25
Present money	29.878	96.81	36.451
Ditto with admission fine	27.965	95.118	36.451
Difference	1.913	1.692	0.0

The annuity that each age should receive, in proportion to ready money paid, without admission fine is

$$\begin{aligned} \text{Age 20, as } 1.6987 : 1 :: 29.878 : 17.588 \\ \text{40, as } 5.343 : 1 :: 96.81 : 18.12 \\ \text{60, as before } \underline{\hspace{2cm}} \quad 13.74 \end{aligned}$$

Divide the total of the 3 annuities by 3 $\underline{49.448}$

The annuity by *equal* dividend is £. $\underline{16.483}$

$$\begin{aligned} \text{Age 60 purchases annuity } \underline{\hspace{2cm}} \quad 13.74 \\ \text{Gains the loss of age 20 } \underline{\hspace{2cm}} \quad 1.105 \\ \text{Ditto of the age 40 annually } \underline{\hspace{2cm}} \quad 1.637 \end{aligned}$$

Receives by *equal* dividend £. $\underline{16.482}$

Or, the ages of 20 and 40 jointly, contribute annually for the deficiency of age 60 £. $\underline{2.742}$.

What

To the PROVIDENT SOCIETY.

What annuity can the society afford to pay? (189.)

To the equal annuity *with* admission 16.0013

Add the same *without* admission fine 16.482

Divide the total of the 2 annuities by 2 32.4833

The mean for one *equal* annuity is £.16.2416

Members would have purchased, and therefore ought to receive £.16.2416 per annum during life (179.) if an *equal* number have been admitted at *all* ages; and if as many have paid *without* admission fine as *with*; and likewise if *all* the members have paid by *the same abstract* as that published in February, 1771: but a difference in *any one* of these will occasion a *proportionable* difference in the annuities; and therefore this is only a *general* calculation, and cannot be *particular*, unless the exact circumstances were known. (199*.)

By an abstract dated May 24, 1770, it appears the members were *then* 500: and by another dated September 6, 1770, they were *upward* of 1000; and as the *last* of these abstracts gives the *first* alteration (that the author knows of) from the proposals in the *first*, it may be supposed that *all* the 1000 members were admitted according

To the PROVIDENT SOCIETY.
to the terms of the *first* abstract, in which
no number money is mentioned.

Age 20 paid no admiffion fine	
First half yearly payment	0.55
60 ditto worth in <i>present</i> money	} 15.745
	£. 16.295

But age 40 paid admiffion fine according
to the *number* of members, as follows.
(205.)

	£. s. d.		
50 paid	0	0	0
50 d° each	1	1	0
50 —	2	2	0
50 —	3	3	0
50 —	5	5	0
50 —	7	7	0
50 —	9	9	0
50 —	12	12	0
50 —	15	15	0
50 —	18	18	0
	} 100 paid each 0 10 6		
	} 100 paid each 2 12 6		
	} 100 paid each 6 6 0		
	} 100 paid each 11 0 6		
	} 100 paid each 17 6 6		

100 paid each	_____	21	0	0
100 ditto	_____	23	2	0
100 ditto	_____	25	4	0
100 ditto	_____	27	6	0
100 ditto	_____	29	8	0
1000 members, of whom <i>ten</i> have paid		163	16	0
			Ten	

To the PROVIDENT SOCIETY.

Ten persons have paid different sums, the total of which is *decimally* 163.8, which being divided by 10, gives £. 16.38 for the share of *each*.

Then it is *certain* the members of 40 have

each paid admiffion fine	——	16.38
First <i>half</i> year's payment	——	2.625
20 ditto worth in <i>present</i> money	—	<u>36.98</u>
		<u>£. 55.985</u>

200 in the 1000 (*d*) of the age of *fixty* have paid one guinea each admiffion: two guineas *decimally* is .21, which being divided by 10, gives .21 for the share of each *collectively*.

Age 60 paid admiffion fine	——	.21
First half year's payment	—	2.625
20 d°. worth in <i>present</i> money		<u>33.576</u>
		£. 36.411

The annuity that each age should receive is

The

(*d*) Which is, in proportion, as *two* to *ten*.

To the PROVIDENT SOCIETY.

The age of 20, as 1.6987:1::16.295: 9.592

40, as 5.343 :1::55.985: 10.478

60, as 2.6522:1::36.411: 13.725

Divideth the total of the 3 annuities by 3) 33.795

Annuity for each by *equal* dividend £.11.265

The age fixty to receive ————— 11.265

Loses, what age 20 gains, ——— 1.673

Loses, what age 40 gains, ——— 0.787

Purchased an annuity of £. 13.725

Thus it has been found what each member *certainly* paid until their number was 1000. What annuities can be ~~afforded~~ ^{afforded} to be divided, depends on the *number* of annuitants at each particular age; if the greatest number was admitted at 60 years old, they can pay *more*; but if the greatest number was admitted either at 20, or 40 years of age, they must divide *less*: and if the numbers were *equal* at the three different ages, and the intermediate ones, they may pay exactly £. 11.265. (199*.) It also appears, that, though sixty pays *now* in the *least* proportion, it *then* paid in the *greatest*.

The abstract published February, 1771, says, " When the number (of *members*) shall
" be

To the PROVIDENT SOCIETY.

“be 1200 :” This is an implication that the society did not *then* consist of 1200; (e) yet let it be supposed that the 1200 is complete; in that case 200 were admitted on the present terms, and 1000 on the former, which is as five to one.

One annuity on the *present* terms is 16.2416
 Five ditto on the *former* terms is $\left. \begin{array}{l} \text{for-} \\ \text{mer} \end{array} \right\} 11.265 \times 5 = 56.325$

Divide the total of the *five* annuities by $\left. \begin{array}{l} \text{five} \\ \text{annuities} \end{array} \right\} \underline{\hspace{1cm}} \quad 6)72.5666$

The society may pay the medium $\underline{\underline{\pounds. 12.0944}}$

By as just a (*though general*) computation as may be deduced, *without knowing* the number of members at *every* particular age, the society may pay $\pounds. 12$ annuity (121.) but those who know the age of every member when admitted, and what they paid, may inform themselves of the *exact* sum. (179.)

Dr. Price in his Observations, &c. page 115, says, “The deficiencies in the schemes of all the other societies, *except the Provident*, are no less considerable :” but it does not

3 L

ap-

(e) The author has been very lately informed that the society *now* consists of upwards of 1200 members.

To the PROVIDENT SOCIETY.

appear on examination, that the PROVIDENT Society merits such a *particular* exception.

In what proportion to those who are members, or to the annuity that can be afforded, must a person *now* pay to become a *member*? (190.)

A person to become a member must pay *now* at the lowest rate, which is, *with* admission fine,

For the age 20 <i>present money</i>	—	27.9649
Five parts in six paid only	—	16.295
	Difference	£. 11.6699

For the age forty	—	95.118
Five parts in six paid only	—	55.985
	Difference	£. 39.133(f)

For the age sixty	—	36.4514
Five parts in six paid only	—	36.411
	Difference	£. .0404

(205.)

Like

(f) A person aged *forty* must pay now to be admitted near 40*£.* more than five parts in six of the present members have paid.

To the PROVIDENT SOCIETY.

Likewise the lowest annuity the age twenty
now purchases is _____ 16.4624

The society can most probably }
pay only _____ } 12.0944

Annual loss £. 4.368

Age 40 purchases now _____ 17.802
The society can most probably }
pay only _____ } 12.0944

Annual loss £. 5.7076

Age 60 purchases now _____ 13.74
The society can most probably }
pay only _____ } 12.0944

Annual loss £. 1.6456

(209.)

There is such a strange perverseness in some dispositions, that they will not, on any account, be brought to a belief of what they *wish* to be otherwise; and look upon every endeavour to convince them, as a *troublesome* impertinence: such, having placed their hope and confidence in receiving their full annuity, will hardly ever be persuaded by the most powerful arguments, that it *cannot* be; nay, some are so very unconscionable as to think they pay an amazing sum *now* for

To the PROVIDENT SOCIETY.

now for the annuity *promised*, and would expect 25*£*. a year for almost *nothing*.

I am very sensible, gentlemen, that the reception this paper may meet with, depends mostly on, whether you and the *majority* of the *members* are of this class of people, or of such as can examine *without prejudice*, and be candidly open to *fair conviction*.

I am, Gentlemen,

With *due* respect,

Your most obedient,

Humble servant,

Sept. 24, 1771.

** ****

There were *two reasons* why the author addressed *this* to the *managers*: First, because it required more attention than could be expected to be given to it at a public reading before the society, and it was hoped they would have had time to peruse it carefully, and recommend or disapprove of it at the general meeting.—Secondly, It appeared, to most, as if the managers *themselves* had the appointment of the secretary,
by

THE PROVIDENT SOCIETY.

by the advertisements directing *all* applications to be made to *them*—yet, that there might not be wanting the respect due to the *rights* of the *members*, an address, *including them*, was left in the form of a petition with the letter.

The intention of the author, by this application, was not so much for the mere office of *Secretary* (a place he could by no means have long continued) as to get an *opportunity* of *explaining* the subject of this essay to the managers, and to endeavour, by their assistance, to persuade the society to *reform* their plan.

The letter was delivered September 25, when the author chanced to meet with some of the *managers*, whom he acquainted with his desire of being a candidate, but did not explain himself farther than by delivering the letter. Being informed that the secretary was to be elected by *ballot*, he hesitated whether to pursue or drop the design, because he was not known to any one member, and knew that his success must entirely depend on the *thorough understanding* of the calculation, which was delivered as a specimen of *one* necessary qualification. But the gentlemen present behaved *then* with such civility, as rather encouraged him to abide the event.—On the 30th, a general court was held, all the candidates summoned to attend
the

THE PROVIDENT SOCIETY.

—the author, in turn, called up—the petition read—after which, a GENTLEMAN reported that a calculation respecting the state of their society had been left with them (the managers) by the candidate present, but that it was pretty *long*, and supposed it might take up too much of their time—It is *too long*, it will take up *too much time*, was echoed and re-echoed all around:—That it was written with *decimals* (g)—that, for *his* part, he did not understand one thing in it: but if there was any gentleman present who understood figures, he would read it—nay, he had indeed given his word to the author, that it should be read before the society, and he was then ready to read it, if they chose to hear it.—It is *too long*, it will take up too much time, *as before*—One gentleman said he had examined it carefully, that he found it pretty explicit, but that it took notice of only *three* ages. (181.)—Others wanted to be informed whether the calculation was in *favor of the society* or not: many more particulars, and some *altercations* passed; but the whole terminated with a *general* invitation, to such as chose to be *informed*, to call upon the author before the day of balloting; and he engaged to *explain* the subject clearly to them.

The

(g) See INTRODUCTION, page 1.

THE PROVIDENT SOCIETY.

The author might have *foreseen* that this was drawing much trouble upon himself; for out of three or four score members present, it would be very natural for the *most part* to be inquisitive about the state of what so *nearly* concerned them.—Whether the reader can believe it or no, there called—*not one*——!!

It may be needless to acquaint the reader, that after such prognostics, the author gave himself no farther concern in the affair; and in publishing this anecdote he does not mean to reflect upon any member, but to produce it as a *second (b)* instance, how little solicitous we *all* are to *reform*, or even to be *made acquainted* with, our *errors*.

(b) See the *Letter* preceding the *Landable Society*.

THE

THE
SOCIETY
OF
LONDON ANNUITANTS,

(ESTABLISHED March 1, 1770.

(182.)

MEMBERS of this society when 50 years old will be entitled to 50*l.* per annum, *but when entitled to the annuity are to pay the HALF yearly sum of three pounds;* and such as become members at upward of 40 years of age must continue such *ten* years before entitled to the annuity.

By an abstract delivered December, 1771, the terms are,

(181.)		Admission fine.	Half yearly.
To pay at the age of 20	£ 2.	£ 1.4	
40	£ 35.	£ 7.35	
60	£ 0.	£ 3.35	

*and exclusive of the above to pay 2*s.* 6*d.* in-rolling fee.*

What

LONDON ANNUITANT SOCIETY.

What doth each member, of the said ages, pay in value of *present* money? (183.)

The age of twenty.

Admission and enrolling fee	—	2.125
First <i>half</i> year's payment	—	1.4
60 ditto worth in <i>present</i> money	—	40.079
		<u>£. 43.604</u>

The age of forty.

Admission and enrolling fee	—	35.125
First <i>half</i> year's payment	—	7.35
20 ditto worth in <i>present</i> money (157.)		103.544
		<u>£. 146.019</u>

The age of sixty.

Admission and enrolling fee	—	0.125
First <i>half</i> year's payment	—	3.35
20 ditto worth in <i>present</i> money	—	42.85
		<u>£. 46.325</u>

What annuities ought to be received, according to the value of the *present* money paid, and the terms of the society? (184.)

Age of 20, as 1.6987:1 :: 43.604:25.668.

40, as 5.343:1 :: 146.019:27.329.

60, as 2.6529:1 :: 46.325:17.46

£. 70.457

3 M

Age

LONDON ANNUITANT SOCIETY.

Age 40 purchased annuity <i>more</i> } than age 60 _____	£. 9.869
Age 20 ditto _____	£. 8.208
Age 40 purchases annuity <i>more</i> } than age 20 _____	£. 1.661

In what proportion doth each age *gain* or *lose* by receiving annuities *equally alike*, instead of that which each age has purchased? (185.)

Divide the total of the *three* above annuities by three.

$$70.457 \div 3 = 23.486$$

The age of 20 purchases _____	25.668
Receives by <i>equal</i> dividend _____	23.486
<u>Losses annually £. 2.182</u>	

The age of 40 purchases _____	27.329
Receives by <i>equal</i> dividend _____	23.486
<u>Losses annually £. 3.843</u>	

The age of 60 purchases _____	17.46
<i>Gains</i> , what the age of 20 loses, —	2.182
<i>Gains</i> , what the age of 40 loses, —	3.843

Receives by *equal* dividend £. 23.485

Or

LONDON ANNUITANT SOCIETY.

Or, the ages of 20 and 40 *jointly* contribute £.6.025 annually, for the *deficiency* of the age of sixty.

What annuity can the society afford to pay? (189.)

The utmost is 27.329; what the age of 40 purchases: but if the society can pay this, every member of it must have purchased at that age; or, if the number at all ages is *equal*, the society might pay £.23.485 (179.) provided *every* member had paid by *this* abstract.

But by a *former* table it appears, that at a general meeting held June 8, 1770, it was ordered that members should be admitted on the *following* terms.

	Age 20	Age 40.	Age 60.
Admission _____	1.	25.	0.
Half yearly payments each	1.2	5.	3.35
Total paid at entrance	2.2	30.	3.35

For the age of twenty.

Amount of the whole at entrance	2.2
60 half year's payments of 1.2 each,	34.353
	<u>£. 36.553</u>

LONDON ANNUITANT SOCIETY.

The age of forty.

Total at entrance	_____	30.0
20 half year's payments 5 <i>l.</i> each	—	70.438
		<u>£. 100.438</u>

The age of sixty.

Total at entrance	_____	3.35
20 half year's payments 3.35 each		42.85
		<u>£. 46.2</u>

Which sums will purchase the following annuities.

The age of 20, as 1.6987 : 1 ::	36.553 : 21.518
40, as 5.343 : 1 ::	100.438 : 18.798
60, as 2.6529 : 1 ::	46.85 : 17.415

Divide the total of the annuities by 3)57.731

The annuity by *equal* dividend £. 19.244

The age of 20 received by equal }
dividend _____ } 19.244

Lost annually by paying to the }
age of 40 _____ } 446

Ditto by paying for the de- }
ficiency of 60 _____ } 1.829

Paid for, and should receive an- }
nuity _____ } £. 21.519

LONDON ANNUITANT SOCIETY.

By the *former* abstracts the age of *twenty* made the *largest* purchase, and by the *latter* abstracts the age of *forty* makes the greatest.

Suppose an *equal* number have purchased by the *former* and by the *latter* abstract. (205)

Annuity, at a *medium*, by the first 19.244

Ditto by the latter abstract — 23.486.

Divide the sum of the two }
annuities by ————— } 2)42.73

The annuity the society could }
afford to pay ————— } £.21.365 (179.)

(121.)

The proportion in which a person must pay *now* to become a member is as follows. (190.)

Age of 20 purchases annuity — 25.668

Can receive only ————— 21.365

Loses annually £.4.303

Age of 40 purchases annuity — 27.329

Can receive only ————— 21.365

Loses annually £. 5.964

The

LONDON ANNUITANT SOCIETY.

The Age of 60 may receive an- } 21.365
nuity _____ }

Although he purchases only --- 17.46

Gains annually, more than purchased £.3.905

Likewise the age 20 pays *now* } 43.604
present money ————— }

present money	_____	43.004
Whereas formerly the same age	} _____	36.553
paid only		

Whereas formerly the same age } 36.553
paid only _____ }

paid only } 30.553

Difference £.7.051

The age of 40 pays now present } 146.019
money _____

money ————— \$ 143.19
Paid by the former abstract only — 100.438

Paid by the former abstract only — 100.438

Difference £. 45. 581

The age of 60 pays now *present* } 46.325
money _____ }

money	_____	}	40.325
Paid by the former abstract	_____		46.2

Paid by the former abstract — 46.2

Difference only £.0.125

(209.)

The

THE EQUITABLE SOCIETY

OF

ANNUITANTS,

(ESTABLISHED April 19, 1779.)

(182.)

MEMBERS 50 years old will be entitled to 50*l.* per annum during life, if they have been members ten years; and such as become members at upward of 40 years of age, must continue ten years before they are entitled to the annuity: *When entitled to the annuity, every member to pay £.1. 10 0 quarterly.*

By an abstract delivered September 10, 1771, the terms are as follow: to pay

	Age 20	Age 40	Age 60
For admission ———	1.4	32.	0.0
Half yearly payment	1.25	6.5	3.35

and two shillings and six-pence each, for registering fee.

THE EQUITABLE SOCIETY.

What doth each member, of the said ages,
pay in value of *present* money? (183.)

The age of *twenty*

Admission and registering fee	—	1.525
First <i>half</i> year's payment	—	1.25
60 ditto worth in <i>present</i> money	—	35.785
		£. 38.56

The age of *forty*.

Admission and registering fee	—	32.125
First <i>half</i> year's payment	—	6.5
20 ditto worth in <i>present</i> money	—	91.569
		£. 130.194

The age of *sixty*.

Admission and registering fee	—	.125
First <i>half</i> year's payment	—	3.35
20 ditto worth in <i>present</i> money	—	43.85
		£. 47.325

What annuities ought to be *received* ac-
cording to the value of the *present* money
paid, and the terms of the society? (184.)

The

THE EQUITABLE SOCIETY.

The age of 20, as 1.6987:1:: 38.56 :22.7
 40, as 5.343 :1:: 130.194:24.367
 60, as 2.6529:1:: 47.325:17.84
£. 64.907

In what proportion doth each age *gain* or *lose*, by receiving annuities *equally alike*, instead of *that* which each age *purchases*? (185.)

Divide the *total* of the *three* above annuities by three.

$$64.907 \div 3 = 21.6356$$

The age of *twenty* purchases annuity 22.7
 Receives by *equal* dividend ——— 21.6356
Annual loss £. 1.0644

The age of *forty* purchases annuity 24.367
 Receives by *equal* dividend ——— 21.6356
Annual loss £. 2.7314

The age of *sixty* purchases annuity 17.84
 Gains, the *annual* loss of age 20, — 1.0644
 Gains, the *annual* loss of age 40, — 2.7314
 Receives by *equal* dividend ——— £.21.6358
3 N What

THE EQUITABLE SOCIETY.

What annuity can this society afford to pay? (189.)

The answer is already found to be

£. 21.6356

The author applied by letter, some time ago, to the secretary of *this* society, to know whether any members had been admitted for *less* admission fine, or half yearly payments: he received *no* answer to *that* question—but has received one, to *another*, since (*page* 155.)

(121. 199*. 203. 209.)

THE

T H E
WESTMINSTER UNION SOCIETY,

(ESTABLISHED in the year 1770.)

(182.)

EVERY member (being a Protestant) will, when *forty-eight* years of age, be entitled to 25*£. per annum*, free from *all deduction*, during life; and *as much more* as shall be agreed to at a general court, provided he or she has paid their *ten years* subscription.

Ordered by a general court of the 28th of January, 1771, That every person, admitted a member of this society, shall pay their *quarterly* payments agreeable to the following table (g), and on admission to pay 5*s.* to defray the contingent expences of this society.

3 N 2

The

(g) It should seem, by this, that the payments *had been* different.

WESTMINSTER UNION SOCIETY.

The age <i>twenty</i> to pay <i>quarterly</i>	£. 0 9 2
<i>Forty</i> —————	£. 2 10 0
<i>Sixty</i> —————	£. 1 0 0
(181.)	

What doth each member, of the said ages, pay in value of *present* money? (183.)

The age *twenty*.

Contingent money —————	0.25
<i>First</i> quarter's payment (28.) ———	0.4588
112 ditto worth in <i>present</i> money	25.6358
	<u>£. 26.3441</u>

Four quarterly, or *one* yearly payment, is 1.8332; and the value of 1£. for 28 years, and then to *cease*, is 13.9842 + (142*. note) which, being multiplied into each other, produce £.25.63583544 for the value, in *present* money, of 112 quarterly payments of £.0 9 2 each, which *would* amount to £. 51 6 8 in the whole.

The age *forty*.

Contingent money —————	0.25
<i>First</i> quarter's payment ———	2.5
40 ditto worth in <i>present</i> money —	70.4381
	<u>£. 73.1881</u>

1£. *immediate* annuity for this age to continue *ten* years, and then to *cease*, is 7.04381 (74.) which, being multiplied by 10£. which

WESTMINSTER UNION SOCIETY.

which is the *yearly* sum paid, produces £.70.4381 for the value, in *present money*, of 100£. which is expected to be paid by this age at 40 *quarterly* payments of £. 2 10 0 each.

The age *fixty*.

Contingent money	_____	0.25
<i>First</i> quarter's payment	_____	1.
40 ditto worth in <i>present money</i>	—	25.582
		<u>£. 26.832</u>

1£. *immediate* annuity, for *this* age, to continue *ten* years, and then to *cease*, is 6.3955 (170.) which, multiplied by 4£. the *yearly* amount of the *quarterly* payments, produces £. 25 582 for the value, in *present money*, of 40£. *expected* to be paid by this age by *forty quarterly* payments.

What annuity for life ought to be received, by the said ages, for the *present money* paid, and according to the conditions of this society? (184.)

The age *twenty*, according to the proposals of *this* society, is to commence annuitant when aged *forty-eight*, and the value of 1£. annuity, on the said conditions, has been found (142*) to be £. 2.02838945. The ages *forty* and *fixty* are to continue *ten* years before entitled to the *annuity*, and therefore the values are, *as before* (174.)—Divide the
sums,

WESTMINSTER UNION SOCIETY.

sums, paid in *present* money, by these values of 1*£.* annuity. As, by the Rule of Proportion,

$$\begin{array}{rcl}
 \text{Age } 20, & -2.028(142^{\circ}): 1 :: 26.3441 : 12.99 & \\
 40, & -5.343 & : 1 :: 73.1881 : 13.698 \\
 60, & -2.6529 & : 1 :: 26.832 : 10.114 \\
 & & \hline
 & & \text{£. } 36.802
 \end{array}$$

Do the ages pay in due proportion to each other—or what is the difference? (185.)?

$$\begin{array}{rcl}
 \text{Age } \textit{forty} \text{ has purchased annuity} & \} & \text{£. } 3.584 \\
 \text{more than age } \textit{sixty} & \text{—} & \\
 \text{Age } \textit{twenty} \text{ has purchased annuity} & \} & \text{£. } 2.876 \\
 \text{more than age } \textit{sixty} & \text{—} & \\
 \text{Age } \textit{forty} \text{ has purchased annuity} & \} & \text{£. } 0.708 \\
 \text{more than age } \textit{twenty} & \text{—} &
 \end{array}$$

What annuity can this society afford to pay? (189.)

Divide the total of the *three* annuities by three.

$$36.802 \div 3 = 12.267$$

If the number admitted of *all* ages be *equal*, (199*.) the society may pay to each member an annuity of £. 12.267, and *no more* (121.)

IN

WESTMINSTER UNION SOCIETY.

In what proportion to those who *are* members, or to the annuity that the capital stock can afford, must a person *now* pay to *become* a member? (190.)

If *all* the members had paid by the *present* abstract, there would be no difference in the proportions otherwise than in the ages: but if *any* have paid *less* sums, the consequence must be in proportion, as in case (205.) Whether *all* have paid *equally*, or not, is *unknown* to the author; but he supposes, as inserted in the note at the bottom of the first page of this Society, that the payments have been different. (209.)

T H E

THE
LONDON UNION SOCIETY,

(ESTABLISHED September 26, 1770.)

(182.)

MEMBERS, when 50 years old, will be entitled to 52 guineas *per annum*, if they have been members *ten* years; and those who become members at upward of *forty* years old, must continue such *ten* years before they are entitled to the said annuity.

By an abstract delivered December, 1771, the terms are, to pay

(181.)	Age 20	Age 40.	Age 60
Premium	8.75	63.	10.
Incidents	.25	.25	.25
Half yearly payment	1.53	8.06667	4.2

N. B. On *admission* 5s. to be paid for *incidental* expences.

What

The LONDON UNION SOCIETY.

What doth each member pay in value of *present* money? (183.)

Age of twenty.

Premium and incidents	_____	9.
First <i>half</i> year's payment	_____	1.5375
60 ditto worth in <i>present</i> money	—	44.6592
		£. 55.1967

Age of forty.

Premium and incidents	_____	63.25
First <i>half</i> year's payment	_____	08.06667
20 ditto worth in <i>present</i> money	—	113.64018
		£. 184.95685

Age of sixty.

Premium and incidents	_____	10.25
First <i>half</i> year's payment	_____	4.2
20 ditto worth in <i>present</i> money	—	53.722
		£. 68.172

What annuities ought to be *received*, according to the value of the *present* money *paid*, and the terms of the society? (184.)

Answers.

Age of 20, as 1.6987:1::	55.1967	:32.4935
40, as 5.343 :1::	184.95685	:34.616
60, as 2.6529:1::	68.172	:25.697
		£. 92.8065

3 O

In

The LONDON UNION SOCIETY.

In what proportion doth each age *gain* or *lose*, by receiving annuities *equally*, instead of *that* which each age *purchases*? (185.)

Divide the total of the *three* above annuities by three.

$$92.8065 \div 3 = 30.9355$$

Age of 20 purchases annuity	—	32.4935
Receives by <i>equal</i> dividend	—	30.9355

Annual los £. 1.558

Age of 40 purchases annuity	—	34.616
Receives by <i>equal</i> dividend	—	30.9355

Annual los £. 3.6805

Age of 60 purchases annuity	—	25.697
Gains, the <i>annual los</i> of age 20,	—	1.558
Gains, the <i>annual los</i> of age 40,	—	3.6805

Receives by *equal* dividend £. 30.9355

Age 40 purchases annuity <i>more</i>	}	£. 8.919
than age 60		

Age 20 ditto	—	£. 6.7965
--------------	---	-----------

Age 40 purchases annuity <i>more</i>	}	£. 2.1225
than age 20		

Whether

The LONDON UNION SOCIETY.

Whether the table *with* premiums is equal to that *without*, and what is the difference? (186.)

The terms *without* a premium are:

	Age 20	Age 40	Age 60
Incidents	.25	.25	.25
Half yearly payment	1.875	12.1	5.25

Age of twenty.

Incidents and <i>half</i> yearly payment	2.125	
60 <i>half</i> yearly payments, value <i>pre-</i>		} 53.677
<i>sent</i> money		
	£. 55.802	

Age of forty.

Incidents and <i>half</i> yearly payment	12.35	
20 <i>half</i> yearly payments, value <i>pre-</i>		} 170.46
<i>sent</i> money		
	£. 182.81	

Age of sixty.

Incidents and <i>half</i> yearly payment	5.5	
20 <i>half</i> yearly payments, value	67.153	
	£. 72.653	

The LONDON UNION SOCIETY.

	Age 20	Age 40	Age 60
Paid <i>without</i> premium	55.802	182.81	72.653
Paid <i>with</i> premium	55.1967	184.96	68.172
Difference	.6053	2.15	4.481

The annuity that each age should receive, in proportion to the *present* money paid *without* premium, is

Age of 20, as 1.6987 : 1 :: 55.802 : 32.85
 40, as 5.343 : 1 :: 182.81 : 34.214
 60, as 2.6529 : 1 :: 72.653 : 27.386

Divide the three annuities by 3 94.45

The annuity *without* premium }
 by *equal* dividend is } 31.48

To which *add* the annuity *with* }
 premium } 30.9355

The total of the *two* annuities is £ 62.4155

The *half* of which is the *medium* }
 for *one* annuity } £ 31.20775

What annuity can the society afford to pay? (189.)

Supposing an *equal* number of members at *all* ages, and of such as have paid *with*,
as

The LONDON UNION SOCIETY.

as of such as have paid *without* premium; the annuity the society might afford to pay, would be £ 31.20775, provided every member had been admitted by the *present* abstract.

But by a *former* abstract members paid admission

	Age 20	Age 40	Age 60
Premium	£ 1.05	£ 37.	£ 0.0
Half yearly payments	£ 1.1	£ 3.5	£ 3.5

Age of twenty.

Premium and first <i>half</i> yearly payment	}	2.15
60 <i>half</i> yearly ditto worth in <i>present</i> money	}	31.49
		£. 33.64

Age of forty.

Premium and first <i>half</i> yearly payment	}	40.5
20 <i>half</i> yearly ditto worth in <i>present</i> money	}	49.3067
		£. 89.8067

Age

The LONDON UNION SOCIETY.

Age of sixty.

Premium and first <i>half</i> yearly	}	
payment _____		3.5
20 <i>half</i> yearly ditto worth in <i>pre-</i>	}	
<i>sent</i> money _____		44.768

£. 48.268

	Age 20.	Age 40.	Age 6
To pay by the <i>present</i> abstract	55.1967	184.957	68.172
Paid by the <i>former</i> ditto	33.64	89.807	48.268
Difference	21.556	95.150	19.904

The annuities by the *former* abstract.

Age 20, as 1.6987 : 1 :: 33.64 : 19.803
 40, as 5.343 : 1 :: 82.8067 : 16.808
 60, as 2.6529 : 1 :: 48.268 : 18.1906

The annuities purchased are,

	Age 20.	Age 40.	Age 60.
By the <i>present</i> abstract	32.85	34.214	27.386
By a <i>former</i> _____	19.803	16.808	18.1906
Annual difference	13.047	17.406	9.1954

Age 20 purchased by a <i>former</i>	}	
abstract _____		19.803
40 ditto _____		16.808
60 ditto _____		18.1906
		£ 54.8016

The LONDON UNION SOCIETY.

Divide the *total* of the *three* }
annuities by ———— } 3)54.8016

Annuity by *equal* dividend ——— 18.2672
Add equal annuity by the *pre-* }
sent abstract ———— } 31.20775

Divide the *total* of the *two* }
annuities by ———— } 2)49.47495

The *medium* of the annuities is £. 24.737475

If the number of members admitted by both abstracts are *equal*, the annuity to be paid may be £.24.737475; (121.) if the *greatest* number be by the *present* abstract, the annuity may be *more*; but if the *greater* number be by the *former* abstract, the annuity must be *less*. (205.)

In what proportion to those who *are* members, or to the annuity that can be afforded by the capital stock, will a person pay to *become* a member?

To pay in <i>present</i> money	Age of 20	Age of 40.	Age of 60.
By abstract <i>with</i> premium	55.1967	184.95685	68.172
By ditto <i>without</i> ———	55.8 2	182.81	72.653
Total of both ———	110.9987	367.76685	140.825
The medium of the two	55.49935	183.8834	70.4125
By the <i>former</i> abstract	33.64	89.8067	48.268
Total of both ———	89.13935	273.6901	118.6805
Medium of what each member hath paid <i>col-</i> } <i>lectively</i>	44.56967	136.845	59.34

The LONDON UNION SOCIETY.

The *two present abstracts, with and without* premiums, are added together, and the totals are *divided* by two, to give the *medium* of the two abstracts: to which are added the payments by the *former abstract*: the totals of these are *divided* by two, to find the *medium* of what each member hath paid *collectively*: whence the *loss*, to such as enter the society at this time, appears to be,

Age twenty.		
Pays by the <i>present abstract with</i>	}	55.1967
premium _____		
No member (<i>collectively</i>) hath	}	44.5696
paid more than _____		
		£. 10.6271

Pays by the <i>present abstract with-</i>	}	55.802
<i>out premium</i> _____		
Each member hath paid (<i>one</i>	}	44.5696
<i>with another</i>) _____		
		55.802
		44.5696
		£. 11.2324

The age forty.		
Pays by the <i>present abstract with</i>	}	184.956
premium _____		
No member (<i>collectively</i>) hath	}	136.845
paid more than _____		
		£. 48.111
		Pays

The LONDON UNION SOCIETY.

Pays by the <i>present</i> abstract <i>with-</i>	}	182.81
<i>out</i> premium _____		
Each member hath paid (one	}	136.845
with another) _____		
		£. 45.965

The age sixty		
Pays by the <i>present</i> abstract with	}	68.172
premium _____		
No member (collectively) hath	}	59.34
paid more than _____		
		£. 8.832

Pays by the <i>present</i> abstract <i>with-</i>	}	72.653
<i>out</i> premium _____		
Each member hath paid (one with	}	59.34.
another) (205.) _____		
		£. 13.313

The differences are by supposition that the members, admitted *with* premium and *without*, are *equal*; and that both numbers together are *but equal* to the number admitted by the *former* abstract.

THE
CONSOLIDATED SOCIETY,

(Instituted Jan. 1, 1771.)

For the Benefit of YOUTH and AGE,

Proposes *Three* different Plans.

PLAN the *First*.

(182.)

MEMBERS - will be entitled to ten pounds per annum (or such sum as the state of the society will admit) for each share, (*i*) till they shall have arrived at the age of fifty; provided that they have paid their full subscriptions for *ten* years continuance, before they commence annuitants, pursuant to the table, by four quarterly payments, till the consolidated numbers are,

Age

(*i*) The annuities are in 10*l.* shares, of which a person may purchase *one, two, three, four, or five.*

THE CONSOLIDATED SOCIETY.

	Age 10.	Age 20.	Age 30.	(181.)
1000: to pay	£.2 10 0	£.1 12 6	£.1 2 6	
100 more ditto	£.2 12 6	£.1 15 0	£.1 4 6	
100 more ditto	£.2 13 6	£.1 16 0	£.1 5 6	
100 more ditto	£.2 14 6	£.1 17 0	£.1 6 6	

Or *with* admission money to pay *quarterly*

1000 —————	£.2 6 6	£.1 9 6	£.1 1 1
100 more —————	£.2 9 0	£.1 12 0	£.1 2 1
100 more —————	£.2 10 0	£.1 13 0	£.1 3 1
100 more —————	£.2 11 0	£.1 12 0	£.1 4 1

The age 10 to pay admission money £ 5.25
 20 ————— £ 4.5
 30 ————— £ 2.

Also TWO SHILLINGS for register money.

What do members pay, in value of *present* money, according to the *fourth* or *last* class *without* admission money? (183.)

The age of ten.
 Register money ————— .1
 First quarter's payment ————— 2.725
 40 ditto worth in *present* money (80.) 85.7973
 £. 88.6223

THE CONSOLIDATED SOCIETY.

The age of twenty.

Register money	_____	.1
First quarter's payment	_____	1.85
40 ditto worth in <i>present</i> money	}	57.0812 +
(94.) _____		
		<u>£. 59.0312</u>

The age of thirty.

Register money	_____	.1
First quarter's payment	_____	1.325
40 ditto worth in <i>present</i> money (99.)	}	39.238
		<u>£. 40.663</u>

What do members pay in *present* money, in the *same* class, and *with* admission money?

The age of ten.

Register	_____	.1
First quarter's payment	_____	2.55
40 ditto worth in <i>present</i> money	}	80.2874 +
(80.) _____		
Admission fine	_____	5.25
		<u>£. 88.1874</u>

The

THE CONSOLIDATED SOCIETY.

The age of twenty.

Register	_____	.1
First quarter's payment	_____	1.7
40 ditto worth in <i>present</i> money	}	52.453 +
(94.) _____		
Admission fine	_____	4.5
		<u>£. 58.753</u>

The age of thirty.

Register	_____	.1
First quarter's payment	_____	1.20415
40 ditto worth in <i>present</i> money	}	35.65922 +
(99.) _____		
Admission fine	_____	2.
		<u>£. 38.96337</u>

(186.)	Age 10.	Age 20.	Age 30.
Paid without admission	88.6223	59.0312	40.663
Ditto with admission fine	88.1874	58.753	38.96337
Difference	.4349	.2782	1.69963
Total of with and without admission fine	176.8097	117.7842	79.62637
Medium between both	88.40485	58.8921	39.813185

The difference between *paying* and *not* paying admission fine may *seem* little, because the annuity is but *small*: if the annuity was *five* of these *ten* pound shares, it would be
50£.

THE CONSOLIDATED SOCIETY.

50*l.* the difference would be in proportion, but more conspicuous, because *five* times as much.

For the age *ten* ——— £ 2.1745
twenty ——— £ 1.391
thirty ——— £ 8.49815

What annuity ought to be received, according to the *medium* of what is paid in *present* money by with and without admission fine, and according to the *fourth* or *last* class in the proposals? (184.)

Age 10, as 8.92023 (86.) : 1 :: 88.40485 : 9.9106 +
 20, as 6.6001768 (97.) : 1 :: 58.8921 : 8.9228
 30, as 3.80195 (100.) : 1 :: 39.813185 : 10.4716

The total of the *three* annuities is ——— 29.305

The total divided by *three* is the *medium* ——— £.9.768

	In 10 <i>l.</i> ann. ^y	In 50 <i>l.</i> ann. ^y
Age 10 purchases <i>less</i> annuity than age 30	0.561	—£ 2.805
20 ditto	1.5488	—£ 7.244
20 ditto than age 10	0.9878	—£ 4.939

What annuity can the society afford to pay?

If *every* member had paid according to the *last* class, the society might pay (excepting for the *refund sums* taken notice of at the end of this Society) the annuity of £.9.768 ——— but the abstract shews there
bar

THE CONSOLIDATED SOCIETY.

has been, or it was expected there *would be*, four different classes (as may be seen at the beginning) and the *medium* of the payments will be found thus:

Because 1000 is *ten* times 100, there must be *ten* of the *first* quarterly payments added to the *three other* in each age, and the totals divided by 13 will give the *quarterly* payments that one member, with another, has paid from the commencement (*if the 1000 paid equally*) until the society consists of 1300, thus:

	Age 10.	Age 20.	Age 30.
<i>Without admission</i>	2.5404+	1.6654+	1.1596
<i>With admission fine</i>	2.3654+	1.5154+	1.07724

The *present* money paid, *collectively*, *with-*
out admission fine.

The age *ten*.

Register	_____	.1
First quarter's payment	_____	2.5404
40 ditto worth in <i>present</i> money (80.)		79.9851
		£. 82.6255

The age *twenty*.

Register	_____	.1
First quarter's payment	_____	1.6654+
40 ditto worth in <i>present</i> money (94.)		51.3854
		£. 53.1508

The

THE CONSOLIDATED SOCIETY.

The age *thirty*.

Register	_____	.1
First quarter's payment	_____	1.1596
40 ditto worth in <i>present</i> money (99.)	_____	34.3599
		£. 35.6195

The *present* money paid, *collectively*, with admission fine.

The age *ten*.

Register	_____	.1
Admission fine	_____	5.25
First quarter's payment	_____	2.3654
40 ditto worth in <i>present</i> money (80.)	_____	74.4752
		£. 82.1906

The age *twenty*.

Register	_____	.1
Admission fine	_____	4.5
First quarter's payment	_____	1.5154
40 ditto worth in <i>present</i> money (94.)	_____	46.7572
		£. 52.8726

The age *thirty*.

Register	_____	.1
Admission fine	_____	2.
First quarter's payment	_____	1.07724
40 ditto worth in <i>present</i> money (99.)	_____	31.9095
		£. 35.07819
		Suppose

THE CONSOLIDATED SOCIETY.

Suppose the number admitted *without* admission money is *equal* to those admitted *with* admission fine, and that the society consists of 1300 members, complete:

Add the sums paid *without* admission to the sums paid *with* admission fine, and divide the totals by *two*, to find the medium of what *each* member has paid *collectively*.

	Age of 10	Age of 20	Age of 30
Paid <i>without</i> admission	82.6255	53.1508	35.6195
<i>With</i> admission fine	82.1906	52.8726	35.07819
Divide by ———— 2)	164.8161	106.0234	70.69769
Paid at a <i>medium</i>	82.408	53.0117	35.3488
Difference of <i>with</i> and <i>without</i> admission } fine	.4349	.2782	.54131

The annuities purchased, at a *medium*, are

Age 10, as 8.92223 (86.) : 1 :: 82.408 : 9.238
 20, as 6.600176 (97.) : 1 :: 53.0117 : 8.0319 +
 30, as 3.89195 (100.) : 1 :: 35.3488 : 9.297

Divide the total of the three annuities by 3)26.5669

The annuity, by *equal* dividend, ——— £. 8.8556

Members purchased, in the last class of 1300, an annuity of £. 9.768; but when that class shall be completely full, the society (*collectively*) will have purchased

THE CONSOLIDATED SOCIETY.

but £.8.8556 for the annuity to the share of each member. (205.)

	Age of 10	Age of 20	Age of 30
Members by the last class } paid at a medium	88.40485	58.8921	39.81318
Collectively, have paid only	82.408	53.0117	35.3488
Difference —	5.99685	5.8804	4.46438

But there has been *another* abstract published, by which members have been admitted on paying

Half yearly at the	Age 10	Age 20	Age 30
The sum of £.	3.3375	2.9166	1.75

And if the 1000 members were admitted by the *former* abstract, the *admission money* has been augmented, each time that the society increased 200 members, at the following rates.

	Age 10.	Age 20.
200 paid each —	8 8 0	2 2 0
200 —	9 9 0	3 3 0
200 —	10 10 0	4 4 0
200 —	11 11 0	5 5 0
200 —	12 12 0	6 6 0
Divide the total by — 5)	52 16 0	21 0 0
Paid at a medium —	10 10 0	4 4 0

Note, Age 30 to pay no admission fine.

What

THE CONSOLIDATED SOCIETY.

What did each member (*collectively*) pay
in *present* money?

The age of *ten*.

Admission money £. 10 10 0 —	10.5
First <i>half</i> year's payment —	3.3375
20 ditto worth in <i>present</i> money (80.)	52.541
	£. 66.3785

The age of *twenty*.

Admission money £. 4 4 0 —	4.2
First <i>half</i> year's payment —	2.91 44
20 ditto worth in <i>present</i> money } (94.) ————— }	44.995409
	£. 52.112009

The age of *thirty*.

No admission— <i>first half</i> year's } payment ————— }	1.75
20 ditto worth in <i>present</i> money (99.)	25.9119
	£. 27.6619

The annuities, these sums purchased,
are

For age 10, as 8.92023 (86.) : 1 :: 66.3785	: 7.4413
20, as 6.600176 (97.) : 1 :: 52.112009	: 7.8955
20, as 3.80195 (100.) : 1 :: 27.6619	: 7.2757

Divide the total by —————	3)22.6125
---------------------------	-----------

The annuity, by <i>equal</i> dividend, is —————	£. 7.5375
---	-----------

What

THE CONSOLIDATED SOCIETY.

What annuity can this Society afford to pay?
(189.)

On supposition that the Society consisted of 1300 members, and that 1000 of them were admitted by the former, and 300 by the latter abstract, then *ten equal annuities* by the former, must be added to *three* by the latter abstract, and the total be divided by thirteen, as thus:

$$\begin{array}{r} \text{£. } 7.5373 \times 10 = 75.373 \\ \text{£. } 8.8546 \times 3 = 26.5638 \\ \hline \end{array}$$

Divide the total by 13) 101.9368

Annuity by equal dividend £. 7.8413 +
(205.)

The Society by *this* plan, and according to *these* suppositions, might pay an annuity of £. 7.8413 to each member; *excepting* the allowance to be made for the money *refunded*, which will be noticed presently.
(121.)

PLAN the Second.

By abstracts printed May 15, 1771, and August 7, 1771, Members will be entitled to 10*l.* per annum (or such sum as the State of the Society will admit) during life, for each share, provided the subscriptions are paid, from age *one to twenty-nine inclusive*, for *twenty* years; from age *thirty to thirty-nine inclusive*, for *ten* years continuance.

By

THE CONSOLIDATED SOCIETY.

By which (it is supposed) the managers mean that the age ten becomes entitled to the annuity when aged *thirty*; the age twenty when aged *forty*; and the age thirty when aged *forty*.

The subscriptions are different as the society shall consist of 1000, or 1100, or 1200, or 1300 members; and also as the person to be admitted shall pay, or *not* pay, *admission money*.

Suppose a person to be admitted when the society shall consist of 1300 members, or nearly; and *without* admission money the conditions are, to pay

	Quarterly	Which is per annum
Age of <i>ten</i>	£.0 18 9	£ 3.75
<i>twenty</i>	£.0 15 6	£ 3.1
<i>thirty</i>	£.2 6 0	£ 9.2

And *two shillings* register money.

What doth each member pay in value of *present* money? (183.)

The age of *ten*.

Register money	_____	..1
First quarter's payment	_____	.9375
80 ditto worth in <i>present</i> money (81.)	_____	47.544
		£. 48.5815

The

THE CONSOLIDATED SOCIETY.

The age of *twenty*.

Register money	_____	.1
First quarter's payment	_____	.775
80 ditto worth in <i>present</i> money	}	37.431 +
(95.) _____		
		£. 38.306

The age of *thirty*.

Register money	_____	.1
First quarter's payment	_____	2.3
40 ditto worth in <i>present</i> money (99.)	68.11128	
		<u>£. 70.51128</u>

What annuity, according to the proposals, do these ages purchase for the sums paid?

Age 10, as 5.1717369 (159.)	:	1	::	48.5815	:	9.3936
20, as 3.938		(160.)	:	1	::	38.306
30, as 6.68589		(147.)	:	1	::	70.51128
						10.5462

Divide the total of the <i>three</i> annuities by	_____	3)29.667
The annuity, by <i>equal</i> dividend, is	_____	<u>£ 9.889</u>

The *equal* dividend is *nearly* what is promised; but the reader will remark, that it is by purchasing at the price the *last* class is rated at, and which is the *highest*: *twelve hundred* have paid *less*, which must *lower* the stock of the society, in the proportion, nearly, as before. (205.)

But,

THE CONSOLIDATED SOCIETY.

But, omitting *these* particulars, and also the purchase *with admission fine*, examine the *same ages* of the *same plan* by a former abstract, supposed to be the first, the conditions then were

	Age 10	Age 20	Age 30
Admission money	£ 3.5	£ 3.5	£ 10.5
Half yearly	£ 2.	£ 1.75	£ 2.5

And *two shillings* for register money.

The age *ten* paid

Register money	_____	.1
Admission ditto	_____	3.5
First <i>half</i> year's payment	_____	2.
40 ditto worth in <i>present</i> money }		50.7137 +
(81.)	_____	
		<u>£.56.3137</u>

The age *twenty* paid

Register money	_____	.1
Admission ditto	_____	3.5
First <i>half</i> year's payment	_____	1.75
40 ditto worth in <i>present</i> money }		42.2615
(95.)	_____	
		<u>£.47.6115</u>

The

THE CONSOLIDATED SOCIETY.

The age *thirty* paid

Register money	<u> </u>	.1
Admission ditto	<u> </u>	10.5
First <i>half</i> year's payment	<u> </u>	2.5
20 ditto worth in <i>present</i> money (99.)	<u> </u>	37.017
		<u> </u>
		£. 50.117

The annuities purchased were

Age 10, as 5.1717369 (159.)	: 1 :: 56.3137	: 10.888
20, as 3.938 (160.)	: 1 :: 47.6115	: 12.09
30, as 6.68589 (147.)	: 1 :: 50.117	: 7.496 +

Divide the total of the three annuities by 3)30.474

The annuity by *equal* dividend £. 10.158

It must be allowed, that *this* plan is the *most extraordinary* of any; for the members paid *more*, consequently purchased *more* annuity, by the *first* abstract than by *any* *since*: yet the difference of the purchase by *each age* is greatest *by this*, and will be found still more so upon examining the age *ten*.

(*To become an annuitant when aged forty.*)

To

THE CONSOLIDATED SOCIETY.

To pay register	_____	.1
Admission fine	_____	2.625
First <i>half</i> year's payment	_____	1.
60 ditto worth in <i>present</i> }		30.792
money (82.)	_____	
		£. 34.517

The age *twenty*

(To become annuitant when aged forty, the same as before)

To pay register	_____	.1
Admission fine	_____	3.5
First <i>half</i> year's payment	_____	1.75
40 ditto worth in <i>present</i> }		£. 42.26
money (95.)	_____	
		£. 47.61

The age *thirty* is also the same as before.

The annuities purchased were

Age 10, as 2.4542 (158.)	: 1 :: 34.517	: 14.064
20, as 3.938 (160.)	: 1 :: 47.6115	: 12.09
30, as 6.68589 (147.)	: 1 :: 50.117	: 7.4964

These are excessive disproportions, and deserved the abolition, it is supposed, they met with, as they have not appeared with the *same* conditions in the abstracts published since.

THE CONSOLIDATED SOCIETY.

PLAN the *Third*,

For the admission of members at all ages,

Who will be entitled to 10*£*. per annum (or such sum as the state of the society will admit) during life, for each share, provided the subscriptions are paid until the age of fifty; except such as become members at forty years of age, or upwards, who are to continue members for ten years before entitled to the annuity.

The subscriptions *without admission*, by an abstract dated August 7, 1771, are different as the society shall consist of 1000, 1100, 1200, or 1300 members, thus:

Until	1000	1100	1200	1300	The medium
Age 20 pays } quarterly }	4 6	5 0	5 6	6 0	.23654 +
40 —	1 10 0	1 12 6	1 15 0	1 17 6	1.5577 +
60 —	13 6	14 0	16 0	17 0	.7

Each member to pay *two shillings* register money.

The subscriptions *with admission* fine are—to pay *quarterly*

Until the society consist of members

Age.	1000	1100	1200	1300	Medium	Admission
20	3 9	4 5	4 9	5 3	.19968	2.1
40	1 7 0	1 9 6	1 12 0	1 14 6	1.4076423	4.2
60	12 9	13 3	15 3	16 3	.6625	1.05

THE CONSOLIDATED SOCIETY.

Each member to pay *two shillings* register money.

N. B. Ten of the first class are added to the three others, and the total divided by thirteen, to find the medium, as was done before.

What doth each member pay in *present* money, and *without* admiffion fine, according to the *fourth* or *last* class? (183.)

The age *twenty*.

Register money	_____	.1
First quarter's payment	_____	.3
120 ditto worth in <i>present</i> money (93.)		17.17662
		£. 17.57662

The age *forty*.

Register money	_____	.1
First quarter's payment	_____	1.875
40 ditto worth in <i>present</i> money		52.8286 +
(74.)	_____	
		£. 54.8036

The age *sixty*.

Register money	_____	.1
First quarter's payment	_____	.85
40 ditto worth in <i>present</i> money (101.)		21.7447
		£. 22.6947

THE CONSOLIDATED SOCIETY.

What doth each member pay in *present* money, *with admission fine*, and according to the *fourth* or *last* class?

The age of *twenty*.

Register money	_____	.1
Admission fine	_____	2.1
First quarter's payment	_____	.2625
120 ditto worth in <i>present</i> money	_____	15.0295425
		£. 17.4910425

The age of *forty*.

Register money	_____	.1
Admission fine	_____	4.2
First quarter's payment	_____	1.725
40 ditto worth in <i>present</i> money	_____	48.6023 +
		£. 54.6273

The age of *sixty*.

Register money	_____	.1
Admission fine	_____	1.05
First quarter's payment	_____	.8125
40 ditto worth in <i>present</i> money	_____	20.785375
		£. 22.747875

N. B. The values of what the three ages pay in *present* money are found, in this plan, by the same references as in first page.

Age

THE CONSOLIDATED SOCIETY.

	Age 20.	Age 40.	Age 60.
Paid <i>with</i> admiffion money	17.57662	54.8036	22.6947
<i>Without</i> ditto ———	17.49104	54.6273	22.7478
Difference ———	.08558	.1763	.0531
Total of <i>with</i> and <i>without</i> } admission ———	35.06766	109.4309	45.4425
The <i>medium</i> between both	17.53383	54.71545	22.72125

The annuities purchafed; *at a medium*,
are

(184.)

Age 20, as 1.6987 : 1 :: 17.53383 : 10.3219

40, as 5.343 : 1 :: 54.71545 : 10.2405

60, as 2.6529 : 1 :: 22.72125 : 8.564

Divide the total of the three an- }
nuities by ——— 3) 29.1264

The annuity by *equal* dividend is £. 9.7088

This class purchafes nearly the annuity that is promifed (*omitting the confideration of refunding*, as mentioned hereafter) but the *laft* abstract fhews that 1200 paid lefs; and the *medium* of what the fociety (collectively) has paid quarterly (admitting it to confift of 1300 members) has been found before in the table; and the value of the whole is as follows.

Without

THE CONSOLIDATED SOCIETY,

Without admiffion money.

The age twenty

Pays register money	_____	.1
First quarter's payment	_____	.23654 +
120 d ^r . worth in <i>present</i> money		13.54319
		<u>£. 13.87973</u>

The age forty

Pays register money	_____	.1
First quarter's payment	_____	1.5577 +
40 ditto worth in <i>present</i> money		43.88857
		<u>£. 45.54627</u>

The age sixty

Pays register money	_____	.1
First quarter's payment	_____	.7
40 ditto worth in <i>present</i> money	—	17.9074
		<u>£. 18.7074</u>

With admiffion money,

The age twenty

Pays register money	_____	.1
Admiffion fine	_____	2.1
First quarter's payment	_____	0.19968
120 ditto worth in <i>present</i> money		11.43276 +
		<u>£. 13.83244</u>

The

THE CONSOLIDATED SOCIETY.

The age forty

Pays register money	_____	.1
Admission fine	_____	4.2
First quarter's payment	_____	1.40764
40 ditto worth in <i>present</i> money	_____	39.6606 +
		<u>£. 45.36824</u>

The age sixty

Pays register money	_____	.1
Admission fine	_____	1.05
First quarter's payment	_____	.6625
40 ditto worth in <i>present</i> money	_____	16.948
		<u>£. 18.7605</u>

	Age of 20	Age of 40	Age of 60
Paid <i>without</i> admission money	13.8797	45.5462	181.7074
<i>With</i> admission	13.8324	45.3682	18.7605
Difference	.0473	.177	.0531
Total of <i>with</i> and <i>without</i> admission	27.7121	90.9144	37.4679
Medium between both	13.856	45.4572	18.73395

The proportions of paying *with* and *without* admission, are in this society so well calculated, that the difference is but very small.

The *medium* of what the members pay according to the last class, which is that consisting of between 1200 and 1300 members, was

THE CONSOLIDATED SOCIETY.

	Age 20	Age 40	Age 60
was _____	17.53383	54.71545	22.72126
Ditto paid collectively	19.856	45.4572	18.73395
Difference	3.67783	9.25825	3.9873

A person aged 40, admitted in the last class, would pay £. 9.258 more than any one member had done collectively, which in five shares would be £. 46.29.

What annuity hath each member purchased, *collectively*, according to the *last* abstracts?

Age 20, as 1.6987 : 1 :: 13.856 : 8.1568
 40, as 5.343 : 1 :: 45.4572 : 8.5078
 60, as 2.6529 : 1 :: 18.73395 : 7.0616

Divide the total of the *three* annuities by _____ } 3) 23.7262
£. 7.9087

It appears now, that if the society consisted of *complete* 1300 members, and *all* had paid by the *last* abstract, and that the number of every age, as well *with* as *without* admission fine, was *equal*, then the members of the society have purchased *collectively*, and therefore should receive, £. 7.9087 *equal* annuity each person.

But the *former* abstract admitted members

THE CONSOLIDATED SOCIETY.

bers upon different terms, viz. The ages 20 and 60 paid no admission money; but the age 40 paid, as the society increased, in these different proportions. (205.)

200 paid each	—	£. 3
200 paid	—	£. 5
200 paid	—	£. 7
200 paid	—	£. 9

Divide by	—	4)24
-----------	---	------

Paid collectively	—	£. 6
-------------------	---	------

It is not known *how many* were admitted by this abstract; but it cannot be supposed that more than 800 were; because every abstract since, *even the last*, proposes admittance into the class of 1000.—The *half* yearly payments were

Age 20	Age 40	Age 60
0 4 0	1 0 0	0 15 0

The age *twenty*

Paid register money	—	.1
First <i>half</i> year's payment	—	.2
60 ditto worth in <i>present</i> money		5.726 +
		£ 6.026

THE CONSOLIDATED SOCIETY.

The age *forty*

Paid register money	_____	.1
Admission fine	_____	6.
First <i>half</i> year's payment	_____	1.
20 ditto worth in <i>present</i> money		14.088 +
		£ 21.188

The age *sixty*

Paid register money	_____	.1
First <i>half</i> year's payment	_____	.75
20 ditto worth in <i>present</i> money		9.59325
		£ 10.44325

The annuities purchased, at a medium,
were

Age 20, as 1.6987 : 1 :: 6.026	: 3.55
40, as 5.343 : 1 :: 21.188	: 3.965
60, as 2.6529 : 1 :: 10.44325	: 3.936

Divide the three annuities by — 3) 11.451

The annuity by *equal* dividend £ 3.817

The *first* members, that have paid in such
small proportion, must reduce the annuity
which latter members have purchased, ac-
cording

THE CONSOLIDATED SOCIETY.

cording to the *number* that were admitted (199*.) of each class. (205.)

There is another consideration, *peculiar to this society*, which must also very much lessen the annuity that it should afford to pay; which is this *refunding* clause, that has been, more than once, referred to:

*“ The heirs, executors, &c. of such members, as shall have died before they become annuitants, may, if expressly authorized by WILL, demand and RECEIVE 10*l.* per cent. for all admission monies and other contributions paid into this society by the deceased, provided that all arrears whatever are paid.”*

On enquiring at the *Queen's Arms, St. Paul's Church Yard*, last *December*, for an abstract, the author was told this society was breaking up, and paying back all monies received, except deducting one guinea from each member for expences incurred.

But Dr. *Price* saying, in his Supplement to his *second* edition of *Observations*, &c. page 33, published since, that the CONSOLIDATED Society requires *now* the full value at 4*l.* per cent. of the annuities it promises, according to the LONDON Table of *Observations*: the author enquired *lately*

THE CONSOLIDATED SOCIETY.

at the *King's Arms, Bridge-Street, Westminster*, and was told they had not assembled *there* a long time.

Whether the society is dissolved or not, the EXAMINATION may be satisfactory to those who *have been*, if there are none who *are* members: and the *different* plans of this society required many more *particular* answers, to questions proposed for its use, than any other society: all which, it is hoped, have conduced to a more perfect knowledge of the subject, and therefore may be a sufficient apology for inserting it.

THE

THE
PUBLIC ANNUITANT SOCIETY, (k)

(Instituted Jan. 1771.)

(182.)

MEMBERS of this society, under the age of forty, will be entitled to 30*l.* per annum at the age of *forty-seven*; and at the age of *fifty*, to an annuity of 50*l.* *during life*; together with such a portion of the *increase* as the fund of the society will admit, *provided it does not exceed SIXTY pounds in any one year.*—Those who commence members between the ages of *forty* and *fifty* must continue such for *seven* years before they are entitled to 30*l.* per annum, and *ten* years before they are

(k) The PUBLIC ANNUITANT Society (Sir RICHARD GLYN, Bart. President) while its number continues between 600 and 700, is not to take at several ages *half* the value of its annuities; and those who have been admitted while the society continued under 500, do not pay a *THIRD* of the true values.—*Supplement to Dr. Price's Observations, &c. page 32, Note*

PUBLIC ANNUITANT SOCIETY.

are entitled to 50*l.* per annum.—Such as become members at 50 years old, or upwards, shall be entitled to the *whole* annuity of 50*l.* at the end of *seven* years; *a circumstance, THOUGH EXTREMELY AGREEABLE, that has not been attended to by any other society* (139.)—Every member shall continue their *quarterly* payments until they are entitled to 50*l.* per annum, and afterwards NO MORE *than the sum of £.1 10 0 QUARTERLY.*—Both the *quarterly* payments, and the *admission* fine, have been, and will be, raised according as the *number* of members *increased* in the following proportions.

	(181.)	Age 20	Age 40	Age 60
100 paid quarterly		.5	2.1	3.1
100 —————		.525	2.5	3.45
100 —————		.55	3.	3.85
100 —————		.55	3.25	4.1
100 —————		.575	3.675	4.55
100 —————		.725	4.5	4.75
100 —————		.95	5.5	5.
100 —————		1.15	6.5	5.5
Divide the totals by 8)		5.525	31.025	34.3
The medium (205.)		.6906	3.878	4.2875

To

PUBLIC ANNUITANT SOCIETY.

To pay admission fine	Age 40.	Age 60.
100. (1) —	26.25	26.25
100 —	27.3	27.3
100. —	28.35	28.35
100 —	29.4	29.4
100 —	31.5	31.5
100 —	33.6	33.6
100 —	35.7	35.7
100 —	37.8	37.8
Divide the total by 8)	249.9	249.9
The medium (205.)	31.2375	31.2375

Each member to pay 5s. 3d. as a provision for contingent expences.

The age *twenty* pays no admission fine.

What doth each member pay, in value of *present* money, and according to the *last* class, or hundred? (183.)

The

(1) The 24 first members are (from the abstract) supposed to have paid no admission fine,—The President, Vice-President, Directors, Secretary, and Solicitor of the society, amount in number to 24 persons.

PUBLIC ANNUITANT SOCIETY.

The age of *twenty*.

For contingent expenses	—	.2625
First <i>quarter's</i> payment	—	1.15
120 ditto worth in <i>present</i> money		}
(93.)	—	65.8437

67.2562

Deduct the value of 3 years an-	—	}
nuit of 30 <i>l.</i> worth in <i>pre-</i>		15.3952
<i>sent</i> money (104.)	—	

£ 51.861

The age of *forty*.

For contingent expenses	—	.2625
Admission fine	—	37.8
First <i>quarter's</i> payment	—	6.5
40 ditto worth in <i>present</i> money		}
(74.)	—	183.139

227.7015

Deduct the value of 3 years an-	—	}
nuit of 30 <i>l.</i> worth in <i>pre-</i>		48.42366
<i>sent</i> money (105.)	—	

£ 179.27784

The age *sixty*.

For contingent expenses	—	.2625
Admission fine	—	37.8
First <i>quarter's</i> payment	—	5.5
28 ditto worth in <i>present</i> money		}
(102.)	—	111.9627

£ 155.5252

PUBLIC ANNUITANT SOCIETY.

What annuity ought to be received for the remainder of the *present* money paid; the said annuity to *commence* when aged *fifty*, and the age *sixty* to commence at the expiration of *seven* years? (184.)

Age 20, as 1.6987	: 1 :: 51.861	: 30.5298
40, as 5.343	: 1 :: 179.27784	: 33.554 +
60, as 3.959 (145.)	: 1 :: 155.5252	: 39.2847

£103.3685

Age 60 purchases more annuity than age 40 _____	}	£5.7307
_____ than age 20		£8.7549
Age 40 purchases more annuity than age 20 _____	}	£3.0242

In what proportion doth each age *gain* or *lose* by receiving *equal* annuities instead of that which each age *purchases*? (185.)

Divide the total of the three annuities by three.

$£103.3685 \div 3 = 34.45616 \text{ (205.)}$

Age 60 receives by <i>equal</i> dividend	34.45616
Loses <i>annually</i> , what age 20 gains	3.92636
Ditto for the deficiency of age <i>forty</i>	.90216

Purchased _____	£39.28468
-----------------	-----------

3 T

On

PUBLIC ANNUITANT SOCIETY.

On supposition that every one had paid according to the rate of the *last* class, which is that consisting of *eight hundred* members; and that the number at all *ages* are *equal*, it appears that the society might pay to each member £ 34.456 annuity: that is, the member, when aged 50, might receive £ 4.456 annuity *more* than he did the *three* years before.

But *so much* annuity cannot be afforded, for it is known by the abstract that *seven* parts in *eight* have paid *less*: and *all* the payments were found to be, *at a medium*, as follows.

The age of *twenty*.

Contingent expenses	_____	.2625
First <i>quarter's</i> payment	_____	.6906
120 ditto worth in <i>present</i> money (93.)	_____	39.54064
		£ 40.4937

The age of *forty*.

Contingent expenses	_____	.2625
Admission <i>fine</i>	_____	31.2375
First <i>quarter's</i> payment	_____	3.878
40 ditto worth in <i>present</i> money (74.)	_____	109.26364
		£ 144.6416

The

PUBLIC ANNUITANT SOCIETY.

The age *sixty*.

Contingent expences	_____	.2625
Admission fine	_____	31.2375
First <i>quarter's</i> payment	_____	4.2875
28 ditto worth in <i>present</i> money	} _____	87.28
(102.)		
		<u>£ 123.0675</u>

What annuity do these ages purchase, to commence when aged *forty-seven*; or the age *sixty* at the expiration of *seven* years?

Age 20, as 2.211876 (146.) : 1 ::	40.4937 : 18.307
40, as 6.957 (137.) : 1 ::	144.6416 : 20.79
60, as 3.959 (145.) : 1 ::	123.0675 : 31.09+

Divide the total of the three annuities by — 3)70.187

The annuity by *equal* dividend — — £ 23.396+

It appears now that the society can 'pay but £23.396 annuity for a continuance, if it commence when aged *forty-seven*— and if it pays 30*£.* for three years, *according to promise*, then the annuity, afterward, must be still *left*, instead of being increased to 44*£.* as pretended.

From the *present* money paid?
by age *twenty* _____ } 40.4937

Deduct the *present* value of }
30*£.* annuity for 3 years } 15.3952(104.)

Remains to purchase an- }
nuity to commence at 50 } £25.0985

3 T 2

From

PUBLIC ANNUITANT SOCIETY.

From the <i>present</i> money paid } by age <i>forty</i> _____ }	144.6416
Deduct the <i>present value</i> of 30 <i>£.</i> } annuity for <i>three</i> years (105.) }	48.4236
Remains to purchase annuity } to <i>commence</i> when aged <i>fifty</i> }	<u>£96.218</u>

These sums would purchase—Annuities.

Age 20, as 1.6987 : 1 :: 25.0985 :	14.775
40, as 5.343 : 1 :: 96.217 :	18.008
60, as before _____	<u>31.09 +</u>

Divide the *total* of the *three* annuities by _____ } 3)63.873

The annuity, by *equal* dividend, is £21.291

After paying 30*£.* annuity for three years to the ages of *twenty* and *forty*, &c. the society must *lower* the annuity to £21.291, if the age *sixty* receives £21.291 annuity at the expiration of *seven* years. (121.)

Some may wonder why the annuities should be *so small* when the members contribute *more* than most of the other societies—the reason is no other than that **EXTREMELY AGREEABLE CIRCUMSTANCE** which the abstract *boasts* of—
for

PUBLIC ANNUITANT SOCIETY.

for if the members were not entitled to *any* annuity till aged *fifty*, or till after *ten* years for those aged upward of *forty*, then the following annuities would be purchased.

Age 20, as 1.6987 : 1 :: 40.4937 : 23.838
 40, as 5.343 : 1 :: 144.6416 : 27.07
 60, as 2.6529 : 1 :: 123.0675 : 46.6 +

Divide the total of the three }
 annuities by _____ } 3)97.508

The annuity by *equal* dividend }
 is _____ } £ 32.503 +

In this case the age *sixty* would purchase more annuity than is promised, but could receive only £ 32.503, because of the deficiencies of others. (205.)

What annuity can this society afford to pay?

It may pay to the *young*, and *middle-aged* members 30£. annuity when aged *forty-seven* till aged *fifty*: and from that time *during life*, the annuity of £ 21.291 only — and the *old* ages may receive £ 21.291 *during life* at the expiration of *seven* years.

In what proportion to those who *are* members, or to the annuity that the capital stock can afford to pay, must a person *now* pay to become a member?

It

PUBLIC ANNUITANT SOCIETY:

It was found how much the members in the *eighth* class have paid, or rather *will* pay, in *present* money; and also how much will have been paid by *each* member *collectively*; and at a medium, when the *eighth* class shall be *completely* full: from whence the difference is known to be

	Age 20.	Age 40.	Age 60.
To pay in the <i>eighth</i> class	67.2562	227.7015	155.5252
Members will have paid <i>collectively</i>	40.4937	144.6416	123.0675
Difference	26.7625	83.0599	32.4577

A person aged *forty* must give more than £ 83 to be admitted into a society whose members *of that age* have paid each *so much* less than himself.

The annuities are

	Age 20.	Age 4	Age 60.
Purchased by the <i>eighth</i> class	30.5298	33.554	39.2847
Purchased <i>each</i> collectively	14.775	18.008	31.09
Difference in the annuity	15.7548	15.546	8.1947

Or, by *equal* dividend the *eighth* class purchase annuity } £ 34.45616

The society can pay only ——— £ 21.291

Annual loss to the *last* purchasers £ 13.16516

The

PUBLIC ANNUITANT SOCIETY.

The calculations are nearer the true state of those societies that have raised the payments by regular classes, than to the real state of others; because the *number of members* admitted by *each degree of payment* is known at a *certainty*; and therefore the true state can differ in nothing from the calculations, but in the *number of members* admitted at *each age*, which are here *supposed* to be *equal*: but the society can afford to pay *more or less* equal annuity, according as *more or less* have been admitted at the ages of *twenty or sixty*; for the aged members under 60 purchase much greater annuities than the *young ones*.

It might be deemed injustice done to this society if the following should be suppressed, which the directors confidently assert—"We have the satisfaction of knowing, that when we become ENTITLED to the annuity, there will be a very ample fund to PAY it with."

THE

THE
RATIONAL SOCIETY,

Being a new Institution, (*m*)

UPON A PLAN INCONTESTABLY DURABLE

(182.)

MEMBERS when *fifty* years of age will be entitled to *thirty pounds* per annum *during life, and clear of all deductions*—Such as are admitted at *forty* years of age, or upward, must continue members *ten* years before entitled to the annuity—The conditions are,

To pay ———	Premium	Half yearly	Half yearly <i>with- out</i> premium
Age of <i>twenty</i>	0.	1.175	1.75
<i>forty</i>	8.4	4.4	4.9
(181.) <i>sixty</i>	0.525	3.05	3.1

What

(*m*) It is not *dated* at what time it was instituted.

THE RATIONAL SOCIETY.

What doth each age pay in value of *present* money? (183.)

The age of *twenty*.

First <i>half</i> year's payment	—	1.175
90 ditto worth in <i>present</i> money		33.6376 +
		£ 34.8126

The age of *forty*.

Premium	—	8.4
First <i>half</i> year's payment	—	4.4
20 ditto worth in <i>present</i> money		61.986 +
		£ 74.786

The age of *sixty*.

Premium	—	0 525
First <i>half</i> year's payment	—	3.05
20 ditto worth in <i>present</i> money	}	39.0126 +
(101.)		
		£ 42.5876

What annuity ought to be received according to the value of the *present* money
id?

THE RATIONAL SOCIETY.

Answers. (184.)

Age 20, as 1.6987 : 1 :: 34.8126 : 20.494 +

40, as 5.343 : 1 :: 74.786 : 13.997

60, as 2.6529 : 1 :: 42.5876 : 16.053

Divide the *total* of the three }
annuities by ——— } 3)50.544

The annuity by *equal* dividend is £16.846

Age *twenty* purchases more an- }
nuity than age *forty* — } £ 6.497

Ditto ——— than age *sixty* } £ 4.441

Age *sixty* purchases more an- }
nuity than age *forty* — } £ 2.056

In what proportion doth each age
gain or *lose* by receiving *equal* annuities,
instead of that which each age purchases
separately?

Age twenty

Purchases ————— 20.494

Receives by *equal* dividend ——— 16.846

Annual loss £ 3.648

Age forty

Receives by *equal* dividend ——— 16.846

Purchases ————— 13.997

Annual gain ——— £ 2.849

Age

THE RATIONAL SOCIETY.

	<i>Age sixty</i>	
Receives by <i>equal</i> dividend	—	16.846
Purchases	<u> </u>	<u>16.053</u>
	<i>Annual gain</i>	<u>£ 0.793</u>

What do members pay in value of *present* money, and *without* admiffion fine?

Age of *twenty*, as *before* — £ 34.8126

	<i>Age of forty.</i>	
First <i>half</i> year's payment	—	4.9
20 ditto worth in <i>present</i> money		<u>69.029</u>
		<u>£ 73.929</u>

	<i>Age of sixty.</i>	
First <i>half</i> year's payment	—	3.1
20 ditto worth in <i>present</i> money		<u>39.652</u>
		<u>£ 42.752</u>

The purchased annuities are

Age 20, as 1.6987 : 1 :: 34.8126 :	20.494 +
40, as 5.343 : 1 :: 73.929 :	13.836
60, as 2.6529 : 1 :: 42.752 :	<u>16.115</u>

Divide the total of the three annuities by } 3) 50.445

The annuity by *equal* dividend is £ 16.815

THE RATIONAL SOCIETY.

The age *twenty* purchases more } £ 6.658
 . annuity than the age *forty* }
 Ditto ———— than the age *sixty* } £ 4.379
 Age *sixty* purchases more an- }
 nuity than the age *forty* } £ 2.279

Is the table *with* admission fine *equal* to that *without* admission fine?

	Age 20.	Age 40.	Age 60.
Paid <i>with</i> admission fine	34.8126	74.786	42.5876
<i>Without</i> ditto ———	34.8126	73.929	42.752
Difference ———	0.0	.857	.1644

What annuity can the society afford to pay?

To the *equal* annuity *with* premium 16.846
 Add the *equal* annuity *without* ditto 16.815

Divide the total of the *two* annui- }
 ties by ————— } 2) 33.661

The *medium* of the *two* annuities is £ 16.8305

If the number of members is *equal* at
all ages, and the number of those who
have paid, and of those who have *not*
 paid premiums be *equal* also; then the
medium

THE RATIONAL SOCIETY.

medium of the two *equal* annuities is the answer——But, to make the PLAN INCONTESTABLY DURABLE, the society *cannot* pay £16.83, much less 30*l.* annuity, for the reasons noticed as *memorandums*, page 182.

It is not known to the author whether any of the members of this society have been admitted on *lower* terms than those mentioned in this abstract or not.—That society in which the *first* members pay *equal* with the *last*, is certainly the most honest in that respect.

THE

THE
FRIENDLY SOCIETY
OF
ANNUITANTS,

(Commenced April 12, 1771.)

THIS society proposed *half* the same annuities as the *Public Annuitant Society* does; and published tables for the *four* first hundreds to be admitted on *just half* the *same* terms as the said society does: nor did the *twenty-four first* members forget to *except themselves* in paying the admission fine.—As there was no difference in the proposals of this society (except the proposing half the annuity per half price, and admitting members by tables *without* admission fine, may be called such) *it may VERY WELL be urged by some* (as they expressed themselves) *that there are a sufficient number of these societies established, without setting up NEW ones.*—This might have

THE FRIENDLY SOCIETY.

have been urged *in their case* indeed, if the directors had not added—*We have the authority of many respectable persons to declare* (THAT BY THE ENCOURAGEMENT OF THE PUBLIC) *the fund will be sufficient to answer the purposes for which it is raised, namely, for a comfortable support or assistance in old age.*

Men who think themselves, and commonly pass in the world by the appellation of, **VERY HONEST**, would immediately see the *injustice* of the *disproportion* in the payments by the *last* and the *first* class in the *Public Annuitant Society*, which are these:

	Age 20	Age 40	Age 60	Admission
Quarterly payments } by 8th class	£.1 3 0	6 10 0	5 10 0	37 16 0
Ditto by the 1st	£.0 10 0	2 2 0	3 2	26 5 0

Yet such *very honest* men, who see very great objections to enter according to the terms of the *eighth* class, find none, but rather an *inclination*, to be admitted by the *first*—Hence this scheme would very naturally suggest itself to such *very honest* men, instead of paying *more* than others, to establish a *new* society, in which the **INSTITUTORS** would pay *less*, and—**BY THE ENCOURAGEMENT OF THE PUBLIC**—*the fund might be sufficient to answer the purposes for which it was raised.*

That such wishes glide imperceptibly into the

THE FRIENDLY SOCIETY.

the hearts of *very honest* men is an undoubted fact well known to the author, how much soever it may be disputed in the world; for the very same scheme has actually, though inconsiderately (it may be read with *doubt* or *astonishment*) been proposed to him by some *really* VERY HONEST MEN, (*n*) with a *very honest* additional improvement, which was to endeavour to obtain the sanction of some *respectable* name as PRESIDENT (*o*) to the society, by which it was hoped the ENCOURAGEMENT OF THE PUBLIC might have been *decoyed*——So much does SELF-INTEREST dim the judgment, that we can scarcely distinguish *common* justice, when we are to judge between SELF and *others*.

But, by whatever principles the institutors of this society were actuated, it may be said to their (it is hoped, *not* PECULIAR) honor, that Two *honest* men were found among them—men both *honest* and *sensible*, whose names have been read, with much
appro-

(*n*) The author pleads guilty, if a charge of a similar nature, from a passage in his Letter to the Secretary of his Society, (page 233, Note) be brought against himself, and submits to the verdict of an impartial public; but hopes every one of the jury will examine his own heart, and condemn or acquit him according to its feelings in the like circumstances.

(*o*) See The Public Annuitant Society, Note (*k*).

THE FRIENDLY SOCIETY.

approbation, in the Public Advertiser; and Whitehall Evening Post; from which last paper the said two *worthy* persons are introduced here, and their proceedings, with those of the *majority* of the members, inserted for an *example* recommended as worthy to be *imitated* by the *directors* and *majority* of the members of *other* societies, *that need reformation to the full as much.*

No. 8, Dove-Court, Lomb'rd-Street.

S I R,

BEING convinced by this letter of Dr. Price's, that the plan of the Friendly Society of Annuitants was not sufficient to answer the end proposed, it was read to the society at a general meeting held on Friday last the 26th inst. A majority of the society having agreed to dissolve the society, we crave the favour of you to publish this letter of Dr. Price's; and we will be your security for so doing, and are

Your most humble servants,

EDWARD HINTON,

JOHN MARGARY,

Directors of the Friendly Society,

October 29, 1771.

Copy of a LETTER from the ingenious Dr.
PRICE, to the *Friendly Society of Annuitants.*

S I R,

Newington-Green, July 13, 1771:

I received a few days ago a letter signed with the initial letters of a name, and together with it, the printed account of the scheme of the Friendly Society of Annuitants. The design of this letter is to acquaint me that some in this society, sensible of the insufficiency of the scheme upon which it is formed, are inclined to give it up; but that the older members, and a few of the younger members, seem resolved to carry it on; he therefore desires that I will write to you an account of my sentiments concerning it. This is the true reason of the trouble I am now giving you.

3 X

The

THE FRIENDLY SOCIETY.

The value, to a person now *forty* years of age, of 9l. (*p*) per annum for life, after attaining to *forty-seven* years of age, and 22l. per annum for life, (or 13l. per annum more) after attaining to *fifty*, is (according to the probabilities of life in Dr. Halley's Table of Observations, and supposing money improved at three and an half per cent. compound interest) at least 21l. 10s. in annual payments, till the age of *forty-seven*; the first payment to be made at admission.

Any person who will calculate in the manner I have directed, in Question 6th of my Treatise on Annuities, or in page 114, may make himself as sure of this as he can be of any thing.

The payments required by the Friendly Society according to the tables for admitting members with admission fines, do not, in the first of these tables, exclusive of the admission fines, come up to a fifth of this value, nor in the fourth to a third.

I leave you, therefore, to imagine in what mischief such a scheme must end. By employing the contributions of the younger members in paying the annuities that will become due to the older members, these last may (if the younger member should continue so infatuated as to go on with their contributions) receive their annuities for several years. But, should the younger members either desert, or insist on some security against such a robbery, the consequence will prove, that even the older members, after receiving their annuities a few years, will lose them, and the disappointment arising from hence will be the more grievous to them, because it will come at a time when they will stand in most need of help, and be attended perhaps with the remembrance of having been led to neglect making other provisions for themselves, by depending on this. The same might be said of all the younger members, were it possible for the scheme to go on till they commence annuitants. But indeed this is not possible, for the disappointment of the older members would soon ruin the society, and it would be necessary perhaps to have recourse to Chancery to decide the disputes that would arise about the division of that part of the fund which would then remain.

In the printed account of the plan it is declared, on the authority of many respectable persons, that the fund will be sufficient to answer the purposes for which it is to be raised. I should be glad to know who these persons are, and on what principles they calculated. They must surely have discovered some new ways of improving money, and some new laws by which the waste of human life is governed.

There

(*p*) The annuity was to have been 15l. but the *quarterly* payments were to be continued until entitled to 25l. which payments amounted to 6l. yearly in the class of the third hundred, and reduced the 15l. annuity to 9l. as above.

THE FRIENDLY SOCIETY.

There are none among my acquaintance who do not believe all these societies to be bubbles, which must do great harm; and it is very melancholy to see the disposition which prevails to encourage them. I will only add, that I think this society will be particularly inexcusable if it does not break up. It has had timely warning; and being but just established, it can find no difficulty in returning the money it has received.

I am, Sir,

Your very humble servant,
RICHARD PRICE.

P. S. Excuse some haste. I have given the value of the annuity promised by the Friendly Society for the age of *forty* only as a specimen; the deficiencies in the younger ages I find to be greater. The payment required at *twenty* does not come up, when the number of members is 400, to a quarter of the true value of the annuity.

Though it may be a duty rather incumbent on the author to beg pardon of Dr. Price for making so freely many extracts from him, and of the public for inserting them; yet there is one place more, in the 36th page of his Supplement, that describes so *justly* the ACTUATING principles of the societies, that the author must entreat permission to quote it here.

“ The principle by which they are up-
“ held is base and iniquitous. The *pre-*
“ *sent* members believe that the schemes
“ they are supporting will last their time,
“ and that they shall be gainers; and as
“ for the injury done to their successors,
“ it is at a distance, and they care little
“ about it.—In conformity to this prin-
“ ciple, the founders of these societies
“ begin low; *so* low; as not to require,

3 X 2

“ per-

“ perhaps, a *fourth* or a *fifth* of the va-
“ lues of the annuities they promise.
“ Afterwards, they advance gradually, just
“ as if they imagined, that the value of
“ the annuities was nothing determinate,
“ but increased with every increase of
“ the society. But, as no ignorance can
“ believe this, the true design appears to
“ be, to form soon as large a society as
“ possible, by leading the unwary to en-
“ deavour to be foremost in their appli-
“ cations, lest the advantage of getting
“ on the easiest terms should be lost.—
“ It is well known, that these arts have
“ succeeded wonderfully, and that, in con-
“ sequence of them, these societies now
“ consist of persons who, for the *same*
“ annuities, make high or lower pay-
“ ments according to the time when
“ they have been admitted; and the gene-
“ rality of whom, therefore, must know,
“ that either more than the values
“ have been required of the members
“ last admitted; or if not, that they are
“ themselves, expecting considerable an-
“ nuities, for which they have given no
“ valuable consideration, and which, if
“ paid them, must be stolen from the
“ pockets of some of their fellow-mem-
“ bers. What scenes, therefore, of *dis-*
“ *honesty* on the one hand, and of un-
“ happy *credulity* on the other, are these
“ societies!—There are many honest men
in

“ in these societies, who, having through
“ misinformation had the misfortune to
“ enter into them, now repent, and would
“ be glad to withdraw; but, having
“ made considerable payments which they
“ cannot get back, they are forced to
“ go on with further payments, in order
“ to avoid losing all their former ones.”

The author having now exerted the utmost of his abilities to explain the true state, and to enforce a reformation of the schemes of THE SOCIETIES FOR THE BENEFIT OF OLD AGE, here takes his leave of them, with earnest wishes that their managers, directors, &c. may take the same into serious consideration, as the subject is not so extremely intricate as they may have imagined; and it certainly is of great IMPORTANCE— Those gentlemen who have inadvertently engaged in schemes which they did not thoroughly comprehend, and who are now convinced that they must prove inadequate, and therefore impositions on the public, will need no further argument to persuade them to relinquish the undertaking, or at least REFORM their plans to RETRIEVE their characters; which conduct may frustrate the designs of the CRAFTY and less HONEST.

The E N D.

A D-

A D D E N D A.

ARTICLE *the First.*

THOUGH the author has the greatest deference for most of the opinions of Dr. Price, yet he cannot entirely agree with him, when he says, (*a*) “ That were these
“ societies indeed formed on durable plans;
“ there would be reason for subjecting them
“ to some regulations (*b*). In all of them
“ the annuities are to commence MANY
“ years *before* OLD age; and in some, at al-
“ most every age. Such annuities, were
“ they to become very common in a state,
“ might have a bad effect, by weakening
“ the motives to industry, and promoting
“ dissipation and idleness.

If FIFTY be not *extreme* old age, yet the author feels for those who are obliged to continue toiling after attaining to such an age, and wishes them to obtain repose from their labour, and some leisure for reflections superior to the cares of this life.—As to the latter
part

(*a*) Supplement to Dr. Price's Observations, &c. page 38.

(*b*) From what Dr. Price says before, these regulations are supposed to be *meant by the interposition of the LEGISLATURE.*

A D D E N D A.

part of this quotation, the author cannot possibly, as yet, subscribe to it.—However, without aspiring to political knowledge, he leaves it to superior capacities—yet, on supposition that most people will think, *until* LEGISLATURE *does interfere*, that they have a right to dispose of their money in a way the most suitable to their circumstances, a scheme for GRANTING ANNUITIES ON SURVIVORSHIP seems to be the most *advantageous* and *equitable* of any for individuals—*advantageous*, because there is no means *so easy* and *less* hazardous of improving money, which is never lost till the time it is no longer needful—*equitable*, because the deceased leaves his share in the society for the benefit of *those* who had associated to benefit *him*, had it been *his* lot to have *survived them*—and yet some may have their reasons for representing the purchase of *life annuities* as a *selfish* scheme, and prejudicial to the annuitant's relations and friends, because the money expended in the purchase is totally lost *to them*, instead of being shared by heirship or legacies: but the reason against the action must be allowed to be founded on the same SELF INTEREST; which, notwithstanding some vain pretensions to the contrary, is so inherent in our nature as to be the spring (however imperceptibly) of most actions—Without entering too deeply into the merits of the question,

A D D E N D A

question, or to decide *for* or *against* it, it may be well supposed, that those who have relations, &c. would still act as *favorably* to them, though the following scheme should take place, either by leaving their substance as heretofore; or, it may be, more beneficially, by providing *annuities* for *them* in the SOCIETY; the establishment of which, it is hoped, would not tend to alter the natural affection of a husband, father, brother, cousin, or a friend.

Not only such objections may be obviated, but also some reasons may be offered in favor of the establishment of a society for granting annuities, which may prove to be not a little cogent.

There are many, of genteel family, who have been bred to neither trade nor labour, and who would pine at the thought of servitude, with such a mediocrity of fortune as sets them above either, and yet not able to subsist agreeably without: this is a situation next to indigence itself, and is truly pitiable. Such might double or treble their income (according as their age may be) by purchasing *immediate* annuities; or, increase it *very greatly* if they purchase annuities to *commence* some *years hence*; and, in the hope of enjoying it, might reconcile themselves to means, that would have been otherwise disagreeable, of present subsistence.

At

A D D E N D A.

At what an *easy* rate PARENTS may provide for their *offspring* has been seen (165.), and merits the attention of the *Clergy*, *Officers*, *Placemen*, and *all* those who, having genteel incomes *for life*, educate their family accordingly; although, when they die, they have nothing to leave them but *distress*.

Many that labour hard, or spend many years in servitude, to acquire two or three hundred pounds, often try to ease their burden by entering into *trade*; and too often, through many difficulties and want of knowledge, *fail*, and are reduced to *want*, at an age that can neither support nor relieve it:—had such provided for OLD AGE by annuities, they might have enjoyed their latter days, secure from want, in peace and comfort.

The pleasing prospect of future repose may be a spur to the industry of both mechanics and servants, who have now nothing but the gloomy prospect of toiling to life's end:—it must likewise promote frugality among this class of people; who, when they know that *much* may be purchased for a *little*, will be more *sparing* than when they could hope for no more than the *simple* interest of a few hundreds, though they hoarded to the utmost; and it might further excite diligence and complaisance in their several callings.

These are some of the beneficial considerations to *individuals*—and these may *jointly* contribute to the *advantage of the PUBLIC*, by reducing the number of the POOR.

A D D E N D A.

The *commencement* of the annuity should undoubtedly be left to the discretion of the purchaser; (*c*) for it has been seen (139.) that it is the *same* thing to the society whether it be *immediate*, or at *any time hence*: therefore there is no need to confine it to any age; for the difference of the values will naturally influence the purchaser to defer it to as *distant* a period as may suit his *circumstances*; or, at least, choose a medium between them and his *inclinations*.

Taking it for granted, that annuities are both of private and public utility, yet how are they to be obtained? If application be made to private persons,—they must make an advantage worth their notice, consequently more than the *just* value must be paid—or, if there are some most willingly ready to grant the full annuity that the purchase money and age requires, the very offer must create a suspicion of doubtful circumstances, and that the payments will be precarious—or, if undeniable *security* be given on these terms, such examples are too rare, and not sufficiently

(*c*) There have been writers (of callous hearts indeed) who have represented it as fully sufficient, if the industrious poor acquire subsistence that will barely preserve the body and soul united; and *five* or *six* years ago our daily papers were stuffed with demonstrations how *luxuriously* they might *regale* for three or four pence each per day—and now some there are, who would not permit them to cease from their labours until they find rest in their graves, which very many do (wearied out with fatigue) long before the age of *sixty*.

A D D E N D A.

sufficiently *general* for *every one* to come at readily (132.)—It may be said, there are *several* SOCIETIES already established for the very purpose—True—but they are every one in an unsettled state; and under the direction and management of persons, who, to be HONEST men, must be *ignorant* that they are inveigling, by delusive promises, the thoughtless and unwary, to give away a great part of their money, to supply the deficiencies of the earlier members, or of different ages—From all which considerations it appears to be *necessary* to institute a NEW *society*, on a plan so *different* from the rest, as to be UPRIGHT and IMPARTIAL.

The author, with the utmost deference to better judgments, most humbly begs leave to submit to PUBLIC consideration the following Plan, of AN IMPARTIAL AND GENERAL SOCIETY OF ANNUITANTS; *which proposes*

I. That the society may grant annuities to the amount of *any* sum (above 10*l*.) which the purchaser requires.

A less annuity than 10*l*. would be but of little service to the annuitant, nor worth the attention of the society.

II. That the annuity may be *immediate*, or to commence at the expiration of *any* term of years, at the option of the purchaser

A D D E N D A.

chafer—but, *at present*, subject to the following restriction.

At the commencement of the society, it may be prudent to allow some time for its establishment, that a *number* may be collected, and a *capital joint stock* be accumulated before any payments be made, after which time the annuities may be IMMEDIATE, and till which time they are continually approaching to it:—Thus suppose the time limited for the earliest annuities to be Christmas, 1776: then the proposal means, that at this time an annuity may be purchased to commence in less time than *five* years; and three years hence an annuity may be purchased to commence in less than *two* years; and at Christmas, 1776, and always afterwards, an IMMEDIATE annuity may be purchased. (d) In which space of *five* years time, the INSPECTOR of the society might have opportunity of making *practical* observations, and deliver a printed report of the state of the society to every member. (65.)

III. That a purchaser may either pay the whole value of the purchase in *present* money, or any part of it, *not less than one tenth*, that is most convenient to himself, and the remainder by *half yearly* payments.

Case (156) shews clearly that it is the *same* to the society whether *present* money, or *half yearly* payments, be paid: but it is worth the members consideration how much

(d) Such as are desirous of *immediate* annuity without hazard-
ing any term of reversion, may be tempted to wait the expira-
tion of these years, rather than become members at *this* time.—
The difference is, that if age *forty* becomes a member now, to
commence annuitant in *five* years, he will by the table (139.)
make above 12*£*. per annum of the purchase money: but if he does not enter till aged *forty five*, and then purchases
an *immediate* annuity, he can by the table (131.) make no
more than *£*. 8 10 8—excepting by the increase of his prin-
cipal, of which a similar case is given (140.)

A D D E N D A.

much he will have paid *more* by *half yearly* payments than by *present* money, if he survives to become an annuitant: the difference may be seen, page 154, and the discount *per cent.* in the Note, page 159— it is not to be supposed that the half yearly sums can be improved *singly* so advantageously as when a number of them are collected in the fund of a society; and when the member becomes an annuitant, the chance of mortality *has been* survived.

IV. Such as pay *half yearly* sums must be punctual in their payment upon, or before, the day it becomes due; on penalty of a fine for the first neglect; which fine to be doubled every fortnight until the payment and fines be paid.

But as *humanity*, as well as *honesty*, forbids the society to thrive by the *poverty* or *distress* of any one; such member as shall plead his INABILITY within *six months*, shall be entitled to such annuity as the money by him already paid doth purchase; provided that, on account of his breach of payments, he shall never afterwards be admitted to purchase any more annuity in the society, otherwise than by *present* money.

V. That the institutors shall determine whether the annuity shall be the sum *certain*, or according as the state of the society shall *admit* of.

If the annuity be certain; that is, *never less* than the nominal value; then, on account of the memorandums (page 182.), an addition must be made to the purchase money, either by a proportionable admission fine or otherwise, to secure a sufficiency to enable the society

A D D E N D A.

society to preserve the FAITH of *promises*: for the table of the values of the annuities is intended by the author to be according to Mr. *Simpson*, with some small alteration, as the Calculations have been all along: which table differs but little from Dr. *Price's*, of the true Probability of Life in London (118.), except that it requires *more* for the older ages—— From the memorandums and other considerations that have been noticed, it is more than probable that the values by the table would prove *deficient*, and therefore must be subject to the REGULATION mentioned (65.) or else the deficiencies must be supplied by *fines* at admission.

VI. That the INSPECTOR keep a clear account of the admission fines, distinctly and separately from the values of the lives by the table: and in case there remain a surplus thereof, after supplying the deficiencies of the table, such surplus to be shared among the annuitants by YEARLY dividends.

VII. That no one member at the commencement shall be admitted for a *less* value than by the rates of the table (which will be prepared), by which all future members will be admitted likewise (*e*).

That is, every member from *first* to *last* will pay by the *same table*, and the purchase differ in nothing but according to the *age*; excepting it should be thought just to make some small difference in the admission fine, on account of the remark in the Note, page 243.

VIII.

(*e*) This may seem to be a proposal very discouraging to the institutors: but gentlemen who are HONEST, (and *no other* are wished to become members) will be fully satisfied to receive the full

A D D E N D A.

VIII. To prevent, as much as may be, *ALL suspicions* of any embezzlements—an exact account of all monies *received, improved, and expended*, shall be kept *constantly open* for the inspection of *any* member.

For which purpose *duplicates* of books to be prepared; the one to be kept undamaged, the other free for the examination of such as choose to inspect it—Every member, intending to examine the accounts, is to signify his intention to the *Secretary*, who must appoint him, in writing, to take, and admit him to employ, the *first* two days that are not pre-engaged by another for the same purpose.—If it should be found that by this mode the book should not be *constantly* engaged; then, *any* member may inspect the said book at *any time*, within the office hours, that it is found so disengaged.

IX. That there may be no cabal, nor any party formed to continue or rechoose a few select members in offices to which *all*, who are *eligible*, have an equal right to be admitted; and that there may be no *reason* to *suspect* such proceeding—One half of the *first* managers to go out, and an equal number to be balloted into office at the expiration of one year: and at the expiration of the second year, the other half to go out of, and
new

full annuity which they purchase; and which has been repeatedly seen is *more* than they could do in the *other* societies, because that the earlier members who purchased *less* annuities, would nevertheless share *equally* with them, which of necessity reduces their annuity in proportion, similar to Case (205.)

A D D E N D A.

new ones to be balloted into, office.—No manager to continue in the office longer than two years; nor can *ever* be RE-CHOSEN into the same, in preference to any one who is eligible, and hath *not* served the office, and is nominated to be balloted for by any *one* member present.

Should this proposal make the members *unwilling* to serve the said office; such as are duly qualified, and are elected into it, and *refuse* to serve it, must pay a fine; and should there be found a deficiency of persons willing to undertake the office, then the *first* managers to be elected again, and each according to the number of his admission be elected, and *fined* on refusal of acting.

X. That, when it shall be thought by the majority of members, that the society consists of a sufficient number to determine upon such an important affair, a general meeting shall be held, to take into consideration every proposal; and such as shall be UNANIMOUSLY approved of as absolutely requisite for the preservation and advantage of the society shall be made so far UNALTERABLE, as that all present shall sign an instrument, purporting that he acknowledges and agrees that the said proposals shall be thenceforth unalterable articles; and that he freely resigns up all his right and privilege to move, or vote for, or by any means to change or disallow of them, excepting that
every

A D D E N D A.

every member should as *unanimously* agree to an abrogation. The articles so agreed to, to be published; and every person must thenceforth sign the said instrument upon admission, as a member of the society (*e*).

XI. That the IMPARTIAL AND GENERAL SOCIETY of *Annuitants* be divided into *four*, or more CLASSES, each of which shall grant annuities as follows.

First class, from 10*£*. to 40*£*. annuity.

Second class, from 40*£*. to 160*£*. ditto.

Third class, from 160*£*. to 640*£*. ditto.

Fourth class, from 640*£*. to 2500*£*. d°. (*f*)

The said classes (though comprehended under *one* name) are to be as *distinctly separate* from all connexion or interference, as though they were different societies, having separate Meetings, Managers, and Members (*g*); but all to proceed upon the same impartial and extensive plan (*b*); and there-

3 Z

fore

(*e*) The articles of the deed of settlement in some of the societies are in the next degree to *none*, being expunged, altered, or evaded, at almost every meeting.

(*f*) By this means any annuity from 10*£*. to 2500*£*. may be subscribed for, to accommodate the infinite variety of different circumstances.

(*g*) Excepting that a person subscribes to more than one class.

(*b*) There are *two* advantages in dividing the society into separate classes—*First*, The members *affiliate* according to their circumstances; so that the HIGH are not ranked with, nor have
any

A D D E N D A.

fore, though the classes are in fact different societies, yet they may be properly included in *one* GENERAL title.

XII. That an INSPECTOR be appointed, whose employment shall be to ASSIST the managers, and to prepare tables, shewing the *present* money and half yearly payments to be paid in all cases at all ages—to keep an exact account of the proportion of the decrements or deaths that shall happen in *practice*, to what is *supposed* by the table would happen—to adjust the annuities accordingly; as also the annual dividend of the admission fines, *if any*—to publish yearly a true and clear state of the society—and to supervise, methodise, and correct, if needfull, all books belonging to the society; for which purpose the *secretary* to be under his direction.

The

any connexion with INFERIORS—*Secondly*, and *chiefly*, the annuities are preserved nearer to an EQUALITY with each other, so that one member can have no more than *four* times the value of any other in the class: for it were best that every member should receive an annuity of the *same* value; but as that would not suit *all* degrees, *some* difference must be admitted; but more than *four* times the value of one to another cannot be prudently allowed: the less the difference, the more would *chance* incline to an *equality*; and there needs not many words to shew how inconvenient it would be for some members to have 2500*l.* annuity from a society that had, for the *most* part, granted only 10*l.* If these large annuitants should be *long* survivors, they would be a fore burden, if not the RUIN, of such a society.—It is true, that, were they to die *soon*, they would greatly benefit the society; but the intention of association is not to *hazard* much, but rather to *reduce* all chance of gain or loss in the society in general to an *equality* as nearly as possible.

A D D E N D A.

XIII. The proposals and articles of the *other* societies to be examined, one by one, at the first general meeting; and such as are generally approved as *reasonable* and *honest* to be adopted,—the others to be rejected with the CONTEMPT they have *long* deserved.

The author does not propose further, nor mean to impose these as Laws; but to be altered, improved, or rejected, as shall seem best to those gentlemen who shall think it expedient to institute a NEW SOCIETY upon such sound principles, as may recommend itself by its *extensiveness*, *impartiality*, and *security*.

Such gentlemen as please to signify their intention of INSTITUTING the said society, by letter (post paid) to the author, inclosed to the publisher, specifying their *address*, *age*, the *annuity* they would subscribe for, the *time* at which they choose the annuity should commence, and *what* sum they would pay in present money, will receive an *answer*, or be waited on *personally* as soon as the tables are prepared; and a meeting will be appointed as soon as a sufficient number may be *depended* upon to form the society IN EITHER CLASS; when they are to select and appoint Managers, Trustees, Auditors, Inspector, and Secretary, among themselves (*i*).

(*i*) No other than this *general* application has been made by the author.

A D D E N D A.

ARTICLE *the Second.*

THE Cases 123. and 127. suggest a very easy method to find the present value of an annuity; which, though omitted then, lest the intricacy that attends it might puzzle and beget doubt, deserves *now* particular consideration.—If the tables of mortality were sufficiently correct (why this is questioned will appear presently) to ascertain *how long* time the age has an *equal* chance to survive; then the *present* value of 2*£.* annuity is immediately found by *inspection* of the Table of Reversion of Annuities *certain.* (49.)

Thus suppose the age has an *equal* chance of living *fourteen* years, and to receive 1*£.* *half* yearly; the equal chance then is, that he will receive 28*£.* Look in the table at the same number of *half* years, that is, 28; and opposite thereto is the value in *present* money, viz. *£.* 21.9869, which will exactly pay 1*£.* *half* yearly for *fourteen* years, in the same manner as *£.* 21.788 in the Table (Case 123.) did pay *nearly* the same time; and therefore is the true value of the said annuity; which, on account of the *equality* of chance, is deemed as *certain.*

This shews likewise in what time the purchaser will receive back his own money with the full interest; which should be always

A D D E N D A.

ways in the time that he hath an *equal* chance to live.—To make this clearer, if needful, Suppose A. applies to B. to grant him 2*£*. annuity: B. finds A. has an *equal* chance, according to his age, for living *fourteen* years; and therefore offers him the said annuity on condition that he pays him down the value in *present* money of *fourteen* years *same* annuity *certain*, which has been seen to be *£* 21.9869.—A. objects to the proposal, observing that during the term of equal chance of life, that is, for *fourteen* years, he should receive *only* his *own* money; and whatever time he should die before the said term, he should leave a proportionable part, of his *own* money, for the gain of B.

—B. acknowledges the justness of the observation, but in reply says, Pray, is there not as much probability of your living as much, *and more, beyond* the *equal* chance of life, as of dying *before*? (127.) and in case you should live to receive the whole of your own principal with the interest, *whose* money must supply your annuity afterwards to life's end? Can you *reasonably* desire to have a *chance* of receiving *my* money, without allowing me a chance likewise of receiving a proportionable part of *your's*? Was you to die immediately after the agreement and payment, it is true I should gain *£* 21.9869 of you: but if, on the contrary, you should live, as there is a *possibility* you may, to an
age

A D D E N D A.

age above 100, then you certainly would receive not only your *own* money with the interest, but also *more* than *equal* to that sum from me and my heirs ; and you may inform yourself (by Case 125.) that you would have an advantage of more than is just, by as much as there is a chance of living beyond the age of 91 $\frac{1}{2}$.

This case is so clear, that A. must accept of the proposal, if he would not expose his ignorance in not understanding it, or his covetousness in desiring *more* than the real value : and this may be considered as a plain, short, and certain method of finding the value in *present* money of an *immediate* annuity.

But the values of *immediate* annuities *for life* have also been found in *another* way (131.) and proved, almost beyond a doubt, to be right ; both by the manner of obtaining the stock, and by the proof of the sufficiency of that stock to pay annuities out of it to the end of life.

Yet with all this certainty, when these methods are examined by an infallible criterion, there will be found a mystery very humiliating to positiveness—the criterion is, that if *both* are right, they will *agree* with *each other*: the mystery is, that upon comparison they do *not* agree; and therefore *both* may be *wrong*, but both *cannot* be right.

The value of 1*£.* *immediate* annuity for life

A D D E N D A.

life for the age *fifty*, by Mr. *Simpson's* Table (131.) has been found to be £10.894; which is proved, page 107, to be insufficient to pay *fourteen* years complete; whereas by the Table (116.) 204 are living aged *fifty*, and 102 aged *sixty-four and a half*; and therefore the equal chance of life is *twenty-nine half* years, and opposite *twenty-nine* in the Table of Reversion (49.) is £22.5916; the *half* of which is £11.2958, and £0.4018 more than the value found by the other method (i).

Some remarks of the same kind have, it seems, been made before; for Dr. *Price*, in his *Observations*, page 175, Note, calls it an “error in those who maintain, that the value of a given life is the same with the value of an annuity certain, for as many years as the life has an equal chance of existing;” and says, “Were this true, an annuity on a life, supposed to be exposed to such danger in a particular year, as to create an equal chance, whether it will not fail that year, would, at the beginning of the year, be worth (k) nothing

(i) The difference for the same age, by the other three calculations, is

	London Bills	Dr. Halley's	Dr. Price's
Value by Table (49.)	11,2892	12,73118	11,5929
Ditto by (131.)	10,7459	11,6356	10,9045
Difference	0,5433	1,09558	0,6884

(k) It may be questioned whether or not this is a case in point; the supposition of an equal chance of life is founded on general events, and taken from the table where *half* only are found

A D D E N D A.

*“ nothing, though supposed to be sure of
 “ continuing for ever, if it escaped that danger
 “ —But there can be no occasion for taking
 “ notice of an opinion, which has been em-
 “ braced only by persons ignorant of MATHE-
 “ MATICS, and plainly unacquainted with the
 “ genuine principles of calculation on this
 “ subject.”*

Notwithstanding the respect due to the authority of so great a master, if it is granted that a person hath an *equal chance* for living a certain term of years, it must be very difficult to overcome an opinion that will immediately and naturally root itself in the imagination; which is, that the said person hath an equal chance of receiving so many years annuity; and so many years annuity must be worth the *present* money as given by the Table 49. if according to its rate of interest.—But it will be said, that it can *not* be so, because it does *not* AGREE with the values found in the Table (116.) and it is for this reason, *as the writer* SUPPOSES, that MATHEMATICIANS explode the said opinion: if the following should induce them to resume the consideration of the subject, it is imagined they will find that they have determined rather too hastily upon

found living of a certain number that *were* living some time before, and consequently must suppose that *one half* are dead also; *each* of whom at the *last* moment had a chance *particular* to himself, but which did not extend to, or farther affect, the *general* chance, but as *one* in a large number.

A D D E N D A.

upon it.—It has been observed, in the Note, page cxvi. of the Introduction, that the decrements (being made arbitrarily, 51.) are not in such exact proportion by *whole* numbers as if they had been given *decimally*: for let them be imagined at age 40 in the Table (62.) to be ten *thousand* instead of *ten*; it is not probable that in the next year the decrement would be exactly *ten thousand* again; but rather *more*, because they have been *increasing* annually from the age of *sixteen* or *seventeen*:—neither is it probable that they would *decrease* in the sudden proportion of *nine thousand* from *ten thousand* at age *forty-four*, but rather by some *intermediate* number; and so on to the end of life.—Is it not possible then, that some such divisions might be found, as would *contract* the term of the equal chance of life, and at the same time *increase* the values of each age in the tables, until they coincide in *both* cases? If there is, then every *deviation* from it would produce the differences now under consideration; and, if the *excess* be in equal proportion to the *defect*, the true value and term may be known by the medium between both.

If the Table (62.) be examined, the equal chance of life for each of the *following* ages will be found *nearly* as in the *second* column; and each equal chance of life has a right to

A D D E N D A.

receive *half* (1) the sum that is opposite the number of those *half* years in the other Table (49.) and which halves are in the third column *here*.—The age may be said to have a *right* to the sum, because all probability of *mortality* is allowed for, and therefore the sum is to be considered as *certain*.

Age		Hf. Yrs.		For 1 <i>l.</i> annuity being $\frac{1}{2}$ as in Table 49.	Reduced equal chance to live Hf. Yrs.
10	{ has an equal chance to live }	66	{ to pay or receive }	19.479	— 61
20	_____	53	_____	17.179	— 50
30	_____	43	_____	15.021	— 41
40	_____	35	_____	13.	— 34
45	_____	32	_____	12.1719	— 31
50	_____	29	_____	11.2958	— 28
55	_____	25	_____	10.054	— 25
60	_____	21	_____	8.723	— 21
65	_____	18	_____	7.663	— 18
70	_____	16	_____	6.925	— 16
75	_____	14	_____	6.161	— 14

N. B. *The fourth column is explained in the next page.*

In Table (162.) the value of an annuity *during life*, to *commence* at age *fifty*, is seen; and the value of 1*l.* *immediate* annuity, to *cease*, when *fifty*, is found at Case 93. for the age of *twenty*; and at Case 97. for the
age

(1) *Half* the sum for 1*l.* annuity, because the table is for 2*l.* annuity.

A D D E N D A.

age of *thirty*.—These values added together is the worth of 1*l*. *immediate annuity during life*, as in the Case of age *ten* (153.) and the worth of the *like* annuity is found in the Table (131.) for the age of *forty* or *upwards*.

Let these values be added to the values found in the last Table, or (49.) and let the *totals* be divided by *two*, and the quotient is the medium between both.

	Age 10	Age 20	Age 30	Age 40	Age 45
Value by Table 131, &c.	17.85	16.0125	14.089	12.386	11.718
Ditto by the Table (49.)	19.479	17.179	15.021	13.	12.172
Divide the totals by 2)	37.329	33.1915	29.11	25.386	23.89
The mediums	18.664	16.5957	14.555	12.693	11.945

	Age 50	Age 55	Age 60	Age 65	Age 70	Age 75
Value by Table (131.)	10.894	10.027	9.048	7.955	7.0714	6.191
Ditto by Table (49.)	11.295	10.054	8.723	7.663	6.925	6.161
Divide the totals by 2)	22.189	20.081	17.771	15.618	13.9964	12.352
The mediums	11.094	10.04	8.885	7.809	6.9982	6.176

Having found these TOTALS, look for the sums, in the Table (49.) that are *nearest* to them; and the number of *half* years opposite to the said sums, are those of the equal chance of life belonging to each age, as in the *last* column of the table *before* this. By comparing the *second* and *fourth* columns of *that* table together, the equal chance of life is found *reduced*; and in this table the values of the annuity are *increased*, (*m*) so as both coincide with each other.

(m) Except age 60 and upward, at which time the values by Table (131.) exceed the values by Table (49.)

A D D E N D A.

Mr. *Simpson* says (*n*), that he *increased* the number living at all ages below *twenty-five*: let it be supposed that he had increased them still more, so as to have 600 living aged *ten*; then, according to the REDUCED equal chance of life, the proportions would be nearly

Living aged 10	—	600
40 $\frac{1}{2}$	—	300
57 $\frac{1}{2}$	—	150
70	—	75
78	—	37.5
84	—	18.75

By the *reduced* chance the age of *ten* has an *equal* chance for living only 30 years and a half; and therefore at age 40 and a half, only 300, which is the half of 600, are found living. Age *forty* has an equal chance for living 17 years; therefore at the age 57 and a half, 150 would be living; and so of the rest.—This *additional* number, that would mostly all be dead before the age of 41, must *increase* the stock proportionably, being to be multiplied into *early* payments; which, when rightly understood, will be found to increase the values of the annuities at every age; tho' at the same time that the values of the annuities are so *increased*, the term for the equal

(*n*) At page ci. of the Introduction to this Essay.

A D D E N D A.

equal chance of life is *diminished*.—It is apprehended a table found and formed by this method, would vary but little from others, except in the ages *below* forty, which are allowed by all writers on the subject to be so vague as to render some alterations thereof *necessary*.—Be that as it may, the reader will please to remember that no use has been made, in the past Calculations, of this article; which, if it proves any thing, proves that the values of the annuities should be *more* than has been supposed; and it was on the foundation of this HYPOTHESIS, if it may be admitted for such, that the reader was referred, in the *memorandums*, page 183, to this article, for *another* cause of DEFICIENCIES.

Though the writer had an idea of this matter some time ago, yet he had not leisure to consider it until now within a few days of publication; and as he believes an immediate publication far more necessary than a *present* farther discussion, he hopes the reader will please to accept the hint as an indigested thought, that may be improved or rejected, as on maturer consideration it shall seem to merit; and *as such* it is submitted to the consideration of better judgments.

ARTICLE

A D D E N D A.

ARTICLE *the Third.*

IT may be satisfactory to examine *wherein*, and *why*, these Calculations have differed in any respect from those made by preceding authors.

The writer is not sensible that there is any difference between these calculations, and the answers to questions of the *like* kind given by Dr. Price in his *Observations*, &c. page 17. except what must consequently arise from different *interest* or *tables*, and the following consideration: the *manner* is also the same, except that the Tables cxii. cxiv. cxvii and cxx. in this Essay, furnish an easier and simpler method.

Dr. Price observes, page 343, that “the mean between the values at 3*ℓ.* *per cent.* and 4*ℓ.* *per cent.* may be taken for the value at $3\frac{1}{2}$ *ℓ.* *per cent.* without any error of consequence.”—But it should be observed, that there is *some* difference, as may be easily seen in Table the *first*, page 309, of Dr. Price; for the mean between 3*ℓ.* and 4*ℓ.* *per cent.* reversion is .966206; but the real value of the same is no more than .966184: and the difference is yet greater in 100 years; for the mean is .035916, whereas the just value is but .03206; and the difference of 1*ℓ.* annuity, forborn 100 years, is, in the *second* table, the mean £ 28.0519, the true value

A D D E N D A.

value £ 27.6554;—for, although *ten* shillings added to 3*£*. increases the interest *one seventh*, and *ten* shillings added to £. 3 10 0 increases it in a *less* proportion, *viz.* *one eighth* only; yet money will increase by compound interest in a quicker proportion by the *last* interest than by the *first*: that is to say, a sum will double itself at 4*£*. *per cent.* compound interest in less than *eighteen* years, and at 3 $\frac{1}{2}$ *£*. *per cent.* very *nearly* in *twenty* years, which is but something more than *two* years difference: but a sum at 3*£*. *per cent.* only, will not double itself in *twenty three* years.

Thus much is premised, because Mr. *Simpson* not having given the value of annuities at the rate of 3 $\frac{1}{2}$ *£*. *per cent.* interest, a *mean* must be taken between the rates of 3*£*. and 4*£*. which he has given with that of 5*£*. at page 38. of his annuities, and which Dr. *Price* has inserted, page 328. of his *Observations*, &c. the incorrectness of which *mean* the reader will please to allow for, and suppose the *true* value something *less* than what the MEAN produces.

Also it has been observed in the Introduction, page lxxiv. that *half* yearly interest reduces the value of each reversion; which, though it is but *little* in *each*, yet when multiplied 109 times into different numbers (as in Table 116.) must reduce the TOTAL of the whole in some degree from what it would

A D D E N D A.

would be by *yearly* interest; and therefore the annuities should be expected for a somewhat *lower* purchase.—The following table will compare the differences between Mr. *Simpson's* Calculations and these.

See Mr. Simpson's Doctrine of Annuities, p. 38.

Age	3 <i>ℓ.</i> per cent.	4 <i>ℓ.</i> per cent.	The MEAN for 3½ <i>ℓ.</i> per cent.	Table 131. page 128.
40	13.2	11.5	12.35	12.386
45	12.3	10.8	11.55	11.718
50	11.4	10.1	10.75	10.894
55	10.3	9.3	9.8	10.027
60	9.2	8.4	8.8	9.048
65	8.0	7.5	7.75	7.955
70	6.9	6.5	6.7	7.071
75	5.6	5.4	5.5	6.191

After giving two reasons; the *one*, that the exact value by 3½*ℓ.* per cent. should be *less* than the MEAN; the *other*, that, had the Table (131.) been found by the equal *rate*, which is by YEARLY interest, it would have *increased* the values; it must be contrary to expectation to find the Table (131.) exceed the values of the mediums of Mr. *Simpson's*, when the very *reverse* should have been the case.

The writer observed, that Mr. *Simpson* had cal-

A D D E N D A.

calculated the values no further than to the age 75, nor carried the decrements beyond the age of 80.—This begat a suspicion that he had not extended *his* calculations to the extreme of life: it was therefore necessary to examine into that particular.—Tables have therefore been carefully composed by 3*℥*. *per cent.* and by 4*℥*. *per cent.* according to the reverſions given in *Mr. Simpson's Treatiſe*, page 112. by YEARLY payments, and the decrements continued forward as in Table (62.): the values have been found to be as follows.

Age	3 <i>℥</i> . per cent.	4 <i>℥</i> . per cent.	The mean
40	12.882	11.485	12.183
45	12.124	10.927	11.525
50	11.205	10.193	10.699
55	10.248	9.397	9.822
60	9.181	8.487	8.834
65	8.006	7.462	7.734
70	7.056	6.631	6.843
75	6.123	5.803	5.966

The difference here is but ſmall, and no other than is ſuppoſed to ariſe from *Mr. Simpson's* method of conſtructing his tables, which may be ſeen, page 7. and 8. of his doctrine of annuities, and which is more liable to ſmall errors than the method uſed

A D D E N D A.

in this Essay, especially when but *one* place of decimals is used: for example, the present value of 100*£*. for age 20 to receive a year hence, is said, page 7. to be 94.7, whereas the just value is but 94.69; and for 100*£*. annuity for life, for age 19, is but

$$1580. \times .9615 \times 1519.17 \times 462 = 701856.54 \div 468 = 1499.69$$

instead of 1499.8 as in Mr. *Simpson*, page 8.
 —The values for every age, from 20 to upwards, are believed to have been found by Mr. *Simpson* in the table he constructed at the rate of 4*£*. *per cent.* for he gives the value of 1*£*. annuity during life for age 20 to be *very near* 1480—and the *total* of all the payments has been found, in the same manner as those in this Essay, to be $£\ 6840.3107 \div 462 = 14.80586$, which is $£\ 1480.586$ for 100*£*. annuity.—But it is thought *by the writer*, that in his table at 3*£*. *per cent.* Mr. *Simpson* did *not* form a table from so *young* an age as 20, but at an older, perhaps 75; and by continually adding the increased fraction to the values of each age as he descended, he also increased the proportions, until at the age 40 he brought out the value to be 13.2 instead of 12.882; which supposition may be warranted by the examples already given above.—It was not the intention of the writer to seek for
 such

A D D E N D A.

uch small blemishes; but, as in searching or greater, he hath discovered these, it may be proper to mention them as *cautions*, and lest cavillers might, *on their authority*, have objected to the accuracy of the tables in these calculations without farther examination of them.

Notwithstanding these little differences, there was sufficient conviction for the author, that Mr. *Simpson* had carried the decrements beyond 80, but not so far as is done in Table (62.) which procured a stock of £ 261.166 for age 75; whereas it is plain Mr. *Simpson* calculated from a stock of £ 243. only for that age; for 45 living, multiplied by 5.4, the worth of 1£. annuity at 4£. per cent. in *his* Table is but £ 243.

But the particular case under consideration still continues inexplicable, since the difference has been rather increased than reconciled.—As the cause is not to be found in Mr. *Simpson*, it must be looked for elsewhere; and if the *half* yearly PAYMENTS be examined, *there* it immediately presents itself; for it is easy to observe, in Table (116.) that the value of 1£. to be received a year hence by the survivors of 294 persons aged 40, is worth in present money, to those

Aged 40 = 294)274.315032(.933044£. each;

A D D E N D A.

but the value of 2*l.* for the same number of persons of the same age, and to be received by *half* yearly payments, is

$$\begin{aligned} \text{Aged } 40\frac{1}{2} &= 289 \times .982801 = 284.029489 \\ 41 &= 284 \times .965898 = 274.315032 \end{aligned}$$

Divide the total by 294)558.344521(1.899131

which quotient being divided by 2, gives .949565 for the worth of 1*l.* This is considerably *more* than by an *yearly* payment, because there will be not only a greater number living to receive the *half* year's payment, but likewise *that* payment is the more valuable for being received at an *earlier* time. The same cause, operating in some degree at every year during life, increases the capital stock, and therefore the value of every age in the proportion that has appeared in the comparison with Mr. *Simpson*.

This circumstance did not occur to the writer at the time he inserted, in page 68. that the Calculations were made at the LOWEST rate: it is now found that calculations by *yearly* payments are *lower*; but those concerned should make no other advantage of this discovery, (*n*) than to observe

(*m*) That half yearly payments increase the present values of the annuity, is remarked both by Mr. *Simpson*, page 78. and by Mr. *De Moivre*, page 123. but the solutions are *algebraical*.

A D D E N D A.

serve how much they would have been de-
ceived by calculations from annual payments,
since there is not one society but proposes
to pay the annuities *half* yearly at least, and
some to pay *quarterly*; and those should re-
member to make *farther* allowance in the
estimate of the value.

Mr. *De Moivre* has, in page 91. of his
Annuities on Lives, given a table of the
present value of 1*£.* annuity *during life* at
 $3\frac{1}{2}$ per cent. interest, of which some ages
are here compared with the values according
to this Essay.

Age	Values according to Mr. De Moivre.	Values according to Table 131. p. 128.
50	11.92	11.6356
55	10.69	10.5292
60	9.36	9.1796
65	7.88	7.6948
70	6.22	6.0777
75	4.38	7.7347
80	2.31	3.9255
85	0.00	3.0076

Two observations may be made here : 1st,
The values by Mr. *De Moivre's* Table, until
the ages above 70 are *more* than those found
by these calculations : 2dly, After the age
of 70, the values by these calculations be-
come greater than those found by Mr. *De*
Moivre.—As both are by Dr. *Halley's* Table
of

A D D E N D A.

of Mortality, and *both* by the *same* rate of interest, whence can proceed the differences, or why should there be any?

It appears from Mr. *Simpson's* Appendix, page 3. that he disputed the *first* observation with Mr. *De Moivre*, and *there* gives a table to shew that Mr. *De Moivre* had estimated the values too *high* for ages more than 40, and too *low* for ages less than 30. Dr. *Price* observes the same also at page 226. of his *Observations* &c.—As to the writer's opinion on the subject, he ingenuously confesses, that though he understands the rule given by Mr. *De Moivre* very plainly, yet he cannot penetrate into the REASONS on which it is founded; and, with due respect for the abilities of that great calculator, very much suspects they have but a weak foundation, the HYPOTHESIS having a part in them: and how much the HYPOTHESIS may be depended upon, will appear by considering the *second* observation.

Though Mr. *De Moivre* says, in his preface, page x. that Dr. *Halley* leaves *twenty* living at the age *eighty-four*, in his Table of Observations, yet it was necessary for his HYPOTHESIS, to suppose “ the extent of “ life confined to a *certain* period of time,” —and which he fixed at “ *eighty-six*; for, as “ to what might be alledged, that by some “ observations of late years, it appears that “ life is carried on to 90, 95, or even 100 “ years,

A D D E N D A.

“ years, he was no more moved by it, than
 “ by the examples of PARR or JENKINS,
 “ the first of which lived 152 years, and the
 “ other 167:”—and he adds—“ that the
 “ age for purchasing annuities for life seldom
 “ exceeds 70, at which term Dr. *Halley*
 “ ends his table of the valuation of lives.”

In consequence of this reasoning, Mr. *De Moivre* makes the value of the life 85 to be NOTHING, for *all* MUST be dead at 86: therefore the age of 80 having *no chance* of living *beyond* 86, the value of the said age by Mr. *De Moivre* is no more than 2.31, and 4.38 is for the *same* reason the value of age 75:—whereas Dr. *Halley's* Table (63.) has been here extended to the age of 93. (Dr. *Price* extends it to 90) and therefore the chances of life for the ages 75 and 80 are much more valuable, and produce the differences seen on comparison of the tables, and which are so much the more as the chance of attaining to the age of 93 is more valuable than *certainly* dying before 86, blended with the *overvalues* at ages above 40, and which below 70 begin to overbalance the *other* consideration of surviving *beyond* 90.

ARTICLE

A D D E N D A.

ARTICLE *the Fourth.*

SOME farther useful observations may be made upon Table xlv. First, Suppose an estate to be mortgaged for 57234*l.* If the interest be only 3½*l.* per cent, and paid half yearly, according to *that* table, it will be 2003*l.* yearly, which is 1001.5*l.* half yearly.—When the *fortieth* half year's interest shall have been paid, an inspection of the table will shew that there will have been *as much* VALUE paid for *interest* as is EQUAL to the whole sum borrowed; for the 1000*l.* paid *half* yearly for interest, may be considered as 2000*l.* ANNUITY; and such an annuity, with *compound* interest, would accumulate, in *forty* half years (that is, *twenty* years) to the sum opposite the said number of half years in the table, which is 57.234*l.* for 2*l.* annuity; or 57234*l.* for 2000*l.* annuity:—yet, although the VALUE of *so much* has been paid, NO part of the mortgage has been REDEEMED.

But if, at the same time, *another* sum, *equal* to the interest, had been set apart to have accumulated compound interest, the said sum at the expiration of *twenty* years would be found also *equal* to *that* for which the estate had been mortgaged; and consequently would REDEEM it.—So that DOUBLE
in-

A D D E N D A.

interest would clear the estate entirely of its incumbrance in *TWENTY* years; while *single* interest might be continued FOR EVER, and the estate remain loaded with the same burthen as at *first*.

Secondly, If the circumstances of the case should be such as that the estate cannot afford *double* interest, it should be remembered that *single* interest MUST be paid: suppose then HALF as much be appropriated to the purpose of discharging the loan, that is 500*£*. *half* yearly. By the table the said sums would be *equal* to the debt in 63 *half* years; for opposite that number of *half* years is *double* the sum of 57.000*£*. that is 114.32, and

$$114.32 \times 500 = 57160$$

very nearly the same as the mortgage:—also *more* than the double of 114.32 is found at opposite 93 *half* years; in which time one QUARTER of the interest would accumulate to the same sum, for

$$230.7 \times 250 = 57675$$

Whence it is evident,—That an interest *equal* to that paid for the loan will discharge the debt in *twenty* years; and 40000*£*. will *seem* to be disbursed for the *inte-*

4 C
rest,

A D D E N D A.

rest, (o) and 40000*£*. to pay off the mortgage; which is 80000*£*. paid for 57234*£*. and its use for *twenty* years—That *HALF* the sum, equal to the interest paid, will discharge the same loan in less than *thirty-two* years; in which case 63000*£*. would be paid for interest, that is, 1000*£*. each *half* year; and *sixty-three* times 500*£*. would be 31520 to discharge the debt; which is 94500*£*. paid for 57234*£*. and its use for near *thirty-two* years—That one *QUARTER* of the sum, equal to the interest paid, will clear the estate of its mortgage in *forty-six* years and a half; in which case 93000*£*. would be paid for interest, and *ninety-three* times 250*£*. would be 23250*£*. appropriated to discharge the debt; which would be 116250*£*. expended for the loan of 57234*£*. for *forty-six* years and a half.—The one is 22766*£*. the other 37266*£*. and the other 59016*£*. *MORE* paid than the *borrowed* sum.

Thirdly, If the owner of the estate would choose, on the contrary, to appropriate *more* than an yearly sum equal to the interest, in order to clear the estate of its incumbrance in *less* time; let it be supposed that he would *apply* *TREBLE* interest for the purpose; *one* part to pay the interest of the sum borrowed; and

(o) See further on.

A D D E N D A.

and *two* parts to discharge the debt—There would be then 2000*l.* each *half* year to accumulate *compound* interest; and the sum *nearest* to the half of 57.234*l.* is opposite *twenty-three* half years, and is 28,02, which multiplied by 2000*l.* is 56040*l.* nearly equal to the loan.—In this case 23000*l.* would be paid for interest, and 46000*l.* would be appropriated to the payment of the mortgage, which would be 69000*l.* expended in the whole (*p*), which is but 11766*l.* more *paid* than was *borrowed*, and the estate is clear nearly in ELEVEN years and a half.—On comparing the several cases, it will be very obvious, that the *longer* time is taken to discharge the mortgage, the *greater* sum will be paid in the *whole*: and, by the reverse, the *shorter* time is taken, the *lesser* sum total will be paid.—In other words, the *more* the appropriated sum for payment is, the *nigher* it approaches to an IMMEDIATE discharge of the loan; and, contrariwise, the *less* it is, the *more* it approaches to the NOT discharging it *at all*:—but even the *smallest* sum, if invariably applied to that purpose, and *sufficient* TIME be allowed for the cause to operate its effect, would effectually clear an estate of *any* mortgage whatever.

(A) Without reckoning *any* interest to either these, or those before.

A D D E N D U M.

The interest made by Mortgagees, for money lent, is generally nearer 5*ℓ.* per cent. than 3½*ℓ.* and the *more* the interest is, the *faster* will the appropriated sums accumulate to pay off the loan.—At 5*ℓ.* per cent. per annum DOUBLE interest will very nearly discharge the debt in *fourteen* years: for the annual interest of 57234*ℓ.* at 5*ℓ.* per cent. would be 2861.7*ℓ.* which, as an annuity, would increase in *fourteen* years to 56089.32*ℓ.* for 19.6 (9) $\times 2861.7 = 56089.32$: that is, nearly the whole debt—and what may seem very extraordinary is, that there will be but little difference in the total of the sums, paid for interest, and to redeem the estate, whether the interest be at 3½*ℓ.* or 5*ℓ.* per cent.—for, by the last rate, the interest of 57234*ℓ.* would be 2861.7*ℓ.* annually: which sum, paid for *fourteen* years continuance, is

$$2861.7 \times 14 = 40063.8$$

and

(9) A Table shewing to what sum 1*ℓ.* annuity at 5*ℓ.* per cent. per annum would amount in any number of years, within *fifteen*, by compound interest.

Year		Year		Year	
1	1.	6	6.801912	11	14.206785
2	2.05	7	8.14201	12	15.917123
3	3.1525	8	9.54911	13	17.712978
4	4.310125	9	11.026565	14	19.598626
5	5.525631	10	12.577892	15	21.578556

A D D E N D A

and the *same sum* was appropriated to discharge the debt; therefore *twice* 40063*l.* is 80127.6*l.*—and it has been seen that 80000*l.* would have been disbursed at the rate of 3½*l.* per cent. so that the difference, in the *total* sums paid, is scarce worth regarding; but in respect to *time* 3½*l.* per cent. would not clear the estate in much less than *twenty* years, but 5*l.* per cent. would do it nearly in *fourteen* years: yet it should be remembered, that in the mean time the *annual* load upon the estate is but 7*l.* per cent. in the first case, and is 10*l.* per cent. in the last.

There can be but little need of adding, that *half* or *double* the annuity, equal to the interest of 5*l.* per cent. would have an effect in proportion, as has been considered in the case of 3½*l.* per cent.—But it may be thought, that though the mortgager must *pay* 5*l.* per cent. interest, yet he may not be able to *obtain* the same interest for the appropriated sum.—Whatever interest the mortgager may pay, he will always obtain the *same* by paying off the mortgage itself. The following Table may illustrate this particular, and the subject in general.

A D D E N D A.

Principal		The interest of the re- maining principal.	Part of the principal paid off annually.
1st year	100 <i>l</i> .		
Paid off	5	—5 <i>l</i> . —	5 <i>l</i> .
2d year	95		
Paid off	5.25	—4.75 —	5.25
3d year	89.75		
Paid off	5.5125	—4.4875 —	5.5125
4th year	84.2375		
Paid off	5.7881	—4.2119 +	5.7881
5th year	78.4494		
Paid off	6.0775	—3.9225 +	6.0775
6th year	72.3719		
Paid off	6.3814	—3.6186 +	6.3814
7th year	65.9905		
Paid off	6.7005	—3.2995 —	6.7005
8th year	59.29		
Paid off	7.0355	—2.9645 —	7.0355
9th year	52.2545		
Paid off	7.3873	—2.6127 —	7.3873
10th year	44.8672		
Paid off	7.7566	—2.2434 +	7.7566
11th year	37.1106		
Paid off	8.1445	—1.8555 +	8.1445
12th year	28.9661		
Paid off	8.5517	—1.4483 —	8.5517
13th year	20.4144		
Paid off	8.9793	—1.0207 —	8.9793
14th year	11.4351		
Paid off	9.4283	—0.5718 +	9.4282
Remaining	£2.0069	£42.0069	£97.9931

A D D E N D A.

Let it be observed—That the interest, and the part of the principal paid off, added together, is always 10*l.* every year: so the first is $5 + 5 = 10$ *l.* the second is $4.75 + 5.25 = 10$ *l.* and so on to the last $0.5718 + 9.4282 = 10$ *l.*—Also, that as the principal is paid off, the interest decreases as in the second column of the table;—and as the annual interests grow *less*, the sums paid off become *greater* yearly, until soon after *fourteen* years the whole principal will be paid; and both the interest and the appropriated sums will then revert to the mortgager.—Likewise, that £ 97.9931 of the principal has been actually paid; which, with the £ 2.0069 remaining unpaid, is equal to the 100*l.* borrowed. Therefore, of the 10*l.* paid for *fourteen* years, only £ 42.0069 has been paid for interest: for it must be plain, that in *fact* the mortgager has not had the use of the *whole* 100*l.* but for the *first* year only, and that afterwards the portion of it became less and less until it vanished.

It may be inferred, from some part of what has been observed, that it is an equal expence to borrow money at 5*l.* as at 3½*l.* per cent.—nay, that it is much preferable, because the debt by *double* interest will be nearly paid in *fourteen* years by the rate of 5*l.* interest, and not in less than *twenty* years

A D D E N D A.

years by the rate of $3\frac{1}{4}\%$. interest, and the total expended will be nearly the same.— But care should be taken to detect a fallacy that lurks beneath the specious appearance. It has been remarked, that the estate is burthened with an annual load of 10% . per cent. in the one case, and but 7% . per cent. in the other; therefore, if interest was but $3\frac{1}{4}\%$. per cent. the estate might pay a TREBLE interest at one half per cent. only more additional burden, than it would pay for *double* interest at 5% . per cent.—By the third case of this article, TREBLE interest discharged a sum at $3\frac{1}{4}\%$. per cent. interest in ELEVEN years and a half, with an expence of 69000% . only: whereas by DOUBLE interest at 5% . per cent. it would take *fourteen* years time, and an expence of 80127.6% . and at the same time the difference of the annual burden to the estate is but *one half* per cent. being $10\frac{1}{2}\%$ per cent. in the first instance, and 10% . per cent. in the latter.

If it should be still insisted on, that the *more* the interest is, the more *advantageous* it would be for discharging a debt in a *short* time; and should it be further urged, that the case of paying off the debt in ELEVEN years and a half, by $3\frac{1}{4}\%$ per cent. interest, was by supposing *treble* interest; and that 5% . per cent. interest, would by *treble* interest

A D D E N D A

terest, discharge the loan in *less* time.—
 The *latter* part would be granted, but the *first* must ever be denied: and there needs no more than recurring to the same observation as before, to confute the argument.—
 If the estate can sustain a burden of *treble* interest at 5*ℓ.* per cent. the annual sum appropriated must be 15*ℓ.* that is, 5*ℓ.* for interest, and 10*ℓ.* towards the discharge of the debt.—With the *SAME* yearly expence to the estate, and interest at only 3½*ℓ.* per cent. the annual sum appropriated to discharge the debt, would be 11½*ℓ.* which, being 1½*ℓ.* more than in the other case, must clear the estate of its incumbrance in *less* time.—
 That it may be clearly indubitable, here follow two tables for demonstration.

Fifteen pounds expended yearly.

	Interest at 5 <i>ℓ.</i> per cent.	Paid off an- nually
1st principal 100 <i>ℓ.</i>		
Paid off 10	—5.	—10.
2d principal 90		
Paid off 10.5	—4.5	—10.5
3d principal 79.5		
Paid off 11.025	—3.975	—11.025
4th principal 68.475		
Paid off 11.576	—3.424 +	—11.576
5th principal 56.899		
Paid off 12.155	—2.845 +	—12.155
44.744		

4 D

6th

A D D E N D A.

		Interest at 5l. per cent.	Paid off an- nually
6th principal	44.744		
Paid off	12.763	— 2.237 —	12.763
7th principal	31.981		
Paid off	13.401	— 1.599 —	13.401
8th principal	18.580		
Paid off	14.071	— 0.929 —	14.071
Remaining after eight } years	4.509	24.509	95.491

Fifteen pounds expended yearly.

		Interest at $3\frac{1}{2}$ per cent.	Paid off an- nually
1st principal	100£.		
Paid off	11.5	— 3.5 —	11.5
2d principal	88.5		
Paid off	11.9025	— 3.0975 —	11.9025
3d principal	76.5975		
Paid off	12.3191	— 2.6809 —	12.3191
4th principal	64.2784		
Paid off	12.7503	— 2.2497 —	12.7503
5th principal	51.5281		
Paid off	13.0965	— 1.9035 —	13.0965
6th principal	38.4316		
Paid off	13.6549	— 1.3451 —	13.6549
7th principal	24.7767		
Paid off	14.1328	— 0.8672 —	14.1328
Remaining after se- ven years }	10.6439	15.6439	89.3561

A D D E N D A.

The estate is not only found clear in *less* TIME by $3\frac{1}{2}\%$. *per cent.* interest than by 5% . but also *much* less, that is, nearly one *third* less, has been expended for the interest; from ALL which it must be *undeniable*, that when an estate is to sustain an *equal* annual burden *be the interest what it will*, the LESS that interest is, the SOONER and *cheaper* the annual expence will pay off *any* mortgage.

There is another mistake likewise that some may inconsiderately fall into:—as it has appeared, page 5. of this Article, that by paying *double* annuity by $3\frac{1}{2}\%$. *per cent.* and by 5% . *per cent.* interest, much the same total sum will be expended; they might think there could be *no* difference, and therefore choose the *longest* time, because it would be done by a LESS *annual* expence. But though the difference in the *disbursements* is but small, the difference in the SAVINGS would be very great: for if the estate be cleared of the mortgage in *fourteen* years, there would be *six years* CLEAR *rent saved* MORE than if the estate had not been cleared until *twenty* years.

When an estate is mortgaged, so much of the rent of it, as is *equal* to the interest paid for the sum borrowed, may be considered as *actually* SOLD, in respect to profit, until such time as it is REDEEMED.—Suppose a certain part yields 2000% . *per annum* rent: if the same annual interest is to be paid for

A D D E N D U M.

a sum borrowed, such a part of the estate can be looked upon as scarcely belonging to the owner; for, though he occupies it, it affords him not the least emolument; nor ever will, unless he appropriates another part towards redeeming it.

Those who sell one part to redeem another, do NOTHING towards clearing the whole estate, or rather worse than nothing, since it would be worse than a PERPETUAL mortgage; for whilst the land is held by the mortgager, he is deemed the owner, and may enjoy the pleasures of it, though not the profit, until he purchases it (for it has been observed, that it is the same as *sold*, until he redeems it) and which he might do with economy and patience to wait *fourteen* years, when or soon after he would find himself in clear possession of the WHOLE, provided that he had appropriated that other part which was intended to have been *sold* towards the purpose.—The difference is, that in the one case, the loss of the rent of the part sold would be FOR EVER; in the other, the loss of *both* rents for somewhat more than *fourteen* years would RESTORE *both* rents for ever.

When the owner will not, or cannot, spare so much rent of his estate, as would annihilate the debt in his own life time, still a part, if but a *little* part, faithfully and invariably

A D D E N D A

variably applied, would in TIME (*q.*) restore the estate clear of incumbrance to some of his descendants: and it may be the DUTY of those who inherit an estate, or any other blessing, from their ANCESTORS, to transmit the same *faithfully* and *entire* to their POSTERITY; for surely every LAW, that hath Divinity, Humanity, or Generosity in it, forbids to throw incumbrances on them, without at least providing, at the same time, the means to rid them of the burden.

When it is thoroughly understood, that *any* rent would, at *any* rate of interest, clear another *equal* rent of its mortgage; or, which is the same thing, that it would purchase an estate of *equal* value in a *certain* time:—then, it is easy to conceive, that *two* rents would become *four* in an *equal* space of time longer; and *four* would become *eight* in the same space of time longer; and *eight* would become *sixteen*, and so on continually at every given period of time:—by which amazing power of doubling, ONE acre might purchase every acre upon the surface of the whole

(*q.*) What the LAWS are, respecting mortgages, the writer does not know; but calculates on supposition, that the mortgager may hold the mortgaged part of his estate in possession as long as he chooses, if he continues paying the *interest* yearly,

A D D E N D A.

whole earth, if constantly and faithfully appropriated to the purpose.

The powers of the like Tables as (42*.) and (45.) have been applied, in a masterly manner by Dr. Price, to the payment of the NATIONAL DEBT, and urged by such forcible arguments as must affect *all*, who believe it to be a national grievance, and have a feeling for the welfare of their country.

When the Doctor says (*page 140. of his Observations*) that “ *it is of little importance what interest a state is obliged to give for money,*” and further on, *page 141.* that “ *reductions of interest would be no great advantage to a state;*” his meaning must not be understood to be as the words there import; for he explains himself more fully at *page 340.* when he says, “ *The disbursements, on account of any loans, will be the same, whatever different interests they bear, supposing a provision made for discharging the principal, by applying to that purpose surpluses* BEARING TO ONE ANOTHER THE SAME PROPORTIONS WITH THOSE INTERESTS.”

Accordingly, in the same page, when he supposes the interest to be paid is but 30.000*£*. he supposes only 3000*£*. for surplus—and when the interest is 60.000*£*. he supposes the surplus to be also double, viz. 6000*£*. which double surplus, with double interest, would almost discharge the principal
in

A D D E N D A.

in half the time that 3000*l.* could, and the disbursements in the whole would be nearly the same.

If 66,000*l.* was to be raised annually to pay interest *be it what it would*, and the surplus of the *same sum* be appropriated to discharge the principal, there can be no reason to suppose, but that a Gentleman so intelligent must know that the value of the interest is of very great importance, and that the LESS it should be, the *sooner* the loan would be discharged, as the *surplus of the fund* would be *greater* in proportion.

ARTICLE *the Fifth.*

IT would be an unpardonable omission to conclude an Essay, that professedly treats of securing FUTURE *provision*, without *remembering the POOR*; and though the general plan has a tendency towards reducing their number, yet their case requires and *deserves* more particular consideration.

It may be a task too difficult to find an *easy* method to provide for ALL.—Let the present enquiry be confined to the search of what may be done for such as have the most immediate claim to pity, who are presumed to be those, who, through the infirmities of AGE, are totally incapable of procuring
nourishment

A D D E N D A

nourishment for themselves in their comfortless and feeble days.

Nature, for a wise and obvious purpose, has implanted in every animal an affection for its young; and the impulse is the strongest while most needful to its infant, tender, and helpless state.—Although the instinct in inferior creatures abates and wears away as the care becomes unnecessary, yet HUMAN souls retain this affection during life.

The *second* helpless state, that of declining Age, Parents cannot live to see without being involved in the same circumstance to a superiour degree, and themselves needing more assistance, than, *at that time*, they have the power to give.—Will not Reason therefore direct the *prudent* FORE-THOUGHT to guard against the possible, though it may be improbable, chance of having WANT added to the other miseries of, that we all wish for, Long life?

If Nature and Reason have not force to establish, suppose Law was to compel, the custom for every Parent to make what might be called a DELIVERY OFFERING—that is, to dedicate a small sum, at the *birth*, to provide a maintenance for the *old age* of the child.

Suppose the sum to be 1*l*. let it be enquired what annuity for life, such a fund would produce for the age *sixty*.

As

A D D E N D A.

As an answer to this question must be of importance, it is proper to examine it by the four Tables of Mortality that are inserted in this volume; for which purpose the number of the living at the age SIXTY, in proportion to 1000 born, are found to be as in this table.

	Born.	Live to age 60.	Live to age 60 out of 1000 born.
By London Bills	1000	140	140.
— Mr. <i>Simpson</i>	1280	130	101.562
— Dr. <i>Halley</i>	1238 *	242	195.476
— Dr. <i>Price</i>	1518	147	96.838

The Table of Compound Interest (42*.) is carried no further than 110 *half* years; whereas there will be 120 required to answer this question: the other ten are as follows.

4 E

Half

* See Dr. *Price's* Observations, &c. page 184. Note (a).

A D D E N D A.

Half years.		Half years.	
110	6.741957 + .117984	115	7.352889 .128675
111	6.859941 .120049 +	116	7.481564 .130927
112	6.97999 .12215 +	117	7.612421 .133219 +
113	7.10214 .124287	118	7.74571 .13555 +
114	7.226427 .126462	119	7.88126 .137922
115	7.352889	120	8.019182

It is now seen by this Table; that 1*l.* in consequence of doubling itself every *twenty* years, would increase in *sixty* years to 8.019182.—If 1*l.* each had been paid for 1000 born *sixty* years ago, the FUND would be found now to be worth 8019.182*l.* and the number living to partake of it is seen in the last column of the first Table in this Article. This fund, divided by the number living at age *sixty*, gives the SHARE of each in *present* money, and according to the different authors.

	Living aged 60 out of 1000 born	100 <i>l.</i> increased by comp. int. in 60 years	Share in present money to each survivor
London Bills	140.	)8019.182	(57.2799 +
Mr. Simpson	101.562	)8019.182	(78.958
Dr. Halley	195.476	)8019.182	(41.0238
Dr. Price	96.838	)8019.182	(82.8102

A D D E N D A.

Let the *present* money be divided by the values of $\text{£}1$. immediate annuity for age *fixty* during life, as given in the Table (131.)

	Values of $\text{£}1$. annuity du- ring life for age 60 (131.)	Present money	The immediate an- nuity purchased
London Bills	8.8514	)57.2799	(6.46 +
Mr. <i>Simpson</i>	9.0484	)78.958	(8.726
Dr. <i>Halley</i>	9.1796	)41.0238	(4.479
Dr. <i>Price</i>	8.66135	)82.8102	(9.33 +

These annuities being very different, and having been found in *another* manner than that the reader has been accustomed to; it is necessary to prove them by comparing them with values found by the *former* method.

In the Table 112. at page 48. there is $\text{£} 2478.396$ stock opposite age *fixty*, for 140 persons then living. As the NEW-BORN will be *fixty* years before they become annuitants, the stock must be multiplied by *so many* years reversion, which by case (46.) must be this *quotient*.

$$(t) 8.019182)100000(.124701$$

The stock being multiplied by this quotient, the product would be 309.058459596, and the

(t) The sum opposite 120 half years compound interest in the last page.

A D D E N D A.

the said product divided by 1000 born, gives .369058 (24.) for 2*l*. which is .154529 for 1*l*. annuity.

If, instead of 1*l*. the immediate annuity found by the *London* bills be required; then this value must be multiplied into that, which was 6.46.

$$.154529 \times 6.46 = .99825734$$

which is 1*l*. (30.) *present* money to be paid to receive 6.46*l*. annuity during life, to commence when aged *sixty*—and agrees with the answer found before, according to the *London* bills.

By the same method *l*. 2352.59 stock must supply 130 persons (*u*) aged *sixty* with annuities for life, as in Table 117. page 78.

Stock.	Reversion.	Product.	Born.
<i>l</i> 2352.59	+	.124701	= 293.37032559
			÷ 1280 =

.22919556 for 2*l*. that is .11459778 for 1*l*. annuity, or .11459778 $\times$ 8.726 = .99998, &c. (30.) that is, 1*l*. for 8.726*l*. annuity, according to Mr. *Simpson's* Table.

Also by Dr. *Halley's* Table, page 61, age 60.

Stock.	Reversion.	Product.	Born.
<i>l</i> 4442.925	$\times$	.124701	= 554.037190425
			÷ 1238

= .447526 for 2*l*. which is .223763 for 1*l*. annuity, or .223763 $\times$ 4.479 = 1.002234477, that is, 1*l*. for 4.479 annuity.

(*u*) The survivors at age 60 in proportion to 1280 born; for neither this, Dr. *Halley's*, nor Dr. *Price's* Table, are in proportion to 1000 born.

A D D E N D A.

And also by Dr. Price's Table, page 92.
age 60.

Stock.	Reversion.	Product.	Born.
$\pounds 2546.442 \times .124701 = 317.543863842 \div 1518$			
$= .209185681$ for 2 <i>l</i> . which is .10459284 for 1 <i>l</i> . annuity,			
or .10459284 $\times 9.56 = .9999075504$ (30.) that is 1 <i>l</i> . for			
1 <i>l</i> . 9.56 annuity.			

The reader, who hath perused the book thus far with the least attention, must understand these values, without farther explanation, to be *equal* to those found before. —And though the annuities, as given by the several Tables, are so very different, yet they *all* serve to shew, that 5*l*. dedicated, at the birth of a child, to its use when it shall attain to *sixty* years of age, would provide a tolerable support to *every* survivor in the decline of life, the *least* being $\pounds. 2:27:11$ per annum.

This is on supposition, that EVERY survivor, at the age *sixty*, should partake of the benefit of this fund: but if, as it were by agreement, *none* should share in it but such whose hard lot leaves them in REAL WANT at that age; then, a *much less* sum would provide the *same* annuity: for it is greatly to be hoped, that four in five would not be so necessitated as to require any support from it, and therefore the *fifth* part that should be in
distress

A D D E N D A.

distress might be relieved by the shares of those who would be in happier situations.

In which case 1*ℓ*. DELIVERY OFFERING would supply the *same* annuity—If it should prove that not more than *one* in *ten* should be so poor as to be entitled to the annuity, then *half* the sum would be sufficient—or otherwise the annuity might be *doubled*—or otherwise it might commence at an *earlier* age—according to which ever proportion the establishers might fix it. (x)

It

(x) It may be readily calculated, what FUND 1*ℓ*. DELIVERY OFFERING would establish in *sixty* years, by adding the *last ten* years compound interest together as follows. (45.)

Half yrs.		Half yrs.		Half yrs.	
111	333.853790 6.859941	114	354.795861 7.226427	117	376.856741 .7.612491
112	340.713731 6.97999	115	362.022288 7.352889	118	384.469232 7.74571
113	347.693721 7.10214	116	369.375177 7.481564	119	392.214942 7.88126
114	354.795861	117	376.856741	120	400.096202

The table being for 2*ℓ*. take *half* the sum for 1*ℓ*. and multiply it by the medium of yearly christenings (Introduction, page xci.) viz.

$$200.048101 \times 15657 = 3132153.117357$$

The product is three millions one hundred thirty-two thousands one hundred fifty and three pounds, within the bills of mortality: and would be *more*, as including *all* BORN as well as *christened*.—Would it not be a real honor to the nation, when it might be said, there is not a person in it aged *sixty* (or of some earlier age) but who has a *comfortable maintenance for life*?

A D D E N D A.

It is apprehended, an easy mode of *collecting* the DELIVERY OFFERINGS might be, to require ALL who practise *midwifery* to pay them, who are to be repaid by the Parents; or, in case of refusal, by the Parish: the Parish to be empowered to pay for such as are not able to pay for themselves; and to distrain for payment, when refused by such as are able, but not willing.

It is not intended to enter into every particular—enough has been said, if the hint be not worth the attention of those in power to carry such a plan into execution—but if it should be adopted, a *farther* consideration would be a matter of course.

It ought to be remembered, that this *offering* should not be considered as a TAX, it being plainly but a provision against an unfortunate contingency—nor should those whose present affluence seems to defy the possibility of their descendants needing the precaution, withhold the tribute due to the Giver of such secure blessings: and surely those who arrive at the aged period in chearful circumstances, will with as chearful hearts give grateful thanks where due, and will scarcely repine to find they need not the use of the preventive offering, which their parents dedicated, at their births, to the benevolent purpose of relieving indigence in age.

With whatever Vices malevolence may seek
to

A D D E N D A.

to reproach the GREAT, it is undeniable that they have *one* Virtue, that of CHARITY, whether public or private : of which there are *so many* instances, that there can be no need to mention *one* to prove it.—They may, and there is little doubt but that they will, assist the charity by *larger* OFFERINGS—and among all ranks there are some of such humane compassionate dispositions, as to find their most supreme delight consist in supplying, or preventing, the want of the necessaries of life, to the infirm and friendless Needy.

Such have a monitor within, and perform their duty without being reminded: not only they, but it is hoped that ALL will give compassionate attention to these concluding words,

REMEMBER THE POOR!



POSTSCRIPT

TO ARTICLE THE SECOND.

IT might have been added to the said Article, that when the term of an equal chance of life can be determined (and until that is done, all other proportions must be upon an unsettled foundation) then the Table (47.) will answer part of this xth question, in Dr. Price's *Observations*. &c. by inspection.—“*What money in hand, and also in annual payments during life, ought a person of an assigned age to give for a sum of money, payable at his death to his heirs?—In other words, What money in hand, and in annual payments during life, ought a person of a given age to pay for an assurance of any given sum on his life?*”

Age *forty-five* in the Table (62.) has an equal chance to live *sixteen* years; and opposite *thirty-two* *half* years in the Table (47.) is £ 57.3982, which is the just value to be paid for 100£. to be paid at the death of a person of the said age—because, should he die at the instant of the expiration of *sixteen* years, there would be loss or gain to neither party, for the 57.3982£. would become 100£. by accumulating compound interest in that time. - What time the assured should die *before* that period, would be gain to his heirs, and what time he should survive that period, would be gain to the assurer.—Also, the most the assurer could possibly lose, would be what the money in hand is deficient of 100£. which in this case is £ 42.6018.—If the age *forty-five* attains to *sixty-one*, that is, lives *sixteen* years, it will be then an equal chance that he lives *ten* years and a half longer, although it was *three* to *one* that he would not attain to age *seventy-one* and a half when he was aged but *forty-five*; and in that time the assurer will have almost gained as much as he could have possibly lost: for *fifty three half* years compound interest is (Table 42.) 2.50793, which multiplied by 57.3982, produces nearly 144£.—and £ 40.6018, which might have been lost, acquires 106.64 in the same time.

P O S T S C R I P T.

But the ~~reduced~~ equal chance of life is only ~~thirty-one~~ half years, by the Table in the Second Article, and the sum opposite thereto in the Table (47.) is 58.4027 for 100£. which is but 41.5973 deficient of the 100£. — If age ~~forty-five~~ survives ~~two~~ equal chances of life, the term would be ~~fifty-two~~ half years, instead of ~~fifty-three~~ as before; and each of these sums multiplied by so many ~~half~~ years compound interest, stand thus: (42.)

$$58.4027 \times 2.465 = 143.9626$$

$$41.5973 \times 2.465 = 102.5373$$

$$£ 41.4253$$

which is very nearly the full sum that could have been possibly lost, gained in ~~fifty-two~~ half years: but as it is ~~three~~ to ~~one~~ that age ~~forty-five~~ does not live so long, the assurer has a farther chance for all the time the age survives beyond ~~seventy-one~~, to compensate for the risk.

To answer the question, Dr. Price directs to “ *subtract the value of the life from the perpetuity; multiply the remainder by the product of the given sum into the interest of 100£. for a year; and this last product, divided by 100£. increased by its interest for a year, will give the answer in a single present payment; and this payment, divided by the value of the life, will give the answer in annual payments, during the continuance of the life.* ”

The interest being here $3\frac{1}{4}\%$ per cent. the perpetuity (a)

is $\frac{100}{3\frac{1}{4}} = 28.57$
 The value of the life for age ~~forty-five~~, is $\frac{100}{11.55} = 11.55$ (b)

$$17.02$$

The given sum, being 100£. multiplied by the rate of interest, is

$$100 \times 3.5 = 350$$

The

(a) “ *That is, the value of the fee simple of an estate found by dividing 100£. by the rate of interest.* ” Dr. Price, page 32. Note.

(b) See the mediums for three and a half per cent. by Mr. Simpson's Table, in the Third Article.

P O S T S C R I P T.

The remainder, of the value of the life subtracted from the perpetuity, multiplied by the product of 100*£*. multiplied by its rate of interest, is

$$17.02 \times 350 = 5957.00$$

which product divided by 100*£*. increased by its interest, is

$$103.5)5957.00(57.55$$

which is *£* 57.55 to be paid in a *single present* payment—or this sum divided by the value of the life, is

$$11.55)57.55(4.982$$

which is about *£* 4 19 8 by annual payments during life.

The worth in present money, when the value of the life is taken out of Table (131.) is 56.987, both which values may be compared with those found by Table (47.)

The age *forty-five* has a chance of living *fewer* years by Mr. *Simpson's* Table, than by any other, except the London Table (60.) for *forty-three* years; therefore the value is *more* when found by it, than any other. If $3\frac{1}{4}$ per cent. be thought too great interest, let the same question be answered by 3 per cent interest.

The perpetuity is $\frac{100}{3} = 33.333$

Deduct the value of the life at 3*£*. per cent. 12.3

Remainder *£* 21.033

The remainder, multiplied by the product of 100*£*. multiplied by its rate of interest, is

$$21.033 \times 300 = 6309.9$$

And the product divided by 100, increased by its rate of interest, is

$$103)6309.9(61.26116$$

This value, in one *single present* payment, divided by the value of the life, is

$$12.3)61.26116(4.98$$

By the *lowest* rate of interest, and *greatest* chance of mortality, the value for age *forty-five* is to pay, in a present single payment, about *£* 61 5 3, or by annual payments *£*. 4 19 7 during life.

From a short account of the Society for Equitable Assurances, in *St. Nicholas Lane*, dated 1771, the annual payments for

* See First Table, in the Third Article of the Addenda.

P O S T S C R I P T.

males (c) of the age *forty-five* on the said conditions is £. 5 11 0, which is £. 0.57 more than has been just found to be the true value, and which, multiplied into the value of the life, is

$$12.3 \times .57 = 7.011$$

which is £. 7.011 more than should be in a *single present* payment; and that is £. 11.44 *per cent.* for

$$61.261 : 7.011 :: 100 : 11.44$$

It is apprehended, that the *low* rate of interest, and the table of mortality, might afford some supply for expences, &c. but if not sufficient, surely 11.44 *per cent.* is a too extravagant contribution towards them; and most certainly, it is every person's duty, at the same time he is careful not to defraud others, to take particular care that he is not imposed upon himself.

Last year, the author attended a friend to the office in *St. Nicholas Lane*, and took that opportunity to ask, how much *per cent.* was required in *present* money to assure the life of a person in his 46th year of age, and was told 70*l.*—As this was but verbal, the author, desirous of better testimony for the present purpose, wrote lately to their actuary, and received the following answer.

"The premium in one payment, for assuring 200*l.* to be paid by this office at the death of a person now aged *forty-five*, is

£. 138 15 0

" Entrance ——— 10 0

" Policy ——— 7 6

£. 139 12 6

The half of this total for 100*l.* is ——— £. 69 16 3

Or more, if the entrance and policy are the same for 100*l.* as for 200*l.* This is £. 13.958 *per cent.* more than the most favorable calculation gives; and the author thinks it his duty to apprise the public of so great a difference.

(c) Females of the same age were to pay £. 5 18 10; but Dr. Price says, page 15, *Note, of his Supplement*, "That he was DESIRED to inform the public, that no such table"—as that for females—"will be admitted into the future editions of that account."



